# Landau Ginzburg Theory of Nuclear Matter at Finite Temperature

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# Introduction

Nuclear Matter at low temperatures - below 3-4 MeV

- Nuclear Structure - pairing phase transition below 1 Mev
- Nuclear Astrophysics - may be a phase transition
  - not very important
  - Fermi gas models or TDHF are adequate

# Finite Temperature Extension

$$E_b(A,Z,T) = \alpha(T)A + \beta(T)A^{\frac{2}{3}} + (\gamma(T) - \frac{\eta(T)}{A^{\frac{1}{3}}})(\frac{4t_\zeta^2 + 4|t_\zeta|}{A}$$

$$+0.8076\frac{Z^2}{A^{\frac{1}{3}}}(1 - \frac{0.7636}{Z^{\frac{2}{3}}} - \frac{2.29}{A^{\frac{2}{3}}}) + \delta(T)f(A,Z)A^{-\frac{3}{4}}$$

where $A = N + Z$, - $t_\zeta = \frac{1}{2}(Z - N)$ ,
$f(A,Z) = (-1, 0, +1)$(even-even,even-odd,odd-odd)
and $\alpha(0) = -16.11$ MeV, $\beta(0) = 20.21$ MeV,
$\gamma(0) = 20.65$MeV, $\eta(0) = 48.00$ MeV, $\delta(0) = 33.0$ MeV

In the Canonical ensemble for a nucleus

$$\mathsf{Z}(A,Z,T) = \sum_{i}^{n} g_i \, exp(-\beta E_i)$$

$$+ \int_{E_n}^{E_m{}^{ax}} d_E \; g_{A,Z}(E) \; exp(-\beta E)$$

$g_i = 2j_i + 1$ - the spin degeneracy factor
$E_i$ - excitation energy of the $i$th state of the nucleus
$g_{A,Z}(E)$ - level density
- at high E - Fermi gas model
- at low E - empirical form (Gilbert and Cameron)

Coefficients in the mass formula determined by a least squares fit to

$$\frac{E_{ex}(A,Z,T)}{A} = \frac{E_b(A,Z,T) - E_b(A,Z,0)}{A}$$

where

$$E_{ex}(A,Z,T) = -\frac{\partial}{\partial\beta}\ln \mathsf{Z}(A,Z,T)$$

for 313 nuclei.

The specific heat is

$$C = \frac{\partial E}{\partial T} = T \frac{\partial S}{\partial T}$$

and the free energy is

$$F = E - TS$$

# Landau-Ginzburg Treatment

Assume a second order phase transition in nuclear matter with critical temperature $T_C$ and an order parameter $\eta$. Necessary to determine the free energy density $f(T)$ in both phases

- subscript 1 refers to the lower T (condensed) phase subscribt 2 refers to higher T (uncondensed or normal) phase

In the uncondensed phase the energy density is

$$W_2(T) = a_2 + k_2 T^2,$$

where $a_2$ and $k_2$ are constants.
From the specific heat can deduce the entropy density

$$s_2(T) = C_2 + 2k_2 T.$$

where $C_2$ is an unknown constant (which later cancels out) and

$$f_2(T) = a_2 - C_2 T - k_2 T^2.$$

In the condensed phase use Landau expansion for f in terms of an order parameter $\eta$ which goes to zero at the transition to the uncondensed phase

$$f_1(T, \eta) = f_2 + A\eta^2 + B\eta^4.$$

where $A$ and $B$ are functions of T
Since $A$ is of opposite sign in the condensed and uncondensed phases, and $B$ is strictly positive

$$A(T) = a(T - T_c)\, 2\sqrt{B(T_c)}.$$

Order parameter determined by requiring condensed phase to be stable below $T_c$ (i.e. $f_1$ should be minimized) which yields

$$W_1(T) = a_1 + k_1 T^2$$

Can estimate the jump in specific heat at the transition to a condensed phase, where there is pairing with an associated energy gap $\Delta$

$\Delta c_V \approx 1.43 c_V$ where $c_V$ is the specific heat per nucleon in the uncondensed phase.

Using $k_2 = 1/6.7$ MeV$^{-1}$ and assuming $T_c \sim .8$ MeV we find $\Delta c_V \sim .3$ MeV, which is consistent with the behaviour of the specifc heat.