

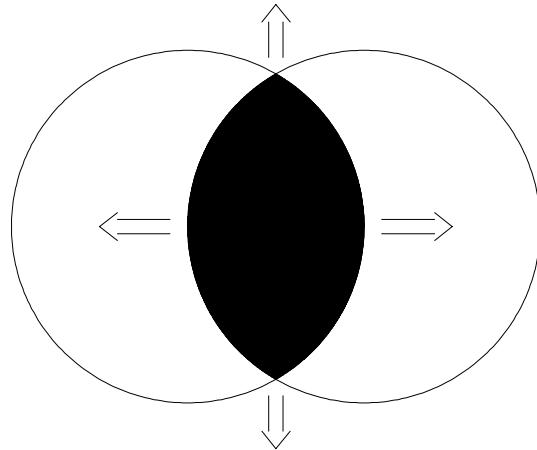
**Topics in Heavy Ion Collisions**  
McGill University  
Montréal, June 25–28, 2003

**ANISOTROPY OF FLOW AND  
HYDRODYNAMICS  
—WHAT HAVE WE LEARNED**

Pasi Huovinen  
University of Minnesota

# Anisotropic particle distribution

Non-central collision:



Anisotropy in configuration space



Anisotropy in particle distributions  
i.e. in momentum space

Characterized by Fourier coefficients:

$$E \frac{dN}{d^3 p} = \frac{dN}{2\pi p_t dp_t dy} \times [1 + 2v_1(p_t) \cos(\phi) + 2v_2(p_t) \cos(2\phi) + \dots]$$

or as averaged over  $p_t$ :

$$\frac{dN}{dy d\phi} = \frac{dN}{2\pi dy} \times [1 + 2v_1 \cos(\phi) + 2v_2 \cos(2\phi) + \dots]$$

# Hydrodynamical description

assumes

- local kinetic equilibrium
- local chemical equilibrium
- no dissipation

Basic quantity is the **energy-momentum tensor**

$$\begin{aligned} T^{\mu\nu} &= \int \frac{d^3\mathbf{p}}{(2\pi)^3 E} p^\mu p^\nu f(x, \mathbf{p}) \\ &= (\epsilon + p) u^\mu u^\nu - p g^{\mu\nu} \end{aligned}$$

where the lower equation holds for **non-viscous** matter.

- relativistic **viscous hydro** is possible in principle but **very complicated** to implement (Muronga: nucl-th/0104064)

Space-time evolution is given by

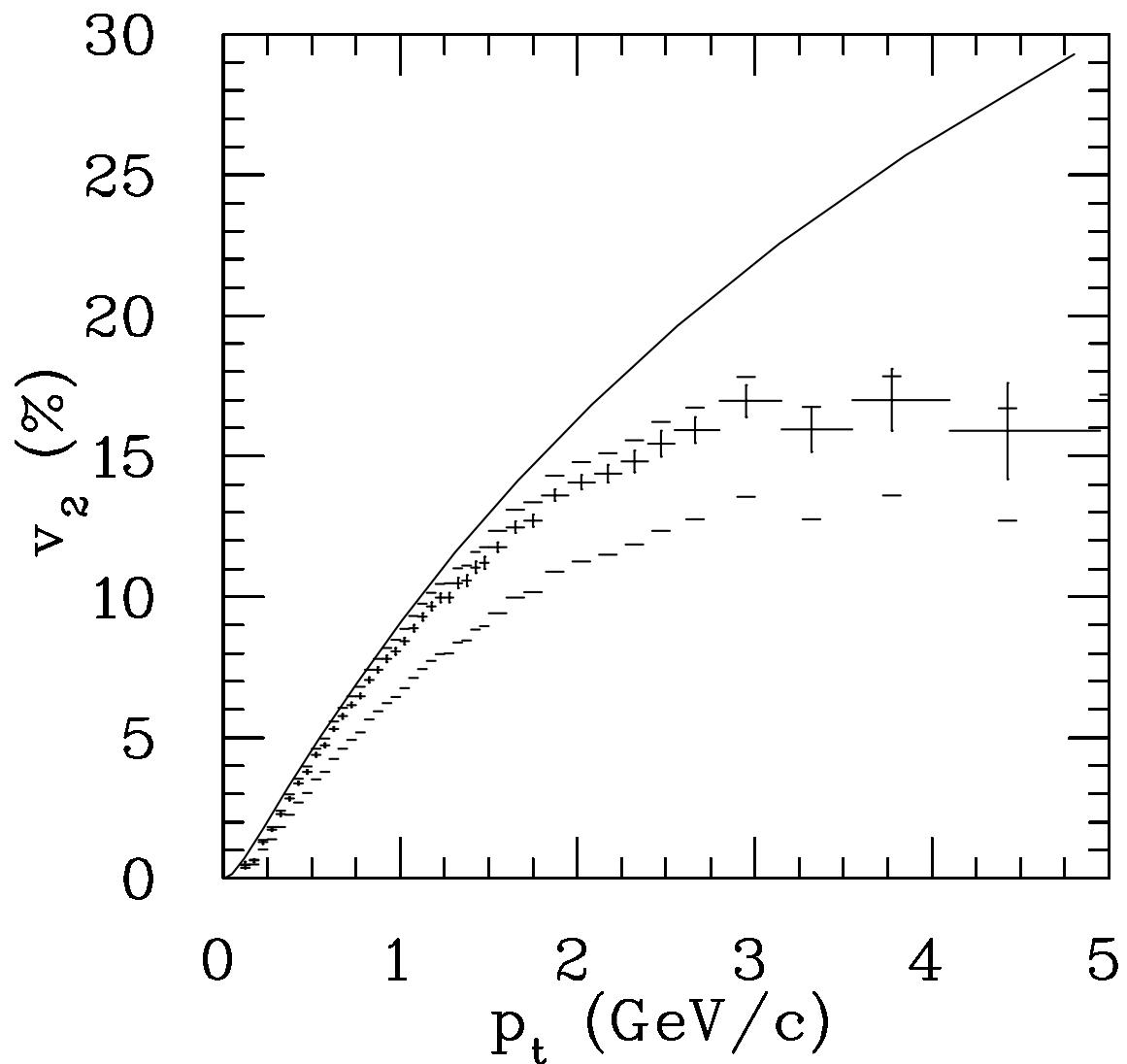
$$\partial_\mu T^{\mu\nu} = 0 \quad \text{and} \quad \partial_\mu j^\mu = 0$$

These equations express **local conservation** of 4-momentum and baryon number

By nature hydrodynamics describes bulk phenomena,  
i.e. low  $p_t$  particles, not high  $p_t$

$v_2(p_t)$ , minimum bias

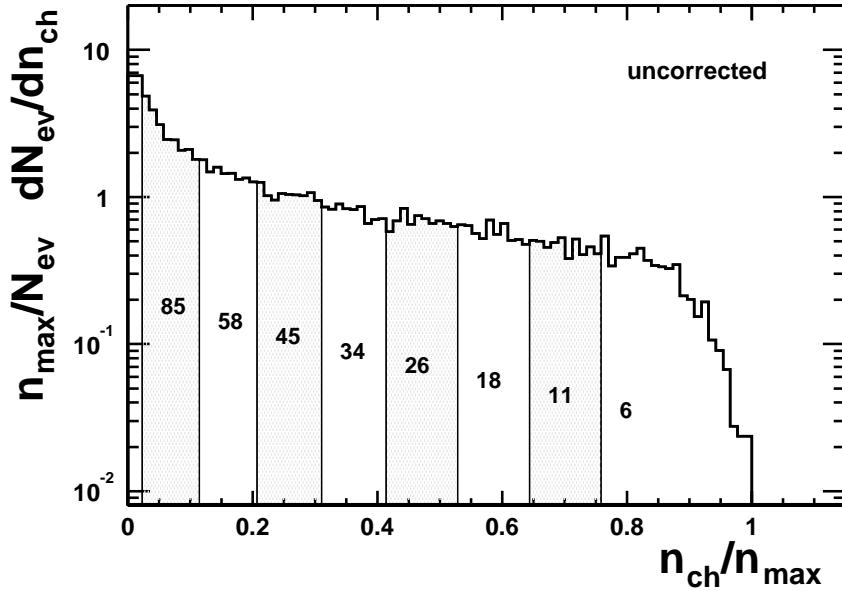
Negative hadrons



Data: STAR, Phys. Rev. Lett. **90**, 032301 (2003)

# Centrality selection

STAR centrality cuts:

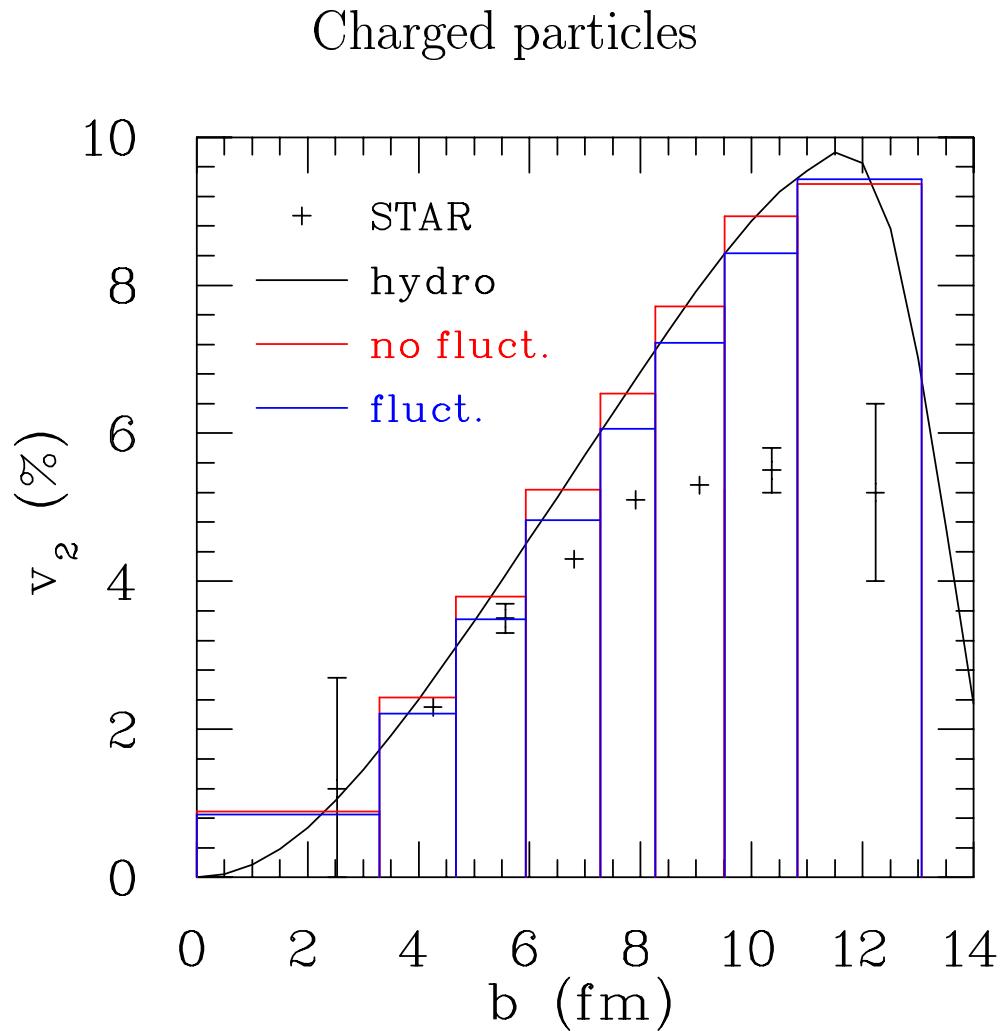


- Multiplicity at fixed  $b$  fluctuates  $\rightarrow$  contributions from wide range of  $b$  in each bin.
- In hydro no fluctuations.
- Assume thermal fluctuations:  $\langle \Delta N^2 \rangle = N$ :

$$\begin{aligned} \mathcal{N}_{\text{events}}(\mathcal{N}_{\text{ch}}, \lfloor \cdot \rfloor \lceil \mathcal{N}_{\text{ch}} \rceil \lfloor \cdot \rceil) \\ \propto \frac{d\sigma}{db} \frac{1}{\sqrt{2\pi \langle N(b) \rangle}} \exp\left(-\frac{(\langle N(b) \rangle - N_{\text{ch}})^2}{2\langle N \rangle}\right) dN_{\text{ch}} db \end{aligned}$$

- Cross section either from black disc ( $R_{\max} = 14.7$  fm) or optical Glauber (difference negligible)

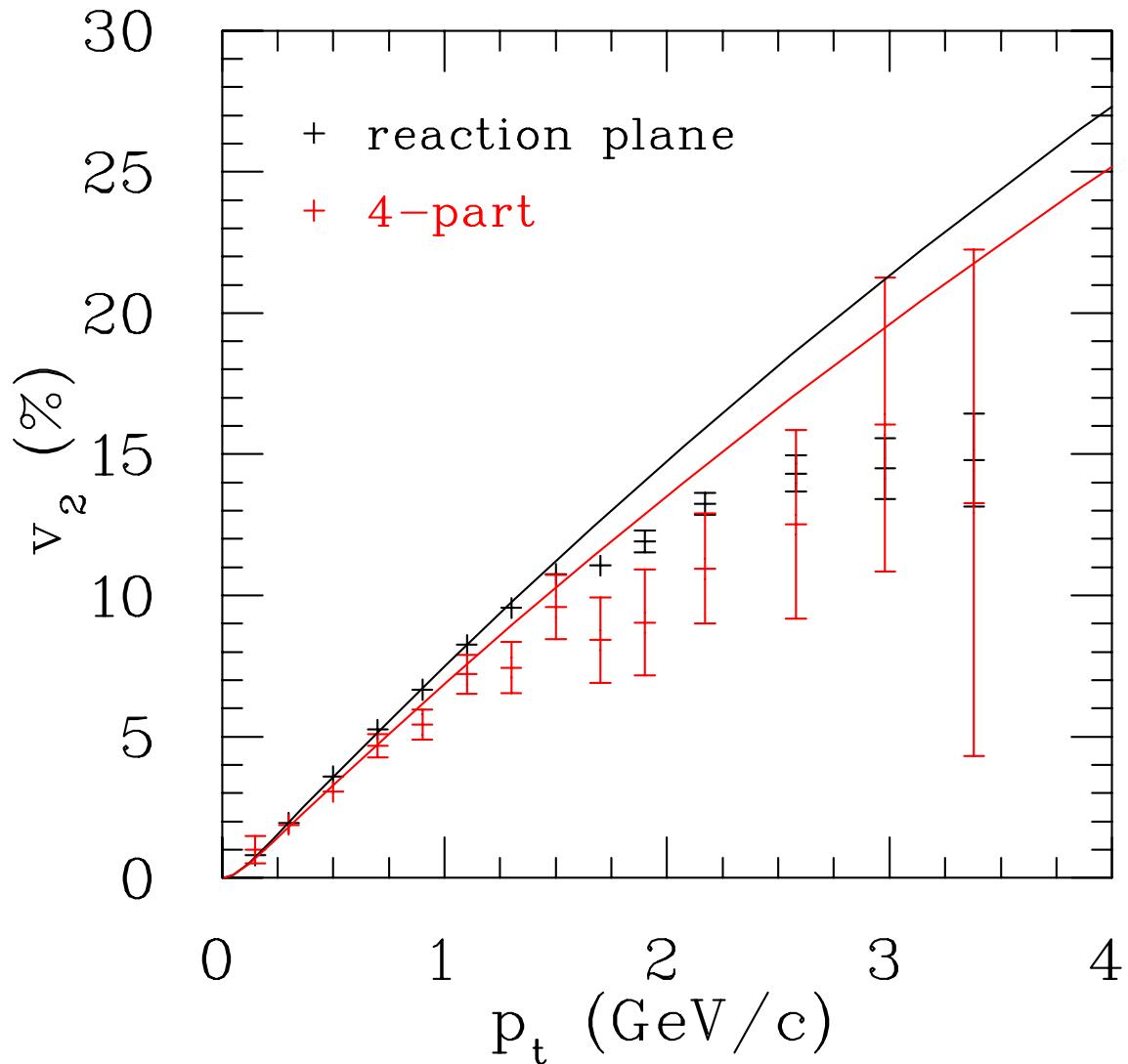
# Elliptic anisotropy $v_2$ as function of centrality



Data: STAR, Phys. Rev. C **66**, 034904 (2002)

## Charged hadrons, $v_2(p_t)$

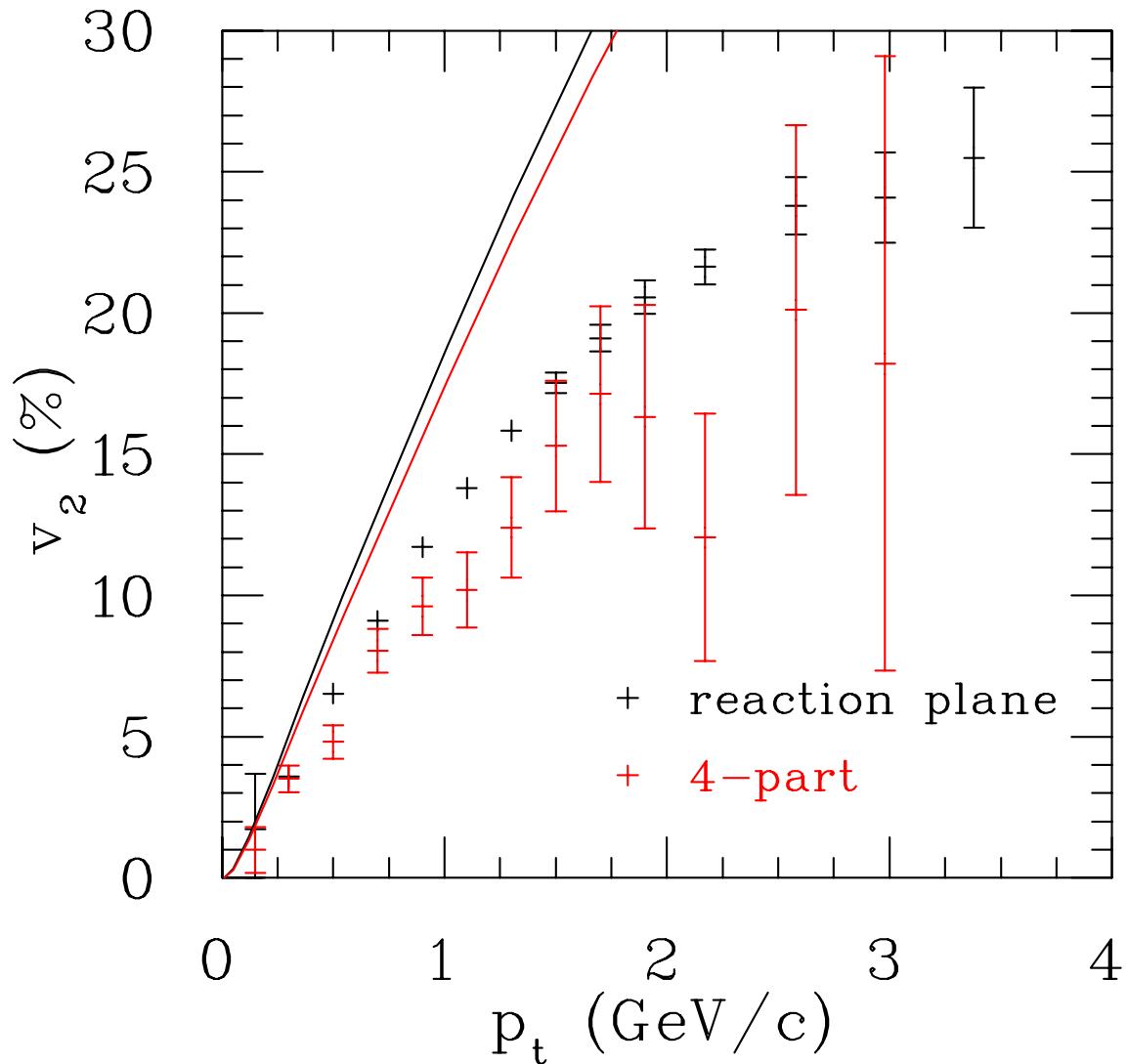
10%–16% centrality,  $4.7 < b < 5.9$  fm



Data: STAR, Phys. Rev. C **66**, 034904 (2002)

## Charged hadrons, $v_2(p_t)$

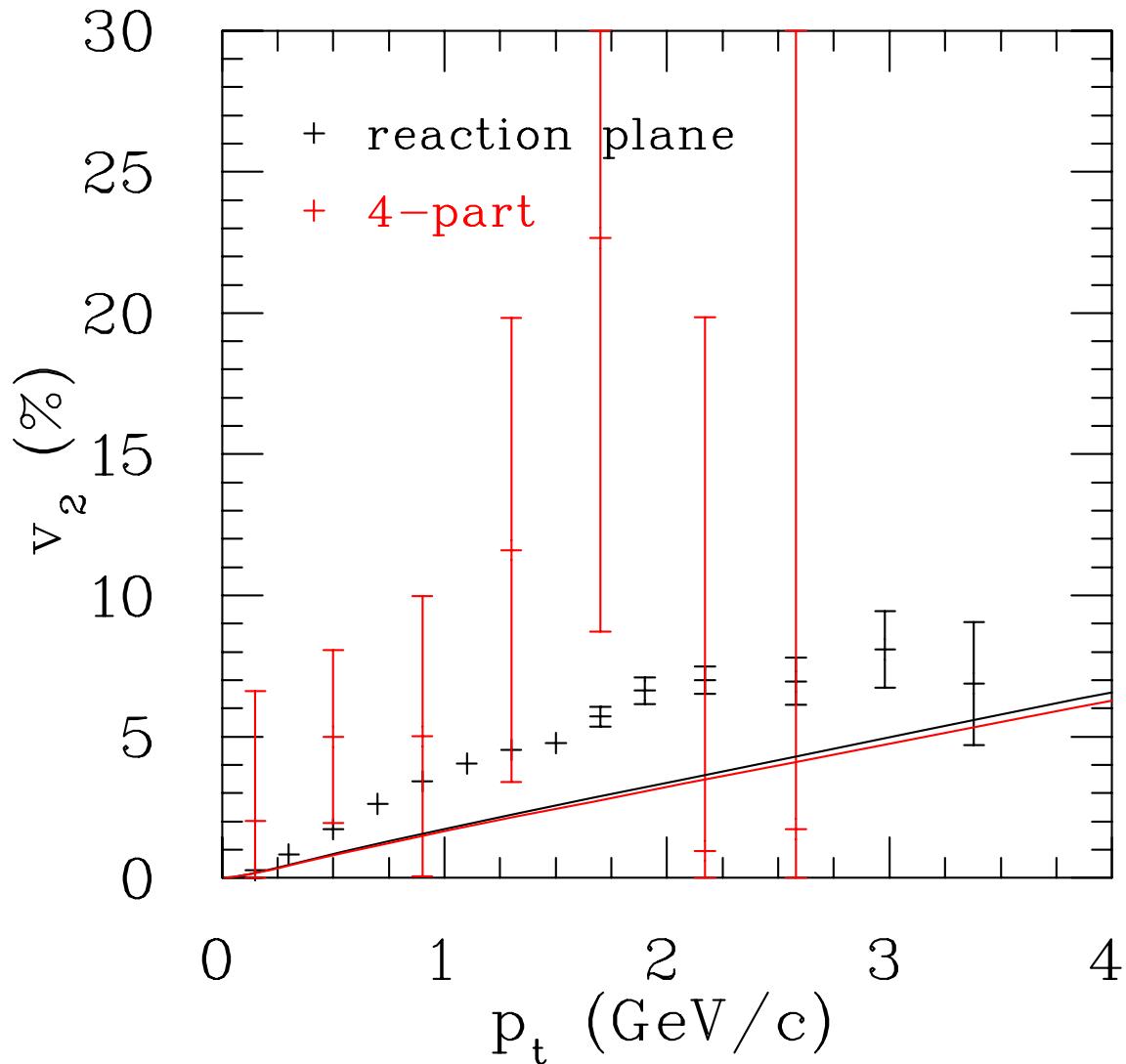
41%–53% centrality,  $9.5 < b < 10.8$  fm



Data: STAR, Phys. Rev. C **66**, 034904 (2002)

## Charged hadrons, $v_2(p_t)$

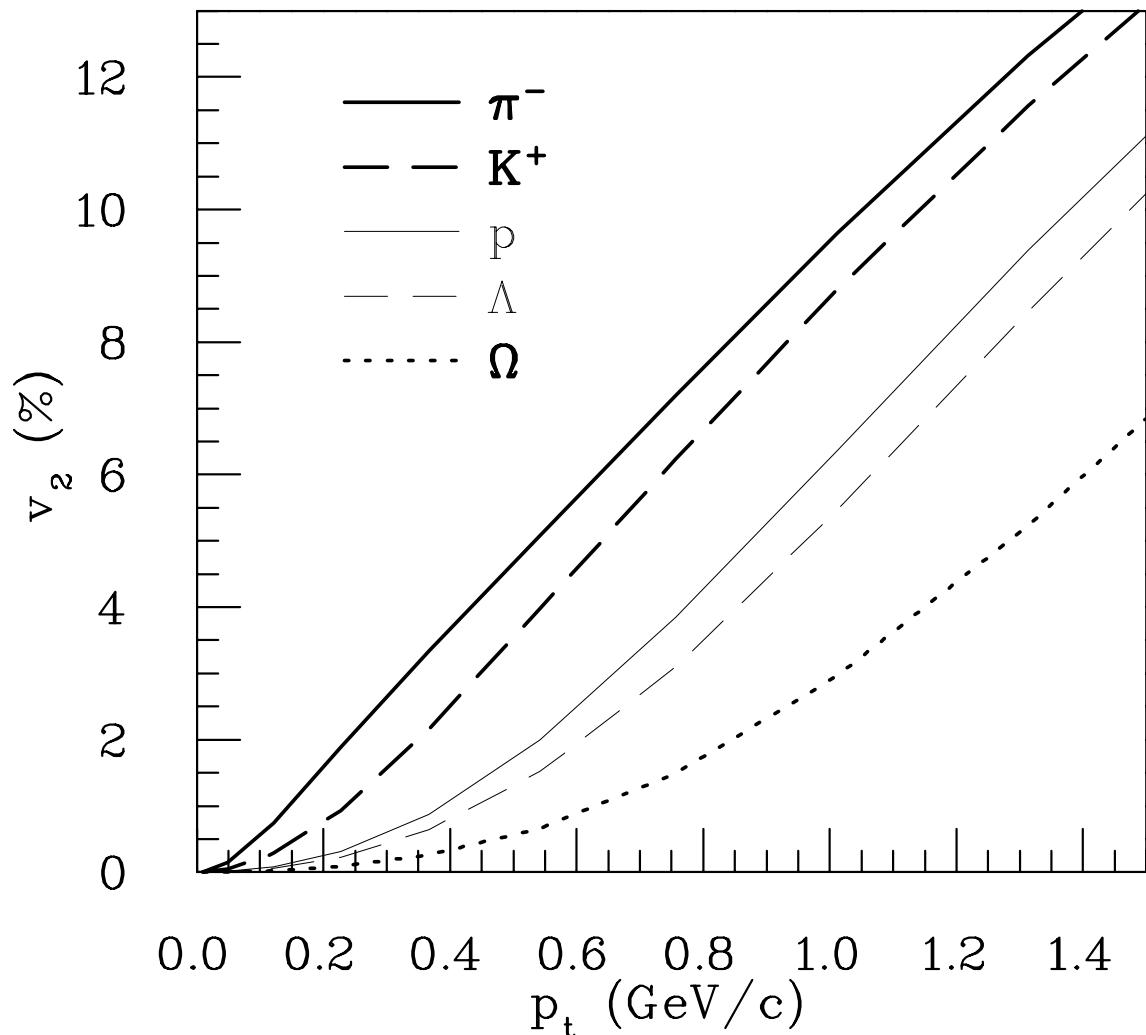
0%–5% centrality,  $0 < b < 3.3$  fm



Data: STAR, Phys. Rev. C **66**, 034904 (2002)

## Identified particle, $v_2(p_t)$

minimum bias

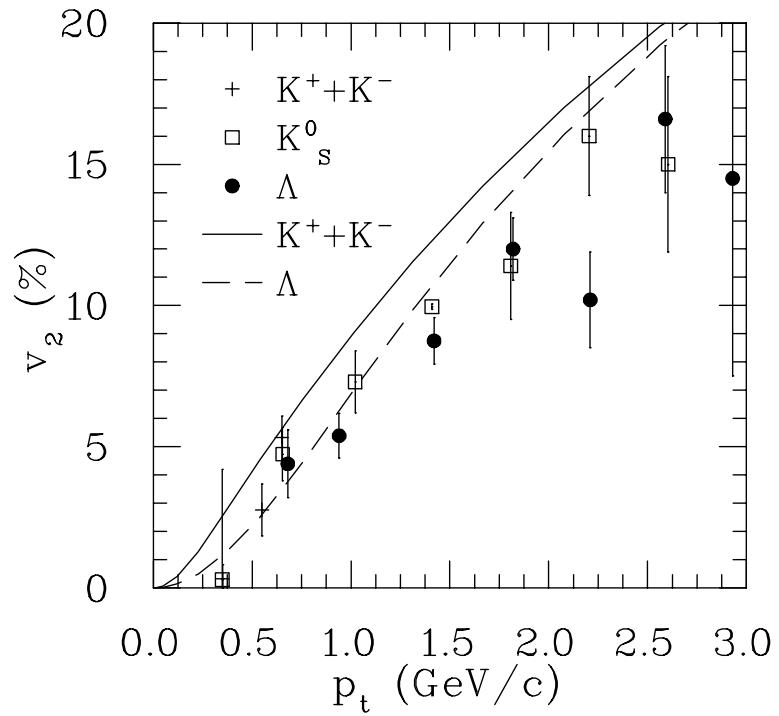


- Hydrodynamical prediction at low  $p_T$ :

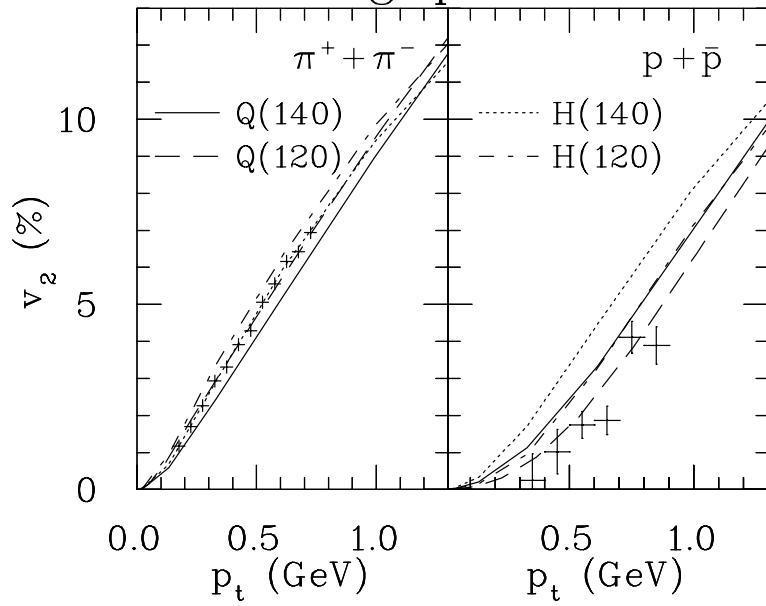
$$m_1 < m_2 \rightarrow v_2(p_T, m_1) > v_2(p_T, m_2)$$

## Identified particle, $v_2(p_t)$

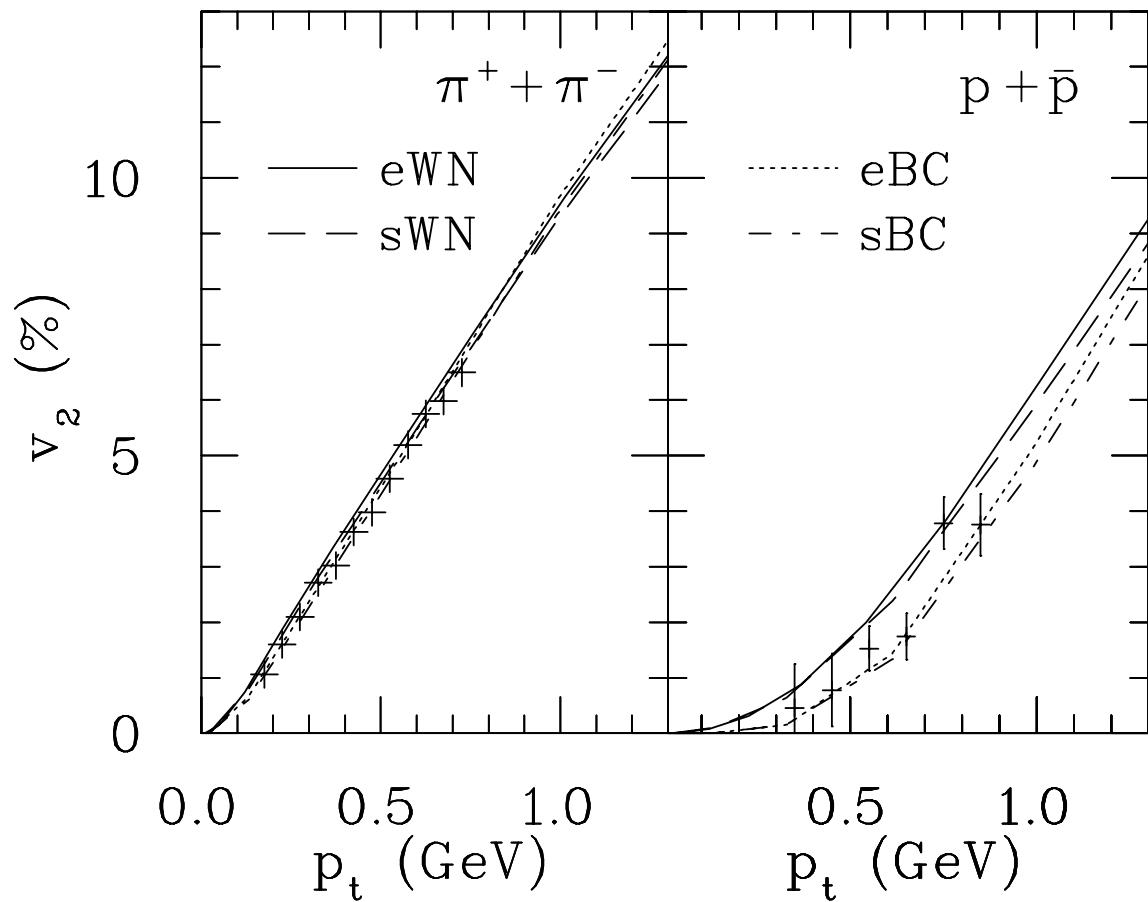
Strange particles:



Non-strange particles:



## Effect of initial shape on identified particle $v_2$

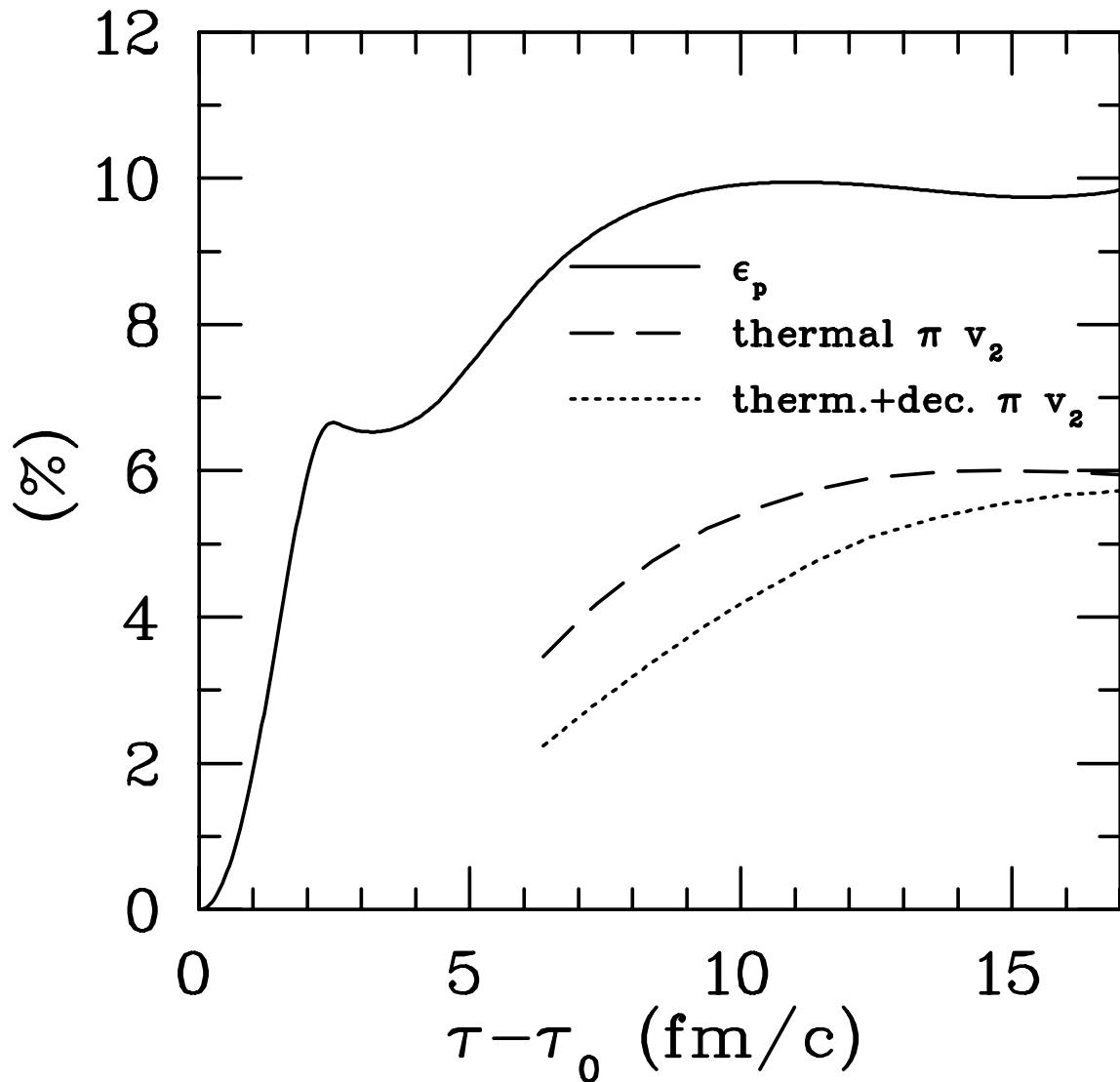


- Additional constraints from  $p_T$  spectra.
- Careful comparison not done yet.

## When is $v_2$ built up?

- Sorge (PRL78,2309) and Kolb et al. (PRC62,054909): “Elliptic flow is a sign of early pressure”
- Momentum anisotropy

$$\varepsilon_p = \frac{T^{xx} - T^{yy}}{T^{xx} + T^{yy}}$$



## Summary

- What works:
  - $v_2(p_T)$  of charged particles
  - Mass dependence of  $v_2(p_T)$  of identified particles
    - Collision system behaves as thermal system at  $b < 6$  fm.
- What needs to be done:
  - Short  $\tau_0 < 1$  fm difficult to prove
  - Constraints for EoS still elusive
  - And HBT is also a puzzle...