

Early Time Dynamics in Heavy Ion Collisions  
McGill University, Montreal  
July 16th-19th 2007

# On the determination of the transport coefficient

$\hat{q}$

# in radiative energy loss formalisms

Néstor Armesto  
*Departamento de Física de Partículas and IGFAE  
Universidade de Santiago de Compostela*

# Contents:

1. Introduction.
2. Models for radiative energy loss.
3. Determinations of the transport coefficient.
4. An exercise (with Carlos A. Salgado, *Rome La Sapienza*).
5. Summary.

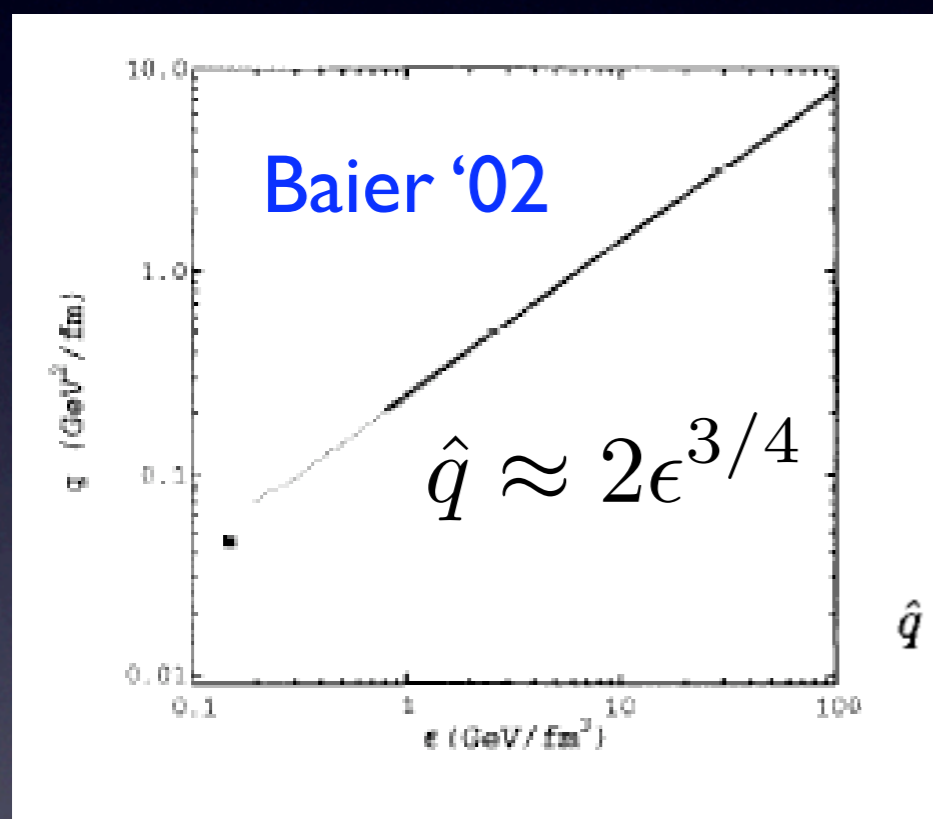
Related theory talks: Baier, Djordjevic, Majumder, Ruppert.

# I. Introduction (I):

- **Radiative energy loss** has become the baseline for explanations of both single particle and back-to-back suppression measured at RHIC.
- **Four formalisms available: BDMPS/GLV, MW and AMY.** They consider the same physical process under different approximations.
- In all of them, medium defined by **two ingredients: geometry/dynamics** (soft), and **medium density** (initial conditions, soft) **times parton-medium cross section** (hard?, soft?). Thus, radiative energy loss explores the medium.

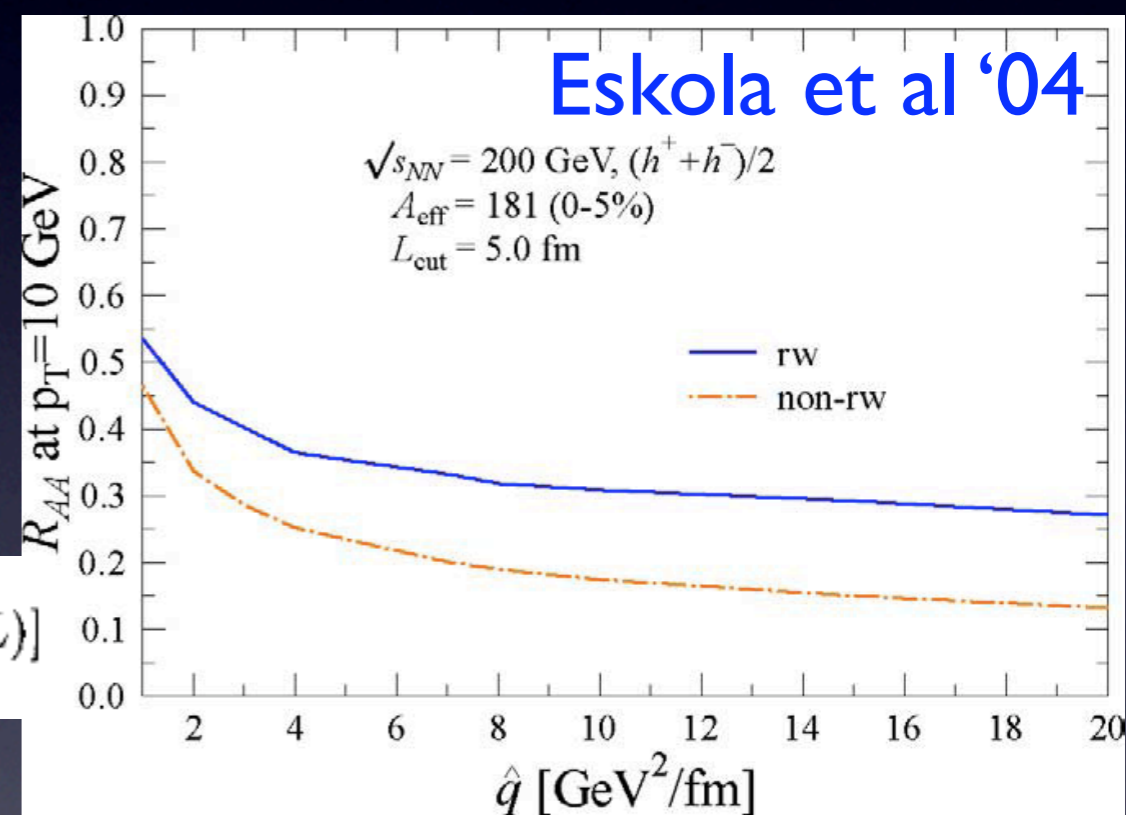
# I. Introduction (II):

- Medium density times parton-medium cross section now standardly discussed in terms of the transport coefficient:



$$\hat{q} = \frac{\mu^2}{\lambda}$$

$$\hat{q} = \frac{4\pi^2\alpha_s N_c}{N_c^2 - 1} \rho [xG(x, \hat{q}L)]$$

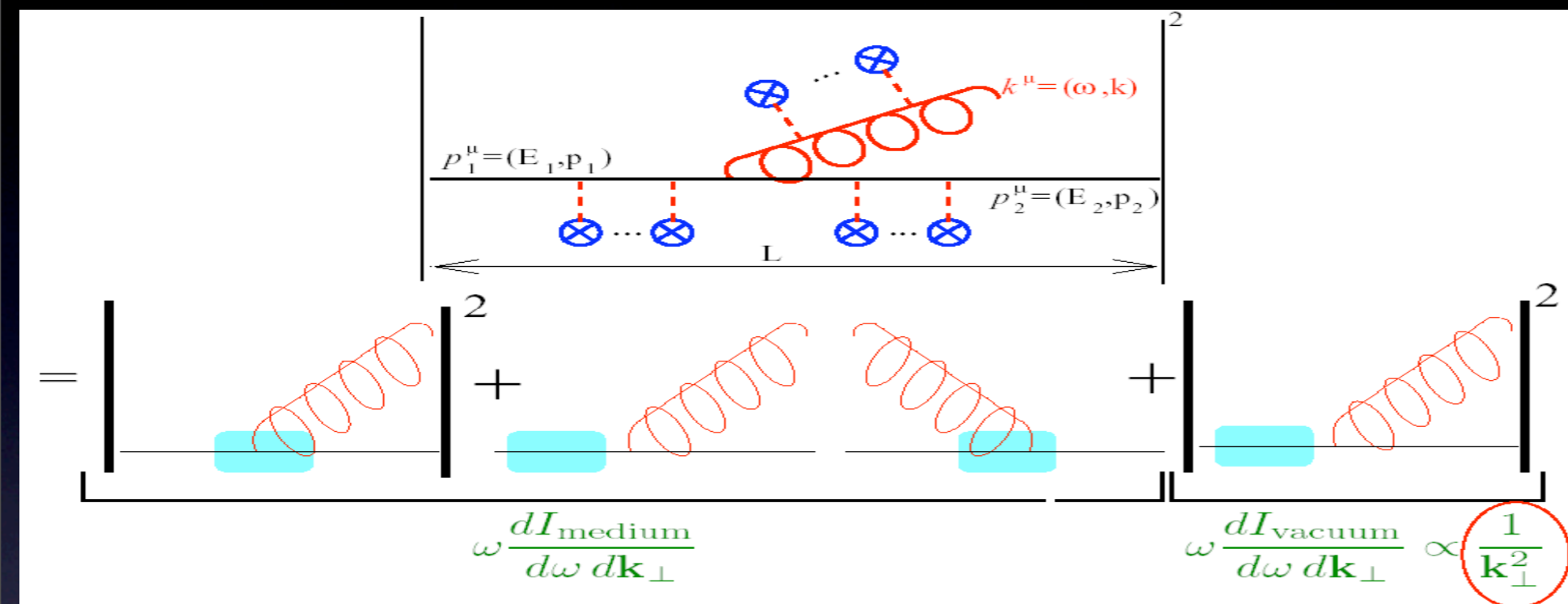


- **Problem:** different implementations/observables give different values:  $\hat{q} = 1 \div 15 \text{ GeV}^2/\text{fm}$

# 2. Models for radiative e loss (I):

(Majumder, nucl-th/0702066)

All models treat the medium modification of gluon radiation through the interference of production and rescattering.



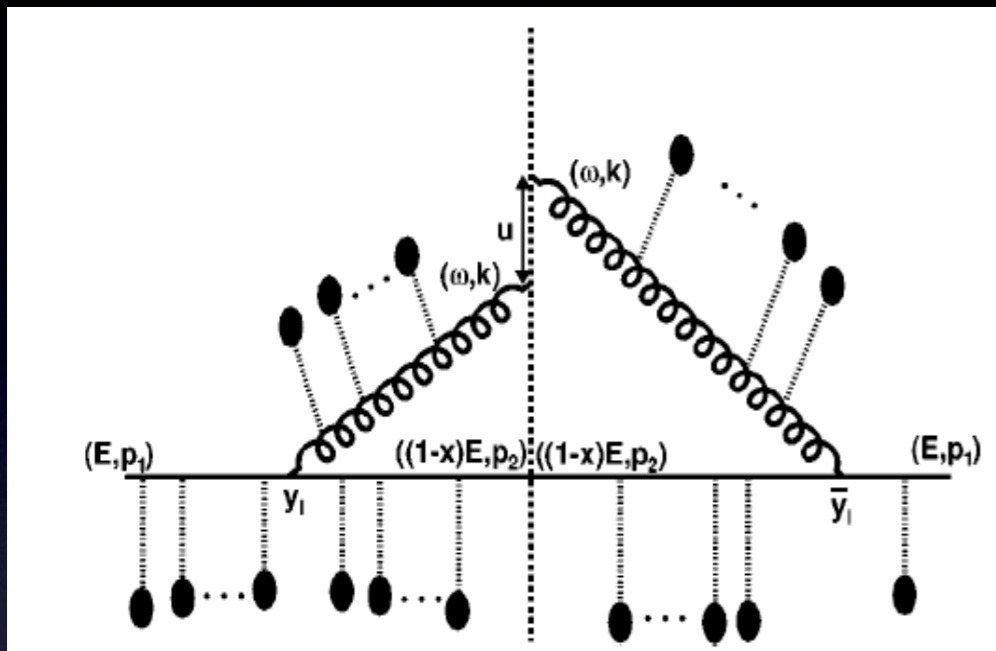
$$\Delta E \sim \int d\omega \omega \frac{dI}{d\omega} \sim \alpha_s C_R \omega_c = \frac{1}{2} \alpha_s C_R \hat{q} L^2$$

$$n(z) \sigma(r) \propto \hat{q} r^2, \quad \hat{q} = \frac{\mu^2}{\lambda}$$

Fragmentation (assumed outside the medium) modified due to the difference in radiation to get rid of virtuality.

# 2. Models for radiative e loss (II):

## 1/2. BDMPS/GLV: static medium.



$$\omega \frac{dI}{d\omega d\mathbf{k}_\perp} = \frac{\alpha_s C_F}{(2\pi)^2 \omega^2} 2 \operatorname{Re} \int_0^\infty dy_1 \int_{y_1}^\infty d\bar{y}_1 e^{i\bar{q}(y_1 - \bar{y}_1)}$$

$$\times \int d\mathbf{u} e^{-i\mathbf{k}_\perp \cdot \mathbf{u}} \exp\left(-\frac{1}{2} \int_{\bar{y}_1}^\infty d\xi n(\xi) \sigma(\mathbf{u})\right)$$

$$\times \frac{\partial}{\partial \mathbf{y}} \cdot \frac{\partial}{\partial \mathbf{u}} \int_{\mathbf{y}=0=\mathbf{r}(y_1)}^{\mathbf{u}=\mathbf{r}(\bar{y}_1)} \mathcal{D}\mathbf{r} \exp\left[i \int_{y_1}^{\bar{y}_1} d\xi \frac{\omega}{2} \left( \dot{\mathbf{r}}^2 - \frac{n(\xi) \sigma(\mathbf{r})}{i\omega} \right)\right]$$

Exact solution unknown, **two approximations**:

1. Harmonic oscillator (Brownian motion):

**multiple soft scatterings.**

$$n(\xi) \sigma(\mathbf{r}) \simeq \frac{1}{2} \hat{q}(\xi) \mathbf{r}^2$$

2. Opacity expansion:  $N=1$ , **single hard scattering**, corrects Brownian motion.

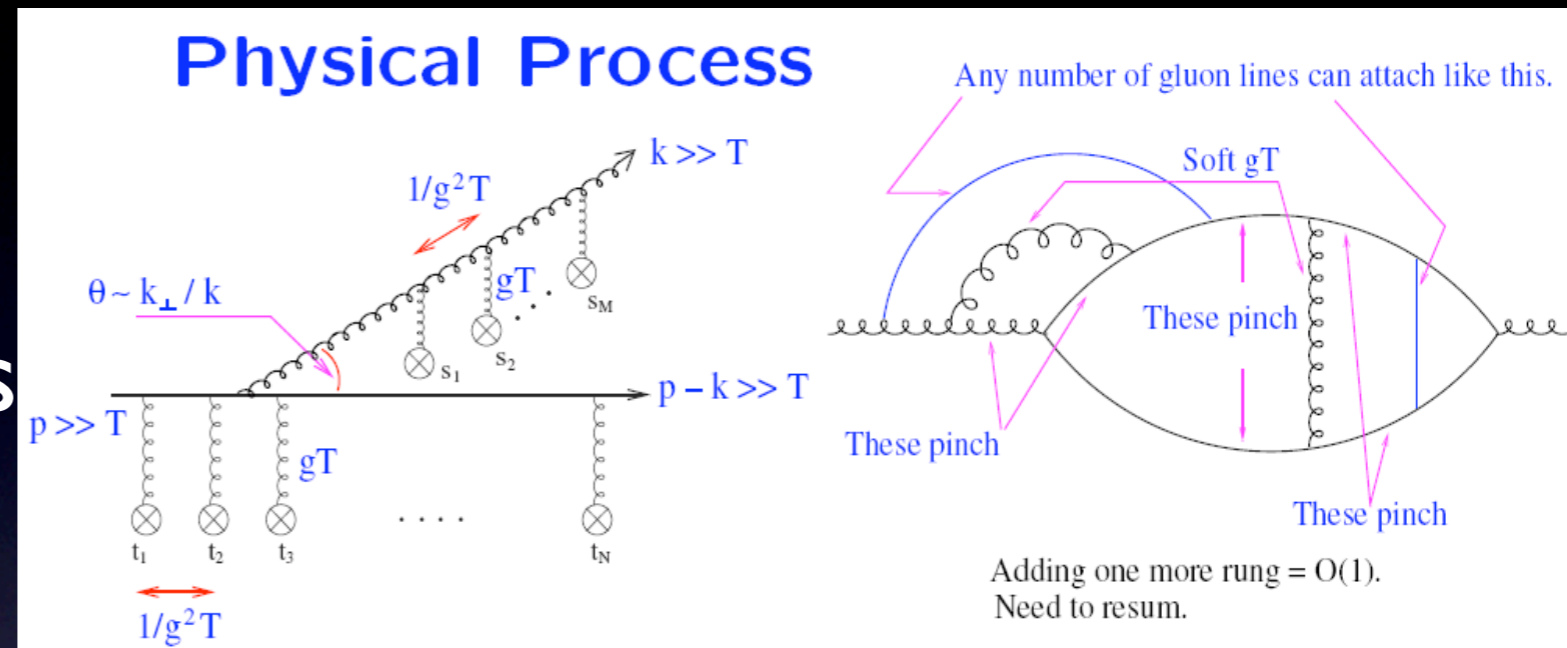
$$[n(\xi) \sigma(\mathbf{r})]^N$$

Comparison for massless and massive: **SW '03, ASW '04.**

# 2. Models for radiative e loss (III):

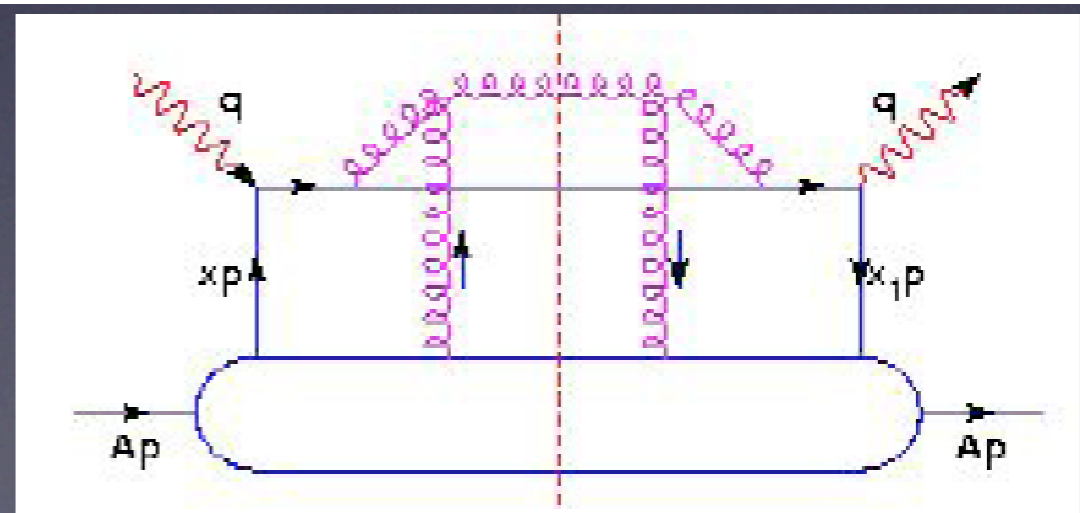
3. **AMY**: rates order  $\alpha_s$ , dynamical medium, no interference of emissions in/out medium, expansion.

4. **GW(M)**: ff in DIS on nuclei, first corrections in  $L/k_T^2$ , modification of DGLAP splitting functions, virtuality.



$$\tilde{D}(z_1, \mu^2) = D(z_1, \mu^2) + \frac{\alpha_s}{2\pi} \int_0^{\mu^2} \frac{dl_{\perp}^2}{l_{\perp}^2} \int \frac{dy}{y} \left( \frac{1+y^2}{1-y} f(x, y, Q^2, l_{\perp}) + V.C. \right) D(z_1/y, \mu^2)$$

$$f = \frac{C_A 2\pi \alpha}{l_T^2 + k_T^2} \frac{\int dy dy_1 dy_2 \langle A | \bar{\Psi}(y) F(y_1) F(y_2) \Psi(0) | A \rangle e^{i \text{ factors}}}{N_c f^A(x)}$$



On the determination of the transport coefficient.

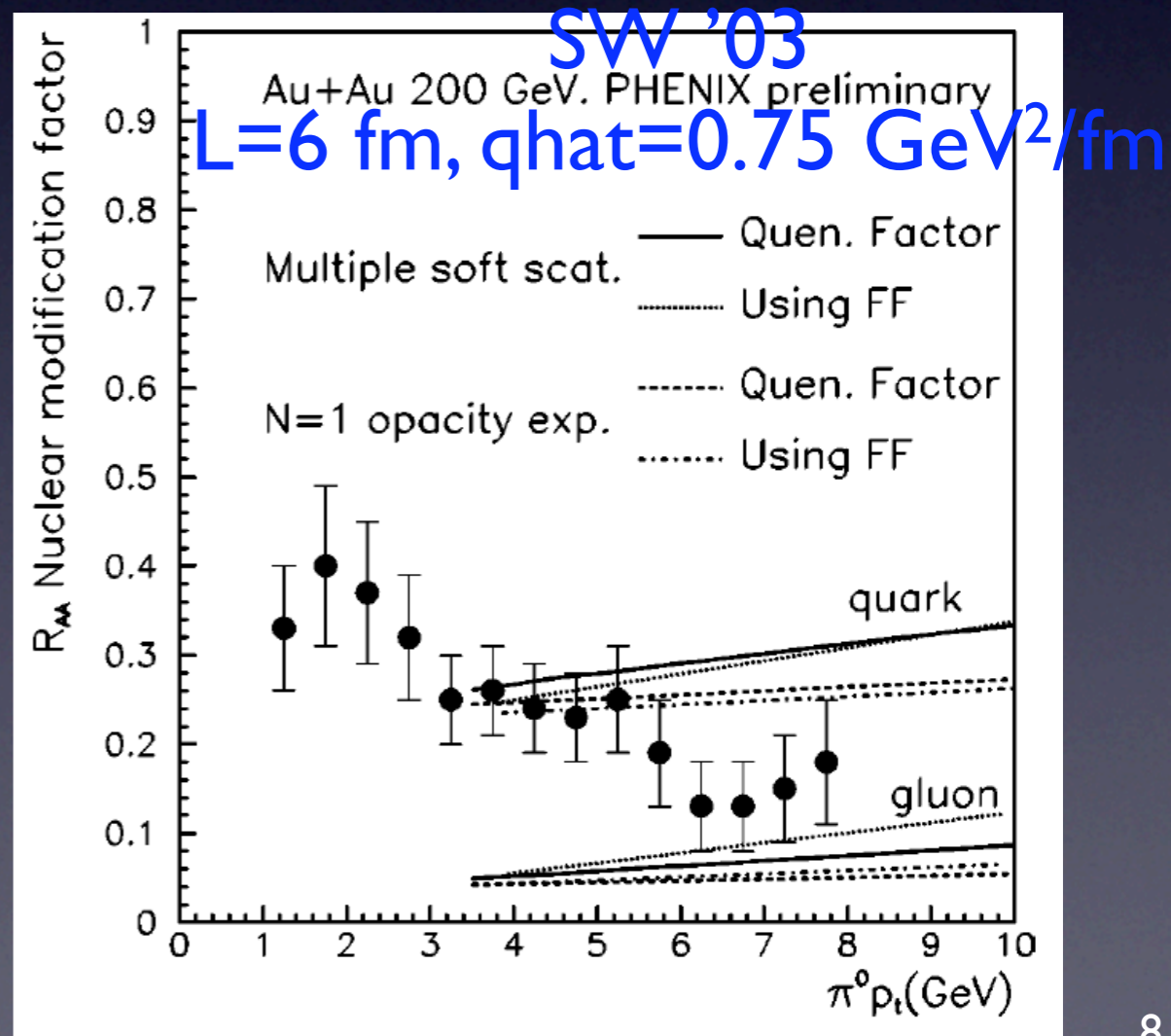
# 3. Determinations of $\hat{q}$ (I):

- $\hat{q}$  is a natural parameter only in **BDMPS**.
- Extraction from a comparison with  $R_{AA}$ .
- Phenomenological implementations are key: mean energy loss rudimentary, distribution of energy losses better: quenching weights (BDMS, GLV '01).
- Fixed length (GLV; Arleo '02; SW '03) gives  $\sim < 1 \text{ GeV}^2/\text{fm}$ .

$$Q(p_{\perp}) = \frac{d\sigma^{\text{med}}(p_{\perp})/dp_{\perp}^2}{d\sigma^{\text{vac}}(p_{\perp})/dp_{\perp}^2} = \int d\Delta E P(\Delta E) \left( \frac{d\sigma^{\text{vac}}(p_{\perp} + \Delta E)/dp_{\perp}^2}{d\sigma^{\text{vac}}(p_{\perp})/dp_{\perp}^2} \right)$$

BDMS '01; Wang et al '96

$$D_{h/q}^{(\text{med})}(x, Q^2) = \int_0^1 d\epsilon P(\epsilon) \frac{1}{1-\epsilon} D_{h/q} \left( \frac{x}{1-\epsilon}, Q^2 \right)$$

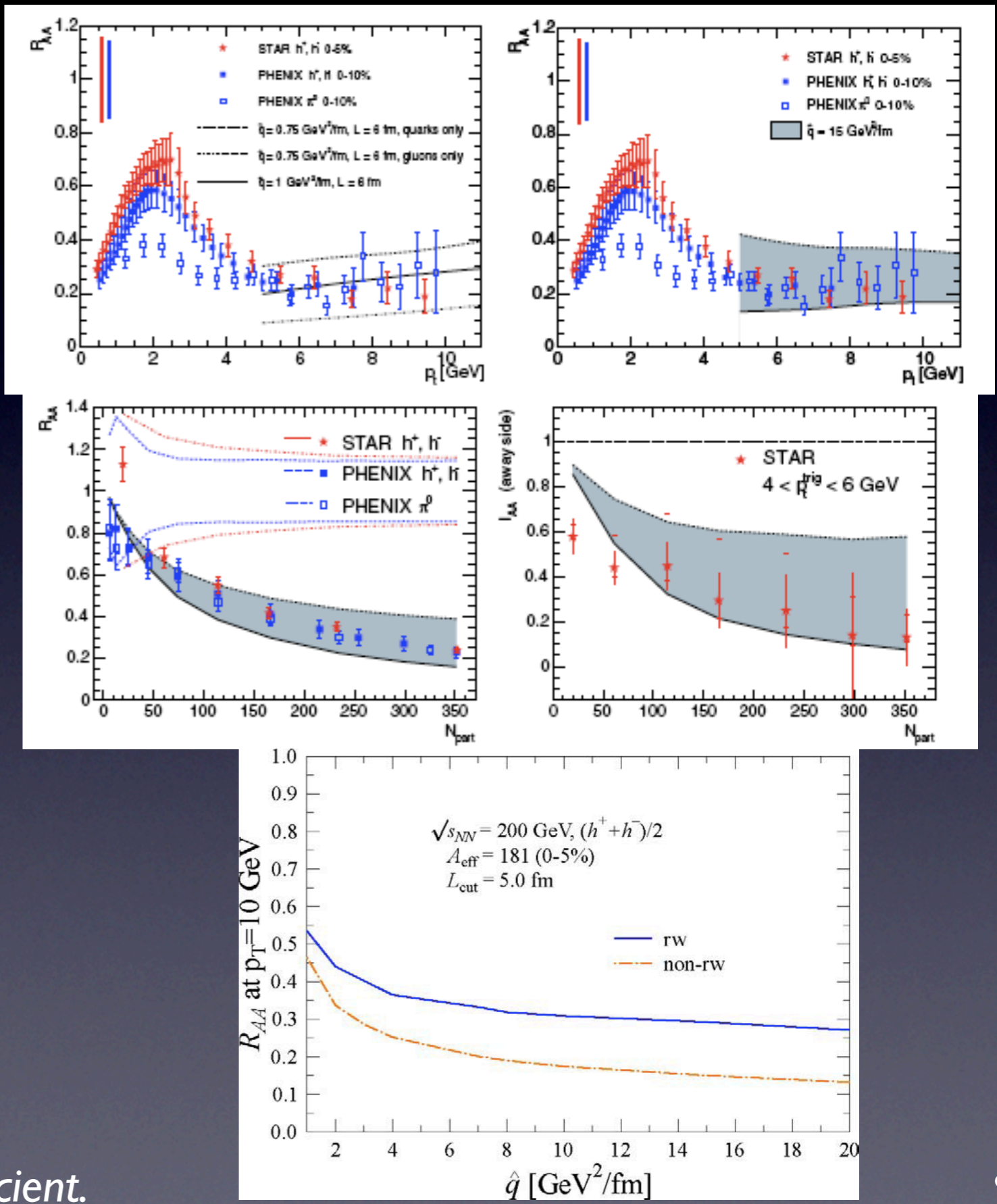


On the determination of the transport coefficient.



# 3. Determinations of $\hat{q}$ (II):

- A **Woods-Saxon** geometry (production plus ‘medium’) gives larger values and leads to saturation: fragility (Dainese et al, Eskola et al ‘04).
- Surface bias (Muller ‘03).
- $\langle \hat{q} \rangle = 4 - 14 \text{ GeV}^2/\text{fm}$ .
- Energy constraints (Baier et al ‘06); energy dependence (Casalderrey et al ‘07).



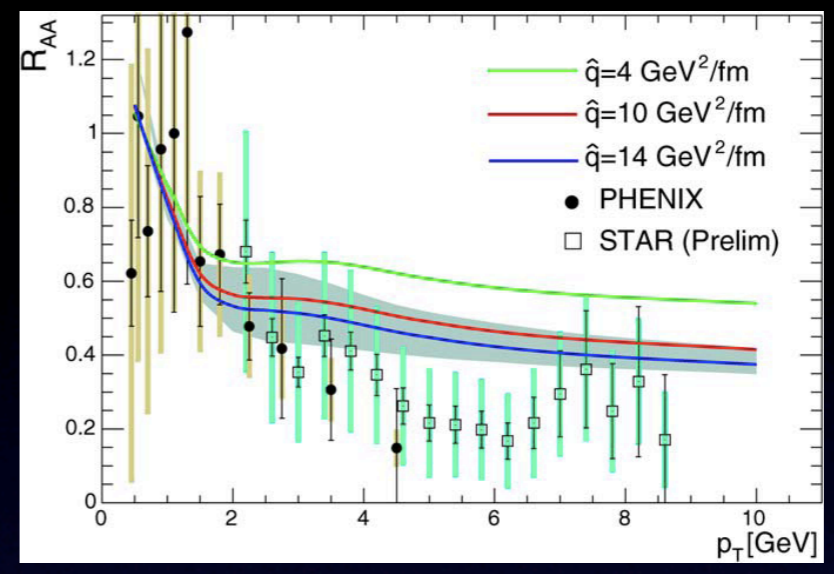
On the determination of the transport coefficient.

# 3. Determinations of $\hat{q}$ (III):

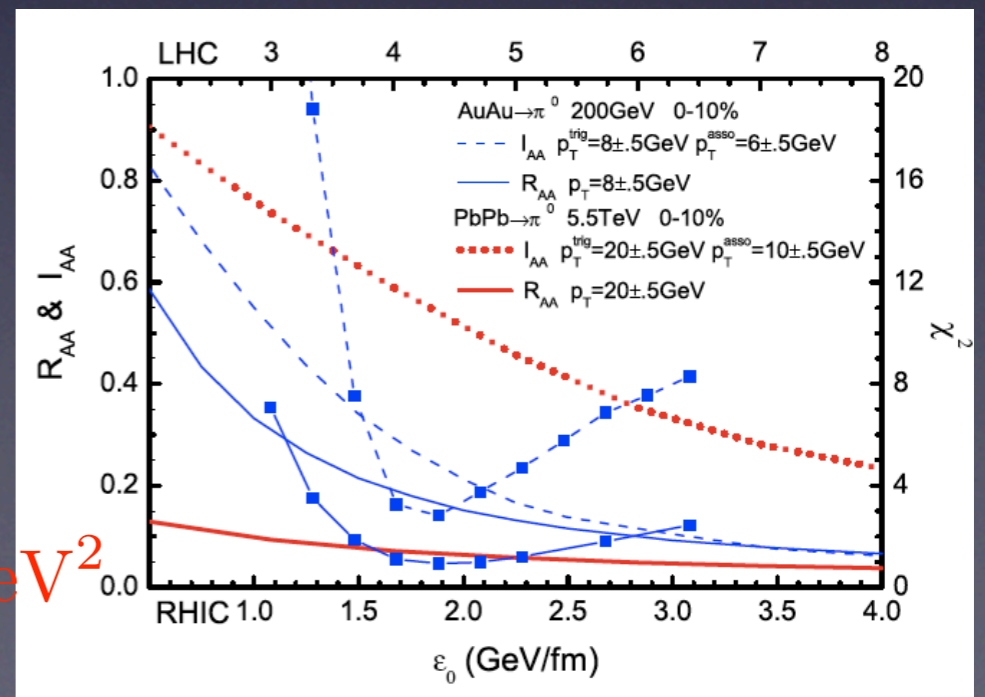
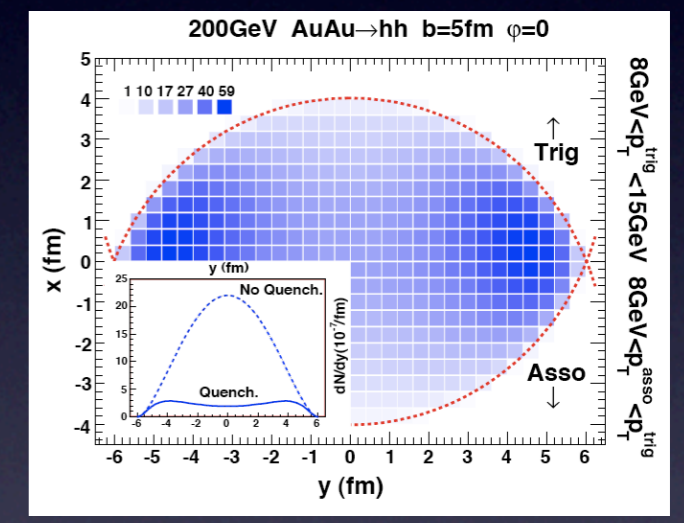
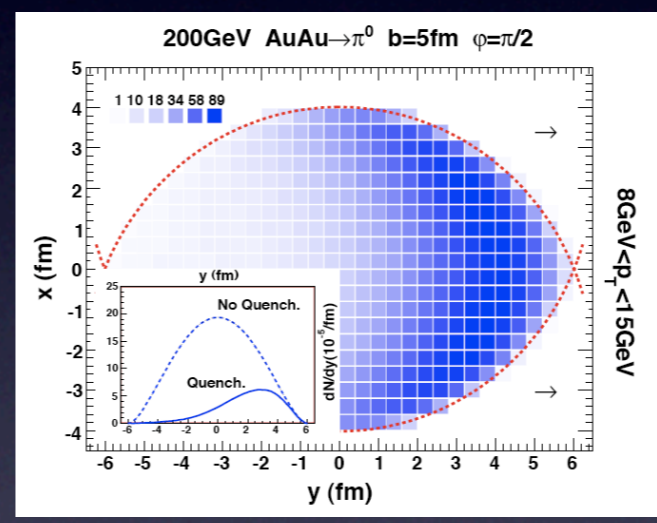
- Hard probes '06: AMY gives 2, GLV gives  $< 1$ , MW give 3-4 GeV<sup>2</sup>/fm: all at initial time.
- Dilution: introduced effectively (GLVW '01, SW '02)
 
$$\langle \hat{q} \rangle = \frac{2}{L^2 - \tau_0^2} \int_{\tau_0}^L d\tau \tau \hat{q}_0 \frac{\tau_0}{\tau} \simeq \frac{2\tau_0 \hat{q}_0}{L} \approx \frac{\hat{q}_0}{2 \div 5}$$
- Flow (Armesto et al '04) doesn't lower  $q_{\text{hat}}$  (Baier et al '06).
- A dynamical medium decreases  $q_{\text{hat}}$  (AMY?, Djordjevic et al '07).
 
$$\hat{q}(\xi) = K \cdot 2 \cdot \epsilon^{3/4}(\xi)$$
- A dynamical expansion (Hirano-Nara '03; Ruppert-Renk '05, '06; Majumder et al '07; Qin et al '07) lowers  $q_{\text{hat}}$  with respect to a static medium; still  $K > 1$ ; late time effect?

# 3. Determinations of $\hat{q}$ (IV):

- **Non-photonic electrons** not conclusive: benchmark (Armesto et al '05), hadronization inside (Adil et al '06), collisional (Djordjevic et al '06)...



- **I\_AA or away side pseudofragmentation function** (Wang '03) tend to favor low values of  $\hat{q}$  (Renk '06; Loizides '06; Zhang et al '07): punch-through.



$\langle \hat{q}_0 \tau_0 \rangle \approx 2 \div 3 \text{ GeV}^2$

On the determination of the transport coefficient.

# 4. An exercise (I):

(with Carlos A. Salgado, Rome La Sapienza)

Quantification of the effect on  $q$ hat of some of the phenomenological ingredients, based on  $R_{AA}$  for central, using a pQCD spectrum and QW.

$$Q(p_{\perp}) = \frac{d\sigma^{med}(p_{\perp})/dp_{\perp}^2}{d\sigma^{vac}(p_{\perp})/dp_{\perp}^2}$$

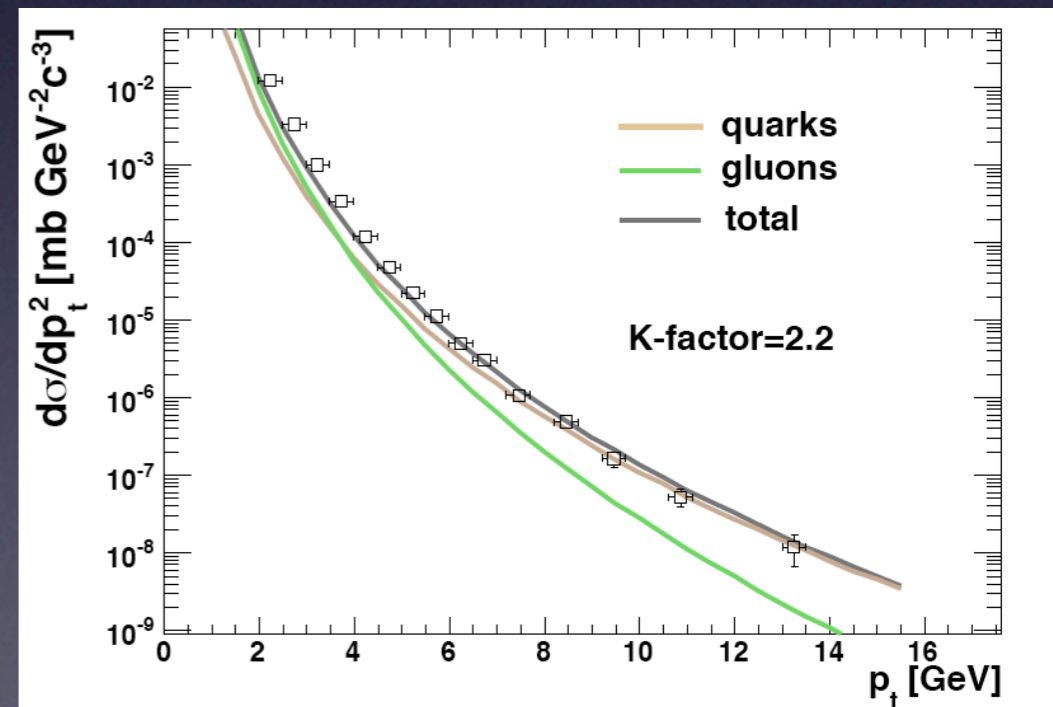
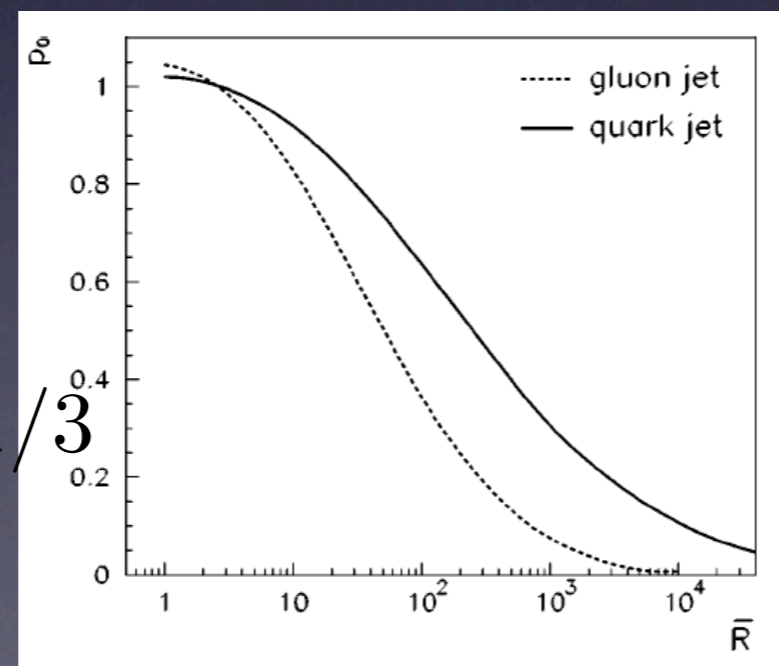
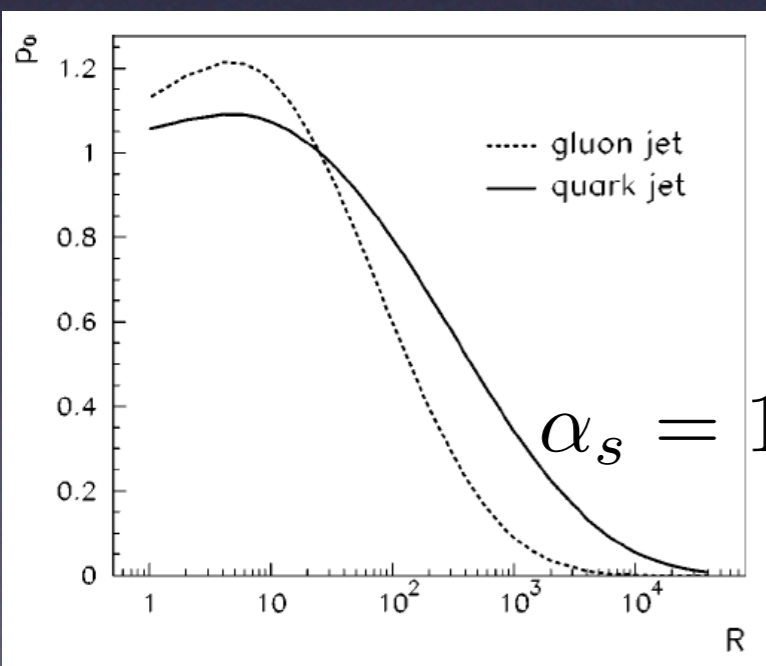
$$= \int d\Delta E P(\Delta E) \left( \frac{d\sigma^{vac}(p_{\perp} + \Delta E)/dp_{\perp}^2}{d\sigma^{vac}(p_{\perp})/dp_{\perp}^2} \right)$$

$$P(\Delta E) = \sum_{n=0}^{\infty} \frac{1}{n!} \left[ \prod_{i=1}^n \int d\omega_i \frac{dI(\omega_i)}{d\omega} \right] \delta\left(\Delta E - \sum_{i=1}^n \omega_i\right) \exp\left[-\int_0^{\infty} d\omega \frac{dI}{d\omega}\right]$$

$$\omega_c = \frac{1}{2} \hat{q} L^2, \quad R = \omega_c L, \quad L/\lambda = 1$$

multiple soft

single hard

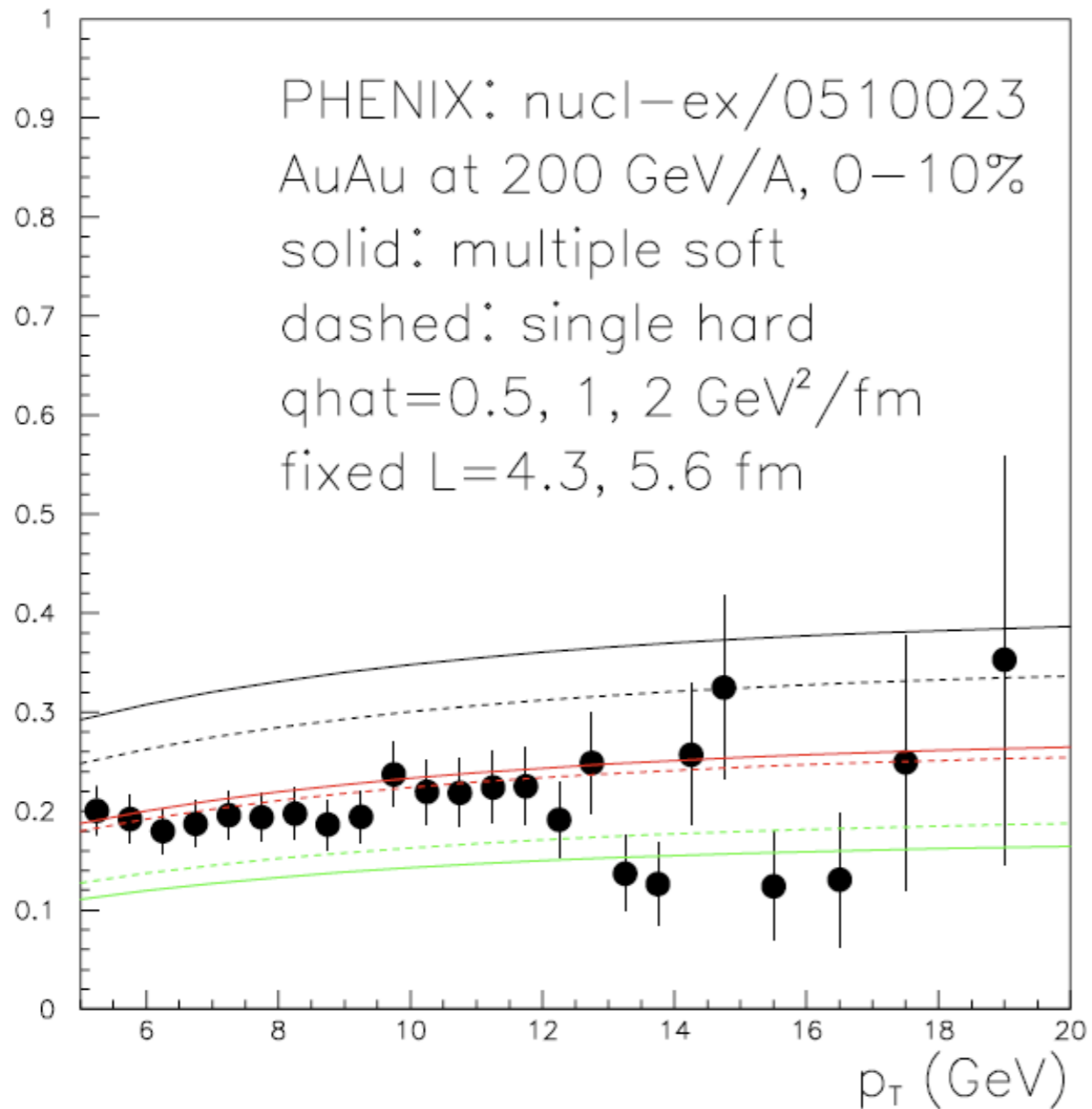


pp@200, PHENIX pi0

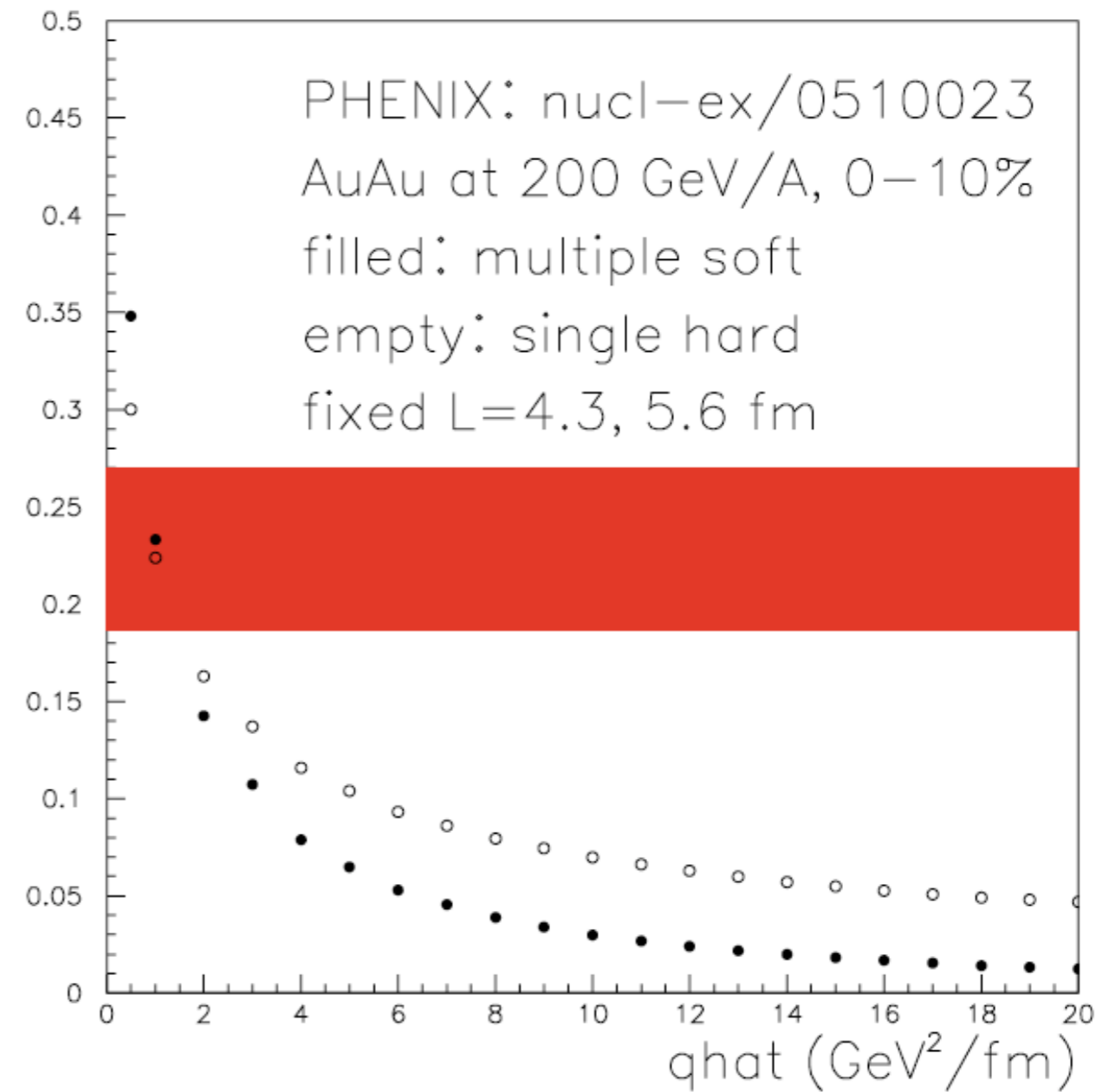
On the determination of the transport coefficient.

# 4. An exercise (II): fixed length

$R_{AA}(p_T)$  for  $\pi^0$  at  $\eta=0$



$R_{AA}(p_T=10 \text{ GeV})$  for  $\pi^0$  at  $\eta=0$



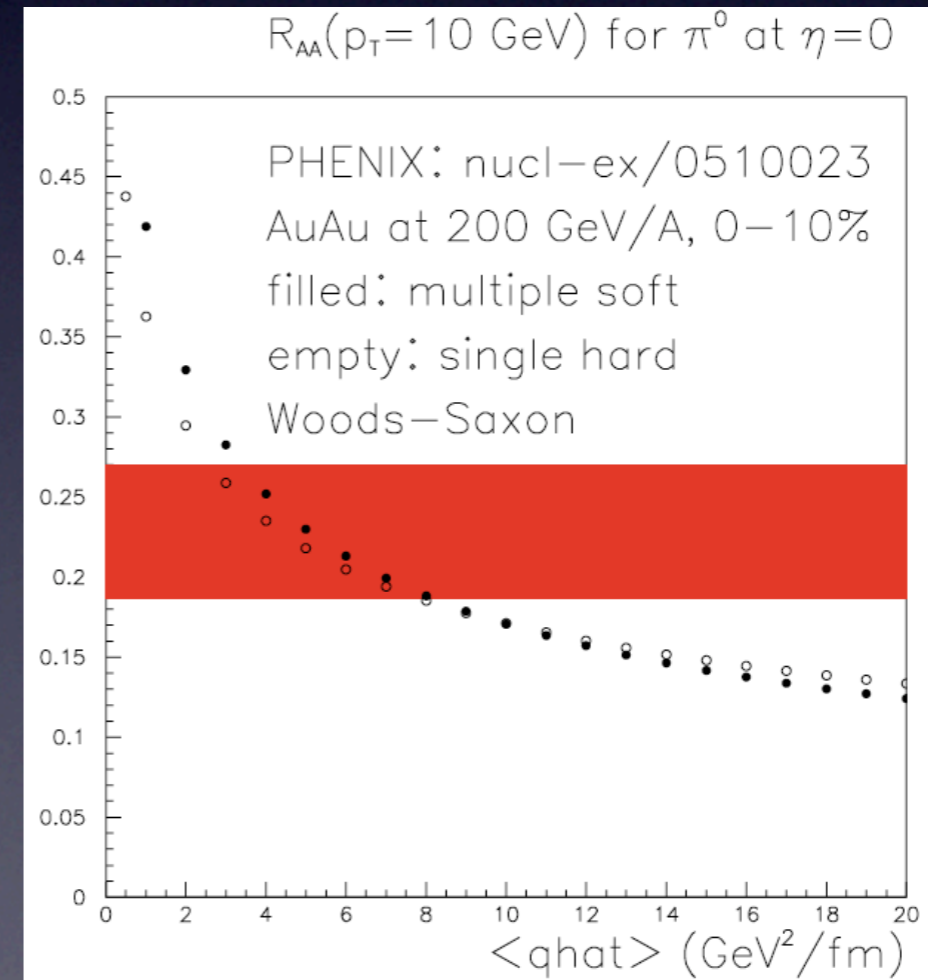
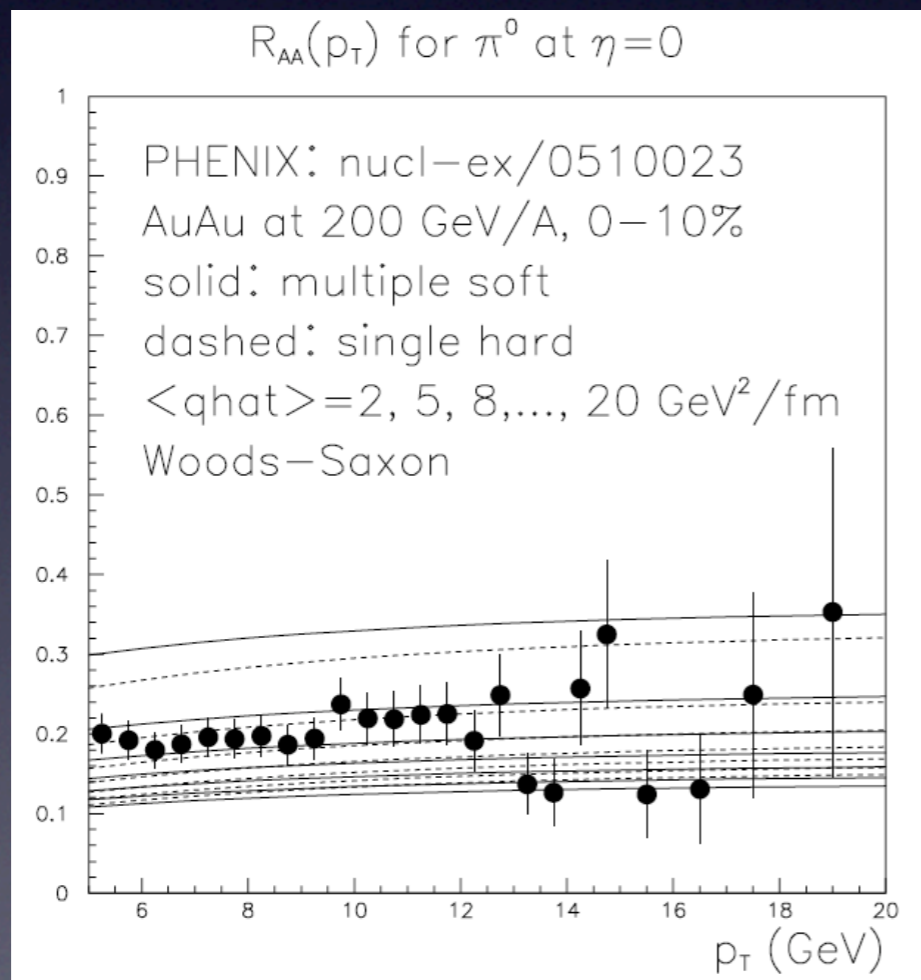
# 4. An exercise (III): Woods-Saxon

$$\omega_c(\mathbf{r}_0, \phi) = \int_0^\infty d\xi \xi \hat{q}(\xi)$$

$$\langle \hat{q} L \rangle(\mathbf{r}_0, \phi) = \int_0^\infty d\xi \hat{q}(\xi)$$

$$\hat{q} \propto T_A T_B (x_0 + \xi \cos \phi, y_0 + \xi \sin \phi)$$

$$R(\mathbf{r}_0, \phi) = 2\omega_c^2(\mathbf{r}_0, \phi) / \langle \hat{q} L \rangle(\mathbf{r}_0, \phi), \quad L = R/\omega_c, \quad \langle \hat{q} \rangle = 2\omega_c^2 / (LR)$$



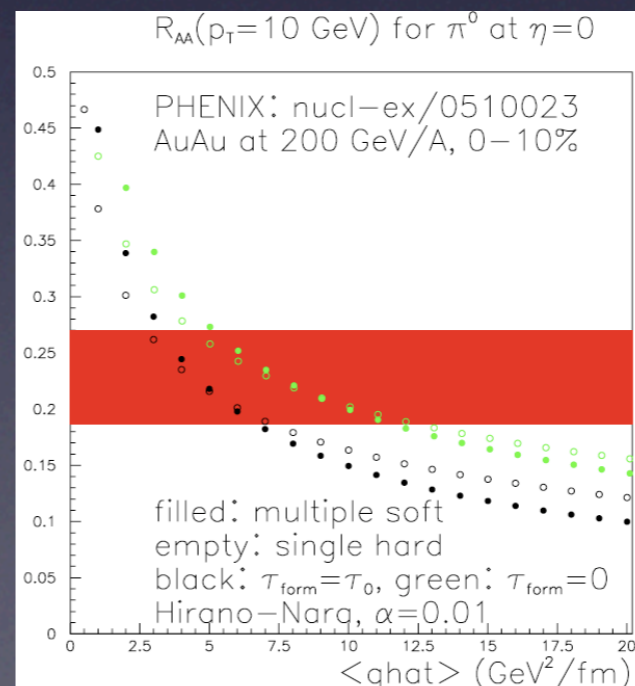
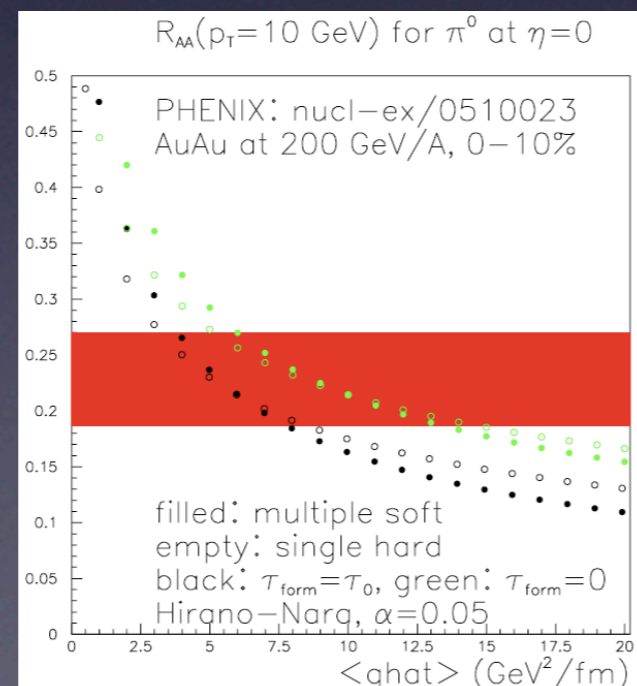
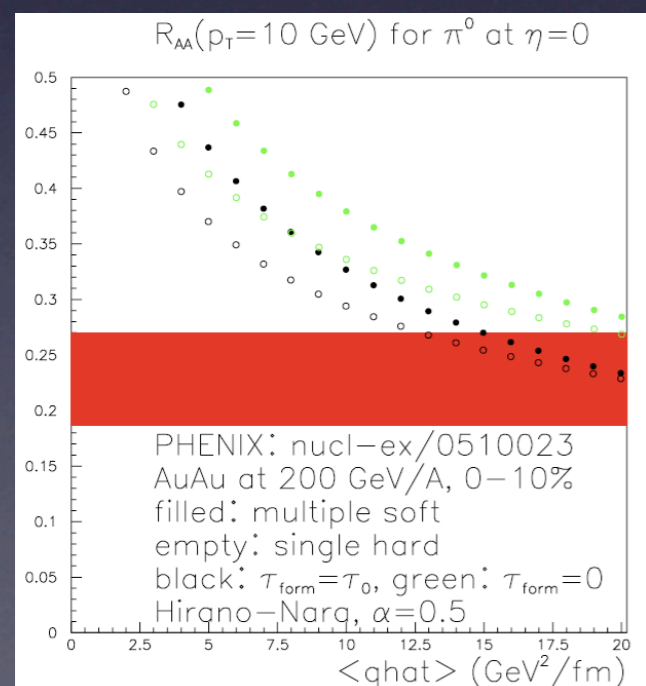
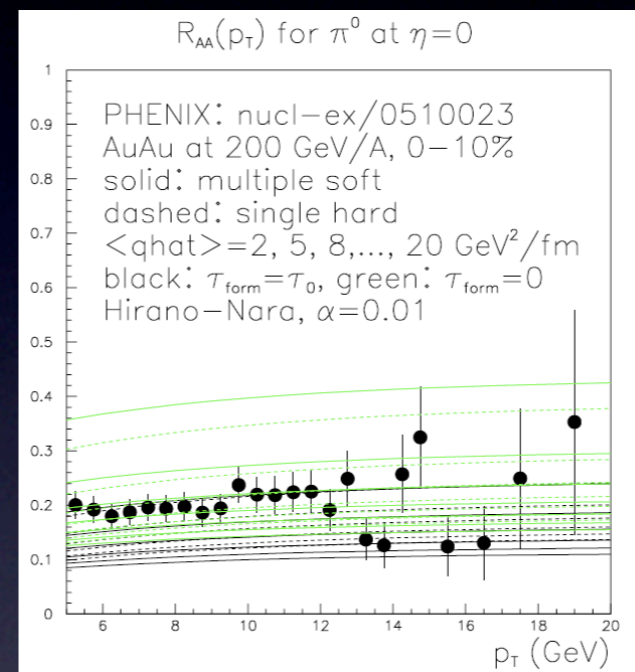
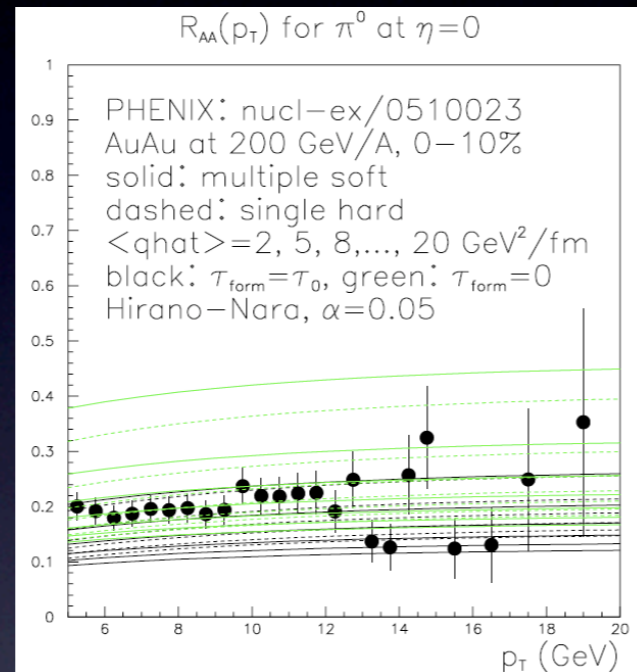
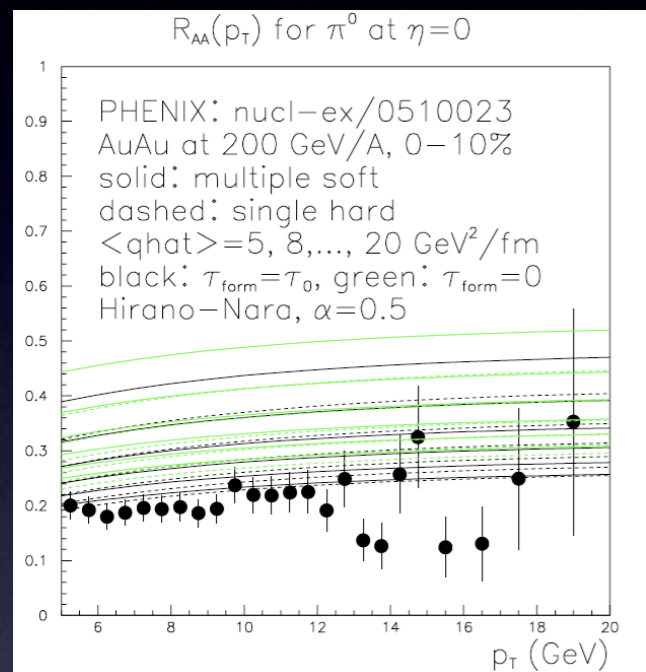
## 4. An exercise (IV): hydro

**Hirano-Nara:** 3+1 ideal hydro,  
 for AuAu@200,  $b=3.1$  fm, ideal  
 EOS with  $N_f=3$ ,  $B^{1/4}=247$  MeV.

$$\langle \epsilon \rangle (\tau_0) \simeq 27(36) \text{ GeV}/\text{fm}^3$$

$$\langle \epsilon^{3/4} \rangle (\tau_0) \simeq 1.5(2.2) \text{ GeV}^2/\text{fm}$$

$$\tau_0 = 0.6 \text{ fm}, \tau_{max} = 10.2 \text{ fm}$$



$$\hat{q}(\xi) = c \epsilon^{3/4}(\xi)$$

We explore:

$$\tau_{form} = 0 \div \tau_0$$

$$0.5 < \alpha < 0.01$$

$$\langle \hat{q} \rangle = 4 \text{ GeV}^2/\text{fm}$$

$$\Rightarrow K \sim 3$$

$$\langle \hat{q}_0 \tau_0 \rangle \sim 5 \text{ GeV}^2/\text{fm}$$

On the determination of the transport coefficient.

# 5. Summary:

Phenomenological  
implementation

	$\hat{q}$ ( $\text{GeV}^2/\text{fm}$ )
fixed length	$\leq 1$ (average)
Woods-Saxon	4-14 (average)
dynamical medium	decreases
flow	no effect
dilution	increases, factor 2-5
hydro	$K \sim 3-4$ , late times
$I_{AA}/p_{ff}$	favors low values
non-photonic electrons	unconclusive
AMY	2 (initial)
MW	2-3 (initial)
multiple soft/single hard	small decrease
GLV	$< 1$ (initial)

Observables

Models

**Future:** heavy flavor ID, more differential observables, LHC.