Correlations and Fluctuations in Heavy Ion Collisions

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Why fluctuations?

- Sometimes physics is in the width.
- Thermodynamically interesting (heat capacity, ...).
- Bulk property : $p_T < 2\,\mathrm{GeV}$

Aren't Correlation functions better?

- Correlation function contains all relevant info. However, to interpret changes in the correlation functions, one needs to think about what fluctuations are doing.

Interesting fluctuations

- 'Charge' fluctuation
  - Electric charge (Fractional charges?)
  - Baryon number (Fractional baryon number?)
  - Strangeness (Gluon fragmentation?)
  - Heavy quark number (Initial wave function?)
- Mean p_T/m_T fluctuation (Temperature? Heat capacity?)
- Multiplicity fluctuations (KNO? Thermal?)
- Energy fluctuation (Heat capacity?)

Fundamental observable

- Probability to have a set of particles $\{\alpha_1, \alpha_2, \cdots, \alpha_N\}$ with momenta $\{p_{\alpha_1}, p_{\alpha_2}, \cdots, p_{\alpha_N}\}$:

$$P(\{\alpha_1, p_{\alpha_1}\}, \{\alpha_2, p_{\alpha_2}\}, \cdots, \{\alpha_N, p_{\alpha_N}\})$$

- Experimentally,

$$P(\Omega_N)\Delta\Omega_N = \frac{\sum \text{Events with } \Omega_N \pm \Delta\Omega_N/2}{\text{Number of all events } (= \mathcal{N})}$$

α_i : Particle id, p_i : Momentum

$\Omega_N = (\{\alpha_1, p_{\alpha_1}\}, \{\alpha_2, p_{\alpha_2}\}, \cdots, \{\alpha_N, p_{\alpha_N}\})$

Correlations

- Relevant to fluctuations: Single particle distributions and 2-particle correlation functions.

- Single particle distribution functions :
  $\rho_\alpha(p)dp$ = Average number of α within dp around p.

$$\int_{\Delta\eta} dp\, \rho_\alpha(p) = \langle N_\alpha \rangle_{\Delta\eta}$$

- 2-particle correlation functions :
  $\rho_{\alpha\beta}(p_1, p_2)\, dp_1 dp_2$ = Average number of $\alpha\beta$ pairs within $dp_1 dp_2$ around p_1, p_2

$$\int_{\Delta\eta} dp_1 dp_2\, \rho_{\alpha\beta}(p_1, p_2) = \langle N_\alpha N_\beta \rangle_{\Delta\eta} - \delta_{\alpha\beta} \langle N_\alpha \rangle_{\Delta\eta}$$

A toy model – "ρ" gas

- $M_\pm$ independently emitted $\pm$ particles "$\rho^\pm$" $\Longrightarrow g_\pm(p_\pm)$
- M_0 neutral clusters "ρ^0"
  $\Longrightarrow f_0(p_+, p_-), \quad g_0(p) = \int dq f_0(p, q)$
  - ∗ Single particle distributions

$$\rho_\pm(p) = \langle M+ \rangle \; g_\pm(p) + \langle M_0 \rangle \, g_0(p)$$

– Two particle correlation functions

$$
\begin{aligned}
C_{++}(p_1,p_2) &\equiv \rho_{++}(p_1,p_2) - \rho_+(p_1)\rho_+(p_2) \\
&= \sum_{a=+,0}\sum_{b=+,0} \langle \delta M_a \delta M_b \rangle\, g_a(p_1) g_b(p_2) \\
&\quad - \langle M_+ \rangle\, g_+(p_1) g_+(p_2) - \langle M_0 \rangle\, g_0(p_1) g_0(p_2)
\end{aligned}
$$

$$
\begin{aligned}
C_{+-}(p_1,p_2) &= \sum_{a=+,0}\sum_{b=-,0} \langle \delta M_a \delta M_b \rangle\, g_a(p_1) g_b(p_2) \\
&\quad + \langle M_0 \rangle \left[f_0(p_1,p_2) - g_0(p_1) g_0(p_2) \right]
\end{aligned}
$$

If Poisson-like, all terms in $C_{\alpha\beta}$ are $O(M)$.
In $\rho_{\alpha\beta}$, the leading term is $O(M^2) \Longrightarrow f_0$ is hidden.

Q fluctuations (also applies to $(s \pm \bar{s})$)

- In full momentum space, fluctuation of multiplicity is a function only of the heights of the corr.ftns.
- Finite acceptance makes local features in momentum space to show up such as the widths.

(Some) Issues

* QGP? $\langle \delta Q^2 \rangle \,/\, \langle N_{\rm ch} \rangle \approx 1/4$?
* Charge conservation corrections?

Charge fluctuation

- What distinguishes a QGP from a hadron gas?
  - Color fluctuation: Hadrons are all color neutral
    $\Longrightarrow$Difficult to observe color fluctuation
  - Charge fluctuation: Quarks have fractional charges
    $\Longrightarrow$Less charge fluctuation per charged degree of freedom
  - There are gluons: Gluons contribute to the entropy but not to the charge fluctuation $\Longrightarrow$Less charge fluctuation per charged degree of freedom

Why charge conservation correction?

- Thermal: $\langle \delta M_\alpha \delta M_\beta \rangle = \langle M_\alpha \rangle \delta_{\alpha\beta}$

$$\left\langle \delta Q^2 \right\rangle_{\Delta\eta}^{\text{therm.}} = \langle N_{\text{ch}} \rangle_{\Delta\eta} - 2 \langle M_0 \rangle \int_{\Delta\eta} dp_1 dp_2 \, f_0(p_1, p_2)$$

('Grand canonical') $\leftarrow 1/4$ results from here

- Charge conservation: $Q_M = M_+ - M_- =$ Constant

$$\left\langle \delta Q^2 \right\rangle_{\Delta\eta}^{\text{cons.}} = \langle N_{\text{ch}} \rangle_{\Delta\eta} - 2 \langle M_0 \rangle \int_{\Delta\eta} dp_1 dp_2 \, f_0(p_1, p_2)$$

$$\boxed{- \left[\langle M_+ \rangle + \langle M_- \rangle \right] \left(\int_{\Delta\eta} dp \, g_\pm(p) \right)^2}$$

('Canonical')

How to correct for charge conservation – I

- Q: When do the two expressions $\langle \delta Q^2 \rangle_{\Delta\eta}^{\text{therm.}}$ and $\langle \delta Q^2 \rangle_{\Delta\eta}^{\text{cons.}}$ become (approximately) equal?

- A-1: When $\sigma_{\text{rel}} \ll \Delta\eta \ll y_{\text{max}}$. The full system is big enough so that $p = \int_{\Delta\eta} g(y) \ll 1$, but the sub-system is large enough to cover many correlation lengths.

  (Canonical $\Longrightarrow$Grand Canonical)

- A-2: When $M_+ = M_- = 0$. That is, all neutral clusters.

How to correct for charge conservation – II

- Trouble - 1: $\Delta\eta$ is smaller than $\sigma_{\rm rel}$
- Answer - 1: Can't do much here.
- Trouble - 2: $\Delta\eta$ is big enough compared to $\sigma_{\rm rel}$ but not small enough compared to $y_{\rm max}$.
- Trouble - 2': Furthermore, we don't know the relative strengths of $M_\pm$ and M_0.

How to correct for charge conservation – III

- Idea I (J.S., V.Koch) :

$$\langle \delta Q^2 \rangle_{\Delta\eta}^{\mathrm{corr.I}} = \langle \delta Q^2 \rangle_{\Delta\eta}^{\mathrm{cons.}} / (1-p)$$

where $p = \langle N_{\mathrm{ch}} \rangle_{\Delta\eta} / \langle N_{\mathrm{ch}} \rangle_{\mathrm{full}} \approx \int_{\Delta\eta} g_{\pm} \approx \int_{\Delta\eta} g_0$

- If $f_0(y_1, y_2) \approx \delta(y_{\mathrm{rel}}) g_{\pm}(y_{\mathrm{cm}})$

$$\frac{\langle \delta Q^2 \rangle_{\Delta\eta}^{\mathrm{corr.I}}}{\langle N_{\mathrm{ch}} \rangle_{\Delta\eta}} \approx \frac{\langle \delta Q^2 \rangle_{\Delta\eta}^{\mathrm{therm.}}}{\langle N_{\mathrm{ch}} \rangle_{\Delta\eta}} = \frac{\langle M_+ + M_- \rangle}{\langle M_+ + M_- + 2M_0 \rangle}$$

for any $\Delta\eta < y_{\mathrm{max}}$. Same result as resonance gas.

- Doesn't work if $M_+ = M_- = 0$.

- Idea II (S.Gavin, C.Pruneau, S.Voloshin)

$$\left\langle \delta Q^2 \right\rangle_{\Delta\eta}^{\mathrm{corr.II}} = \left\langle \delta Q^2 \right\rangle_{\Delta\eta}^{\mathrm{cons.}} + p \left\langle N_{\mathrm{ch}} \right\rangle_{\Delta\eta}$$

where $p = \langle N_{\mathrm{ch}} \rangle_{\Delta\eta} / \langle N_{\mathrm{ch}} \rangle_{\mathrm{full}} \approx \int_{\Delta\eta} g_\pm \approx \int_{\Delta\eta} g_0$

- If $f_0(y_1, y_2) \approx \delta(y_{\mathrm{rel}}) g_\pm(y_{\mathrm{cm}})$

$$\frac{\langle \delta Q^2 \rangle_{\Delta\eta}^{\mathrm{corr.II}}}{\langle N_{\mathrm{ch}} \rangle_{\Delta\eta}} \approx \frac{\langle \delta Q^2 \rangle_{\Delta\eta}^{\mathrm{therm.}}}{\langle N_{\mathrm{ch}} \rangle_{\Delta\eta}} + 2p \frac{\langle M_0 \rangle}{\langle M_+ + M_- + 2M_0 \rangle}$$

Grows as p grows. In the $p \to 1$ limit, $\dfrac{\left\langle \delta Q^2 \right\rangle_{\Delta\eta}^{\mathrm{corr.II}}}{\langle N_{\mathrm{ch}} \rangle_{\Delta\eta}} \to 1$, same as uncorrelated pion gas.

- Doesn't work if $M_+ = M_- = 0$.

Charge Transfer Fluctuation

- PP charge transfer fluctuation (70's Fermi Lab)

  Define charge transfer at η (forward minus backward):

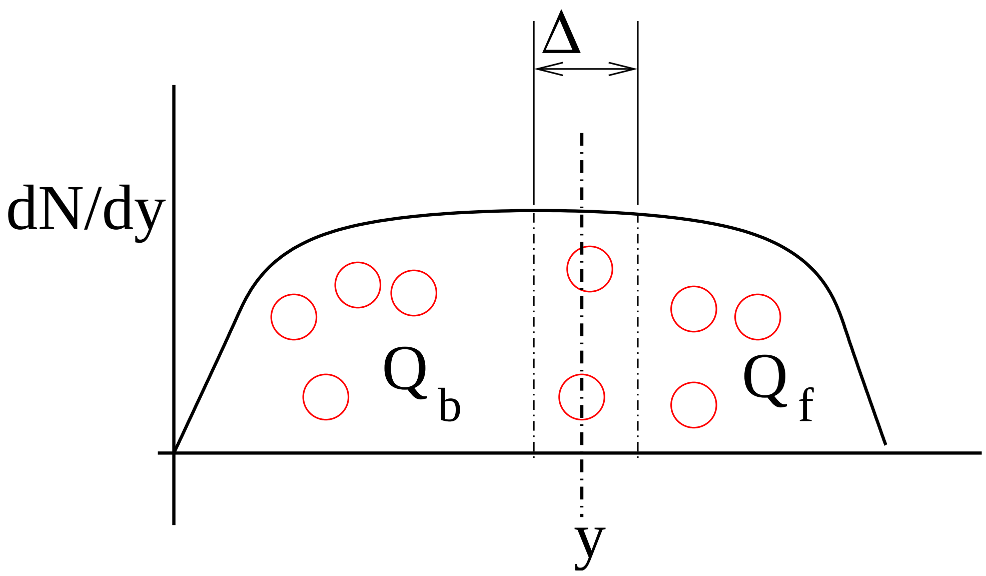
  $$u(\eta) = [Q(y > \eta) - Q(y < \eta)]/2$$

  * No need for charge conservation corrections
  * Experimentally, at $\sqrt{s} = 20\,\text{GeV}$,

    $$\left\langle (\delta u(y))^2 \right\rangle \approx C \frac{dN_{\rm ch}}{dy} \quad \text{with} \quad C = 0.6 - 0.7$$

    for all y [Quigg-Thomas, Kafka et.al.].
  * Non-trivial relationship between the single and 2–particle functions.

Simple picture



* If all hadrons come from neutral clusters, fluctuation of $u = (Q_f - Q_b)/2$ can only come from within the correlation length Δ around y.
* $\left\langle \delta u^2 \right\rangle \propto$ Number of clusters within $\Delta \propto \Delta dN/dy$

- In our toy model:
  - $*$ $g_{\pm}(y)$ can be completely determined, but inconsistent with experimental $dN/dy \Longrightarrow \boxed{\langle M_{\pm}\rangle \ll \langle M_0\rangle}$
  - $*$ $\boxed{f_0(y_1, y_2) \propto \exp(-y_{\rm rel}^2/2\sigma_{\rm rel}^2)\, dN_{\rm ch}/dy|_{y=y_{\rm cm}}}$

    yields

    $$\boxed{\left\langle (\delta u(y))^2 \right\rangle / (dN_{\rm ch}/dy) \approx 0.4\sigma_{\rm rel}}$$

Interesting thought...

- PP experiments actually claimed that $\langle M_0 \rangle \gg \langle M_\pm \rangle$ is most consistent with their results.

- Charge transfer fluctuation is sensitive to correlation length in $y_{\rm rel}$

- QGP makes $\sigma_{\rm rel}$ smaller by a factor of up to 2 if a QGP forms [Bass,Danielewitz,Pratt].

- HIJING gives $\left\langle \delta u^2 \right\rangle / \langle N_{\rm ch} \rangle \approx 0.2 - 0.25$ within $-1 < y < 1$.

- Can we do this at RHIC?

Summary

- Correlation function is the best way to go. A host of dynamical information coded in the heights, width, etc.
- Fluctuation studies (theoretical and experimental) hold the key to the encoded messages.
- Corrections such as the charge conservation effect can be carried out by considering simple models of correlation functions (consulting reality, of course).
- Worth measuring charge (strangeness) transfer fluctuation.
- Many more to come – Energy, Transverse momentum, ... (have to overcome b fluctuations, though)