A TRANSITION CENTRIC APPROACH TO LEVEL SCHEMES

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THE 8π

Radioactive nuclei produced by TRIUMF's ISAC facility are deposited on a tape located at the centre of the 8π, which includes 20 high purity germanium detectors as well as many auxiliary detectors.

These nuclei then β^+ or β^- decay to an excited state in the daughter nuclei.

The excited daughter nuclei emit γ rays as they decay to its ground state.

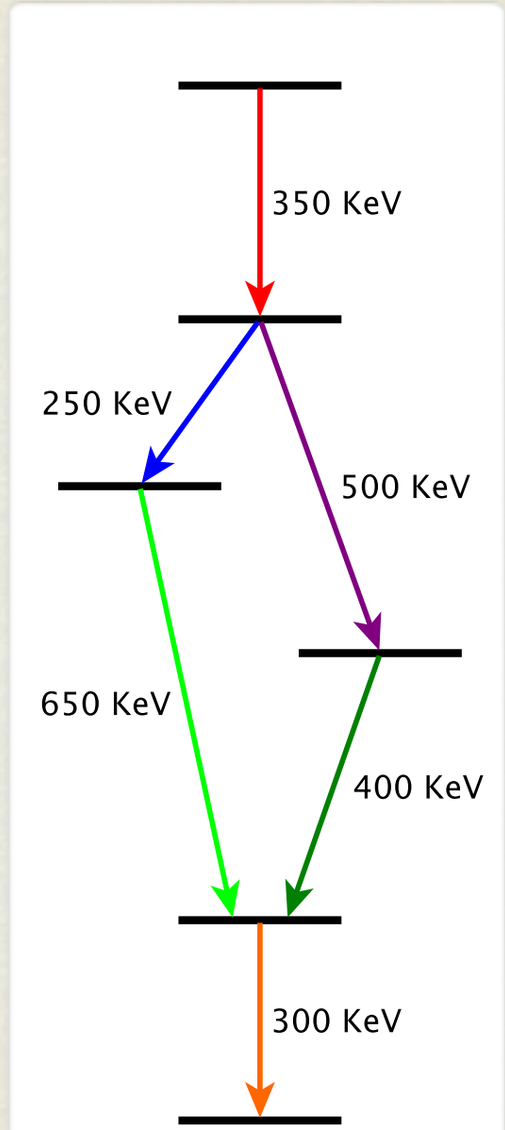
γ-RAY COINCIDENCES

Characteristic half-life of nuclear energy levels is shorter than our detector time resolution.

γ rays from the same decay path are detected in coincidence (at the same time)

We can reconstruct level schemes with hundreds of transitions in a year of analysis

All of our experimental data relates to transitions between energy levels.

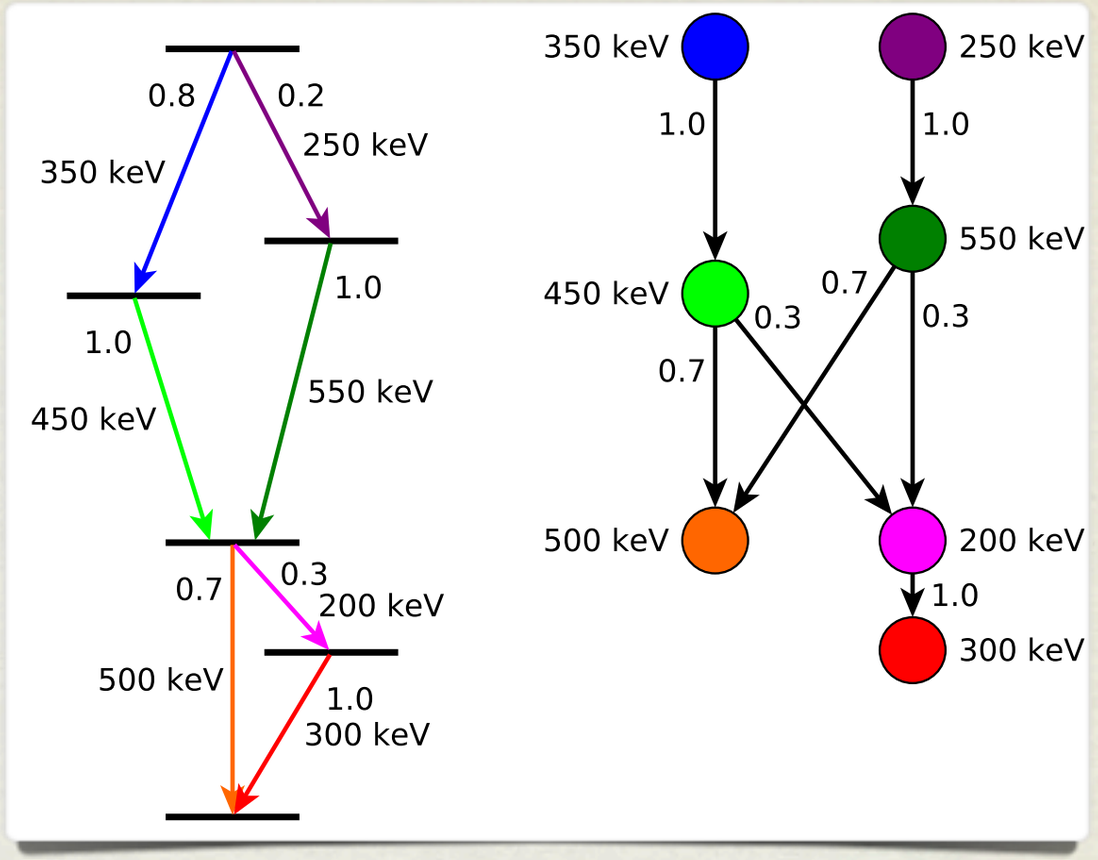


TRANSITION SPACE

We will use a representation that mirrors the form of our experimental data.

Nodes represent transitions. Edges represent abstract connections between transitions, (and can be related to energy levels)

Mathematically, transition space is the line graph of the level scheme.



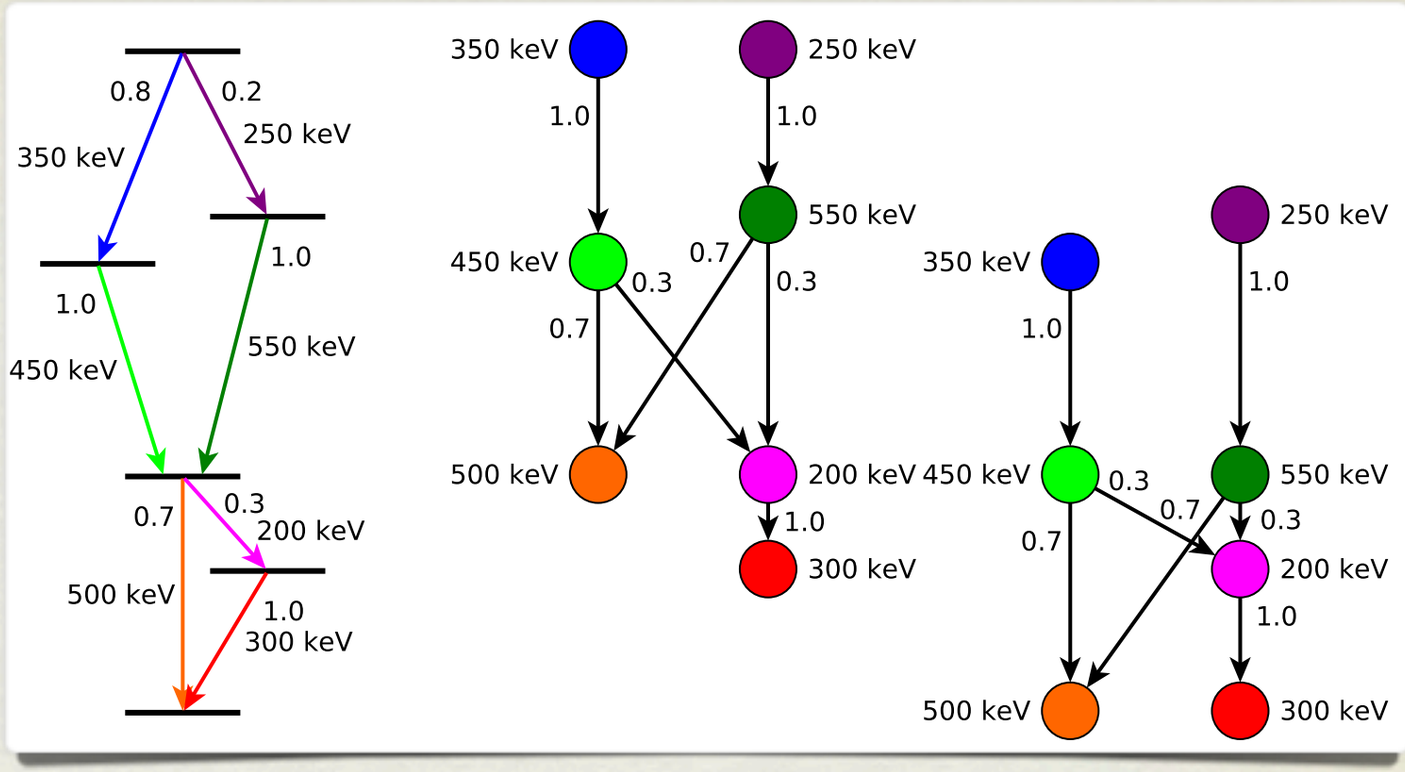
$$A = \left(\begin{array}{c|ccccccc} keV & 200 & 250 & 300 & 350 & 450 & 500 & 550 \\ \hline 200 & 0 & 0 & 1.0 & 0 & 0 & 0 & 0 \\ 250 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0 \\ 300 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 350 & 0 & 0 & 0 & 0 & 1.0 & 0 & 0 \\ 450 & 0.3 & 0 & 0 & 0 & 0 & 0.7 & 0 \\ 500 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 550 & 0.3 & 0 & 0 & 0 & 0 & 0.7 & 0 \end{array}\right)$$

TRANSITION SPACE

We can associate transitions with either their source or destination energy.

For this presentation, I will use the source energy representation.

In this representation, transitions with a shared source energy are located at the same height in the transition space level scheme.



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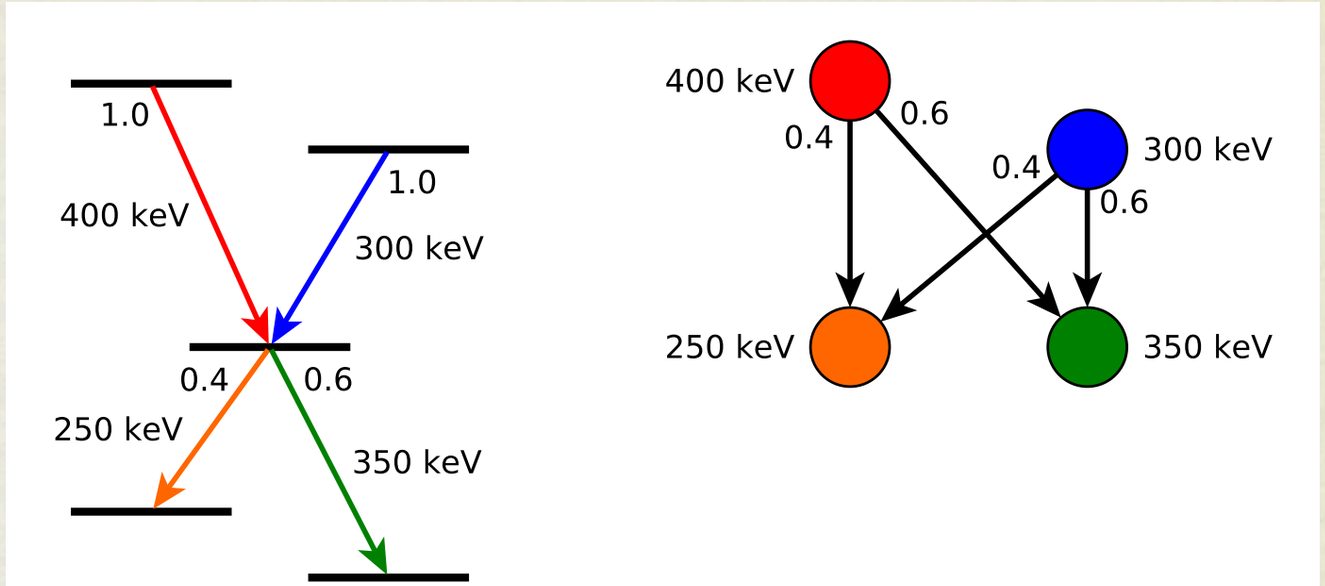
LEVEL IDENTIFICATION

Energy levels are not explicitly represented in transition space

Due to the Markovian nature of energy levels, all transitions feeding a level have the same probability of decaying via any of the transitions emanating from that level

This signature allows us to identify energy levels



$$A = \left(\begin{array}{c|cccc} keV & 250 & 300 & 350 & 400 \\ \hline 250 & 0 & 0 & 0 & 0 \\ 300 & 0.4 & 0 & 0.6 & 0 \\ 350 & 0 & 0 & 0 & 0 \\ 400 & 0.4 & 0 & 0.6 & 0 \end{array} \right)$$

DIRECTED COINCIDENCES

The number of events in element $D_{i,j}$ of a directed coincidence matrix is a product of:

N_o Number of Parent Nuclei

P_i Probability of Observing γ_i

$P_{i,j}$ Probability of Observing γ_j Given that γ_i was Observed

$$D_{i,j} = N_o P_i P_{i,j}$$

Note: P_i and $P_{i,j}$ include detector efficiencies and branching ratios

EXPERIMENTAL SIGNIFICANCE

The product of the number of parent nuclei and the probability of detecting γ_i is experimentally measured as the singles spectra, *S*.

$$S_i = N_o P_i$$

The probability of observing γ_j given that γ_i was observed is an efficiency weighted infinite sum of powers of the adjacency matrix

$$P_{i,j} = (A + A^2 + A^3 + \dots)_{i,j} E_j$$

which can be rewritten using the geometric series sum identity as:

$$P_{i,j} = ((I - A^{-1}) - I)_{i,j} E_j$$

ANALYTICAL FORMULA

Combining everything together and rewriting in matrix form

$$D = S((I - A)^{-1} - I)E$$

where: D is the directed coincidence matrix,

S is a diagonal singles matrix

E is a diagonal efficiency matrix

A is the transition space adjacency matrix

This equation directly relates experimentally relevant quantities to level scheme structure

UNDIRECTED COINCIDENCES

One challenge with this method is that the formula uses directed coincidences, whereas experimentally we must symmetrize our coincidence matrices, as our detectors cannot determine the time ordering of detecting γ_i and γ_j

$$C = D + D^T$$

However, the formula must be combined with an optimization method in order to determine nuclear level schemes.

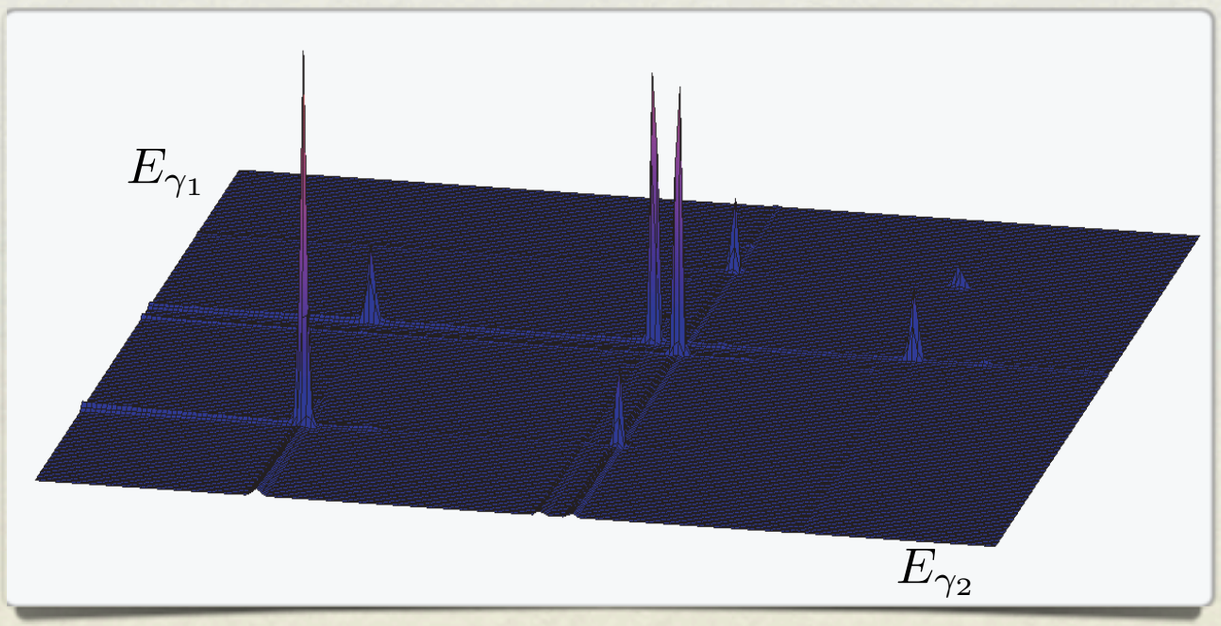
$$A = I - (S^{-1}D + I)^{-1}$$

TRANSITION BINNED EXPERIMENTAL DATA

In the preceding derivation we used $D_{i,j}$, S_i and E_j, where the subscripts referred to arbitrary transitions.

In essence, we assumed that our histograms were binned by transition instead of by energy range.

Histograms can be converted from energy range binning to transition binning through fitting.



OPTIMIZATION

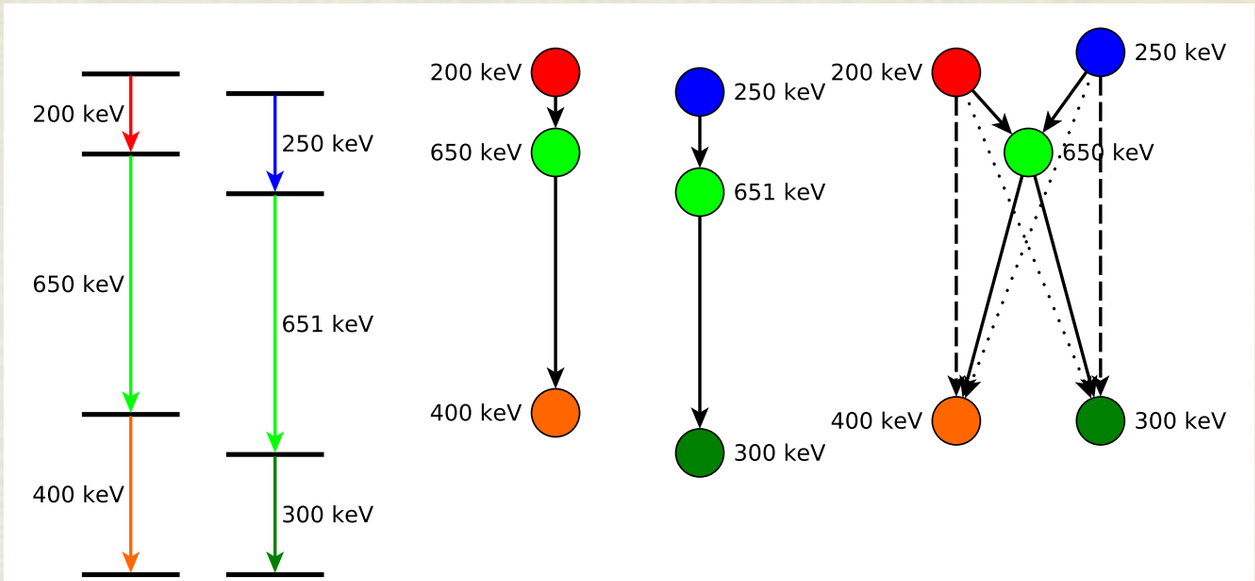
As a proof of concept, I have determined level schemes for simulated data with 150 distinguishable transitions and 60 energy levels.

I optimized the source level energy ordering of the transitions, as this combined with $C = D + D^T$ allows the calculation of D from C.

UNRESOLVED MULTIPLETS

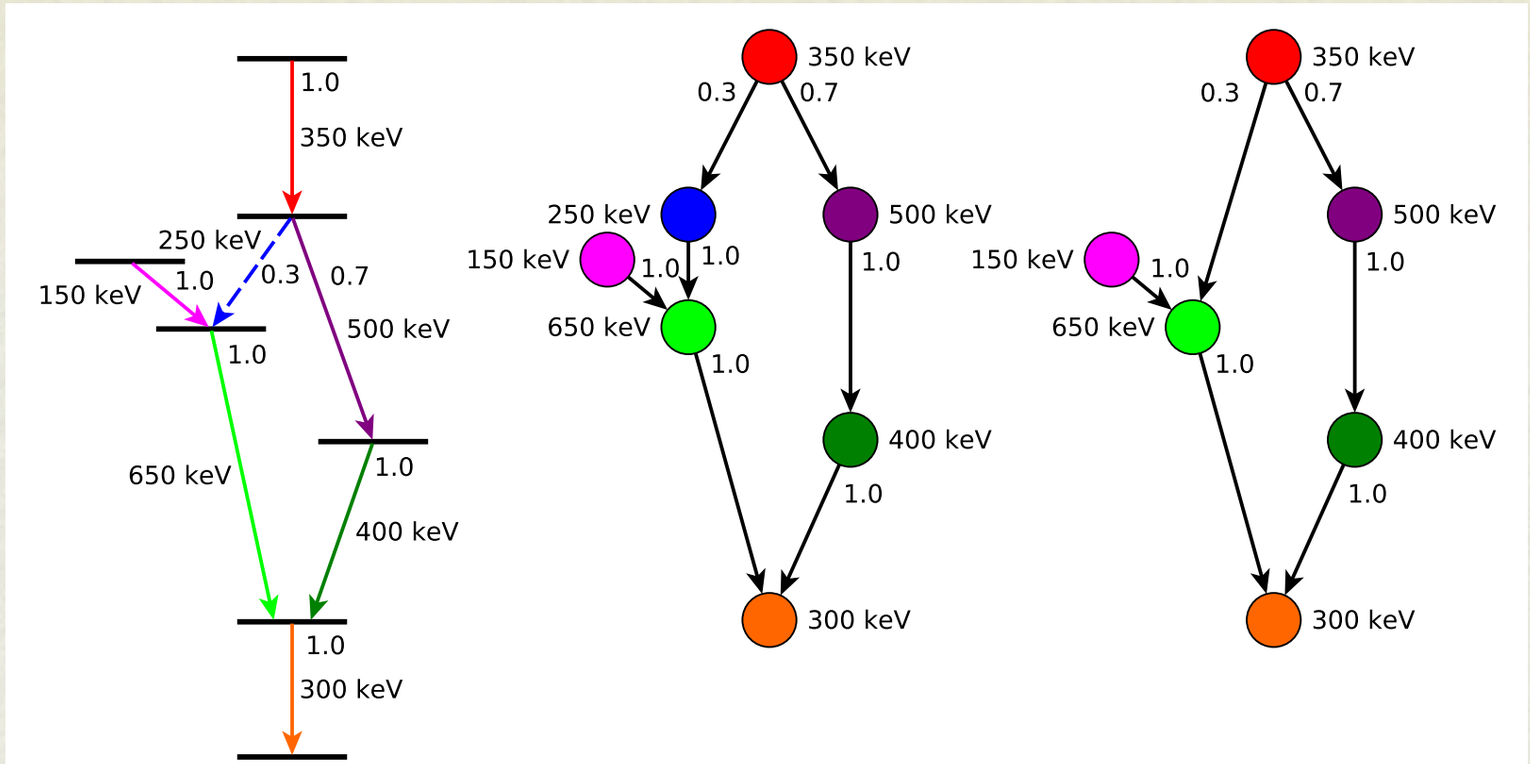
Failing to resolve multiplets introduces negative ‘cross’ transitions (dotted lines) and positive two-step transitions (dashed lines)

This signature can be used to identify unresolved multiplets



$$A = \left(\begin{array}{c|ccccc} keV & 200 & 250 & 300 & 400 & 650 \\ \hline 200 & 0 & 0 & \frac{-b}{a+b} & \frac{a}{a+b} & 1 \\ 250 & 0 & 0 & \frac{b}{a+b} & \frac{-a}{a+b} & 1 \\ 300 & 0 & 0 & 0 & 0 & 0 \\ 400 & 0 & 0 & 0 & 0 & 0 \\ 650 & 0 & 0 & \frac{b}{a+b} & \frac{a}{a+b} & 0 \end{array} \right)$$

MISSING TRANSITIONS



Missing transitions can result in energy conservation violations and can violate level identification rules

These characteristics can be used as a signature to identify missing transitions

SUMMARY

Transition space is a transition centric level scheme representation

We derived an analytical formula relating experimentally measurable quantities to level scheme structure

$$A = I - (S^{-1}D + I)^{-1}$$

Approaches to dealing with irresolvable multiplets and missing transitions were presented.

Future work will involve using this method on high statistics source data to demonstrate the validity of the method, followed by its use on experimental data from the 8π Spectrometer

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