Scaling of v_2 from AGS to RHIC (nucl-th/0702013 ,accepted for publication, PRC)

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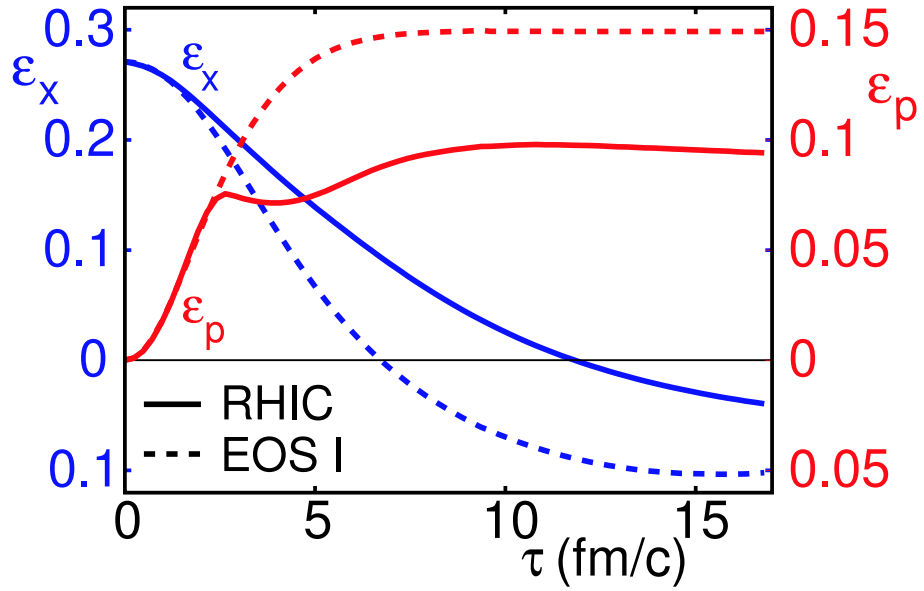
Why is v_2 such a good observable I?

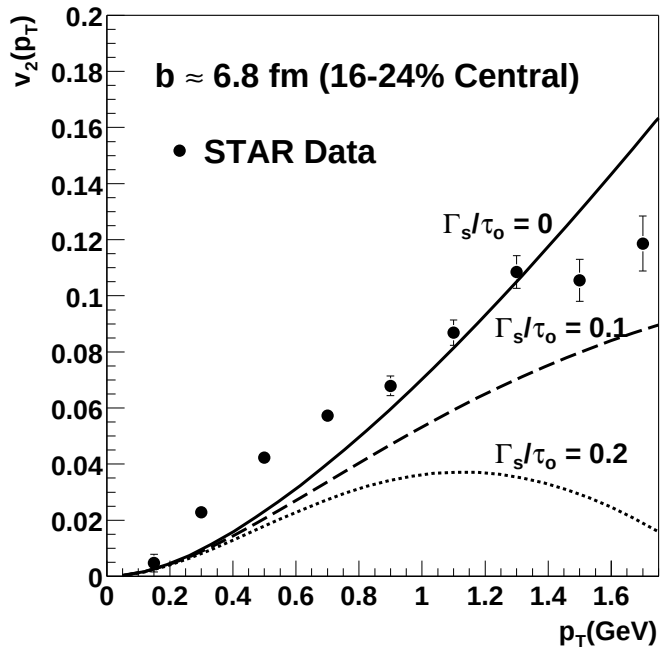
It is self-quenching (Heinz,Kolb, nucl-th/0305084)



and therefore depends only on the initial stages, and not on the (messy and not understood) freeze-out.

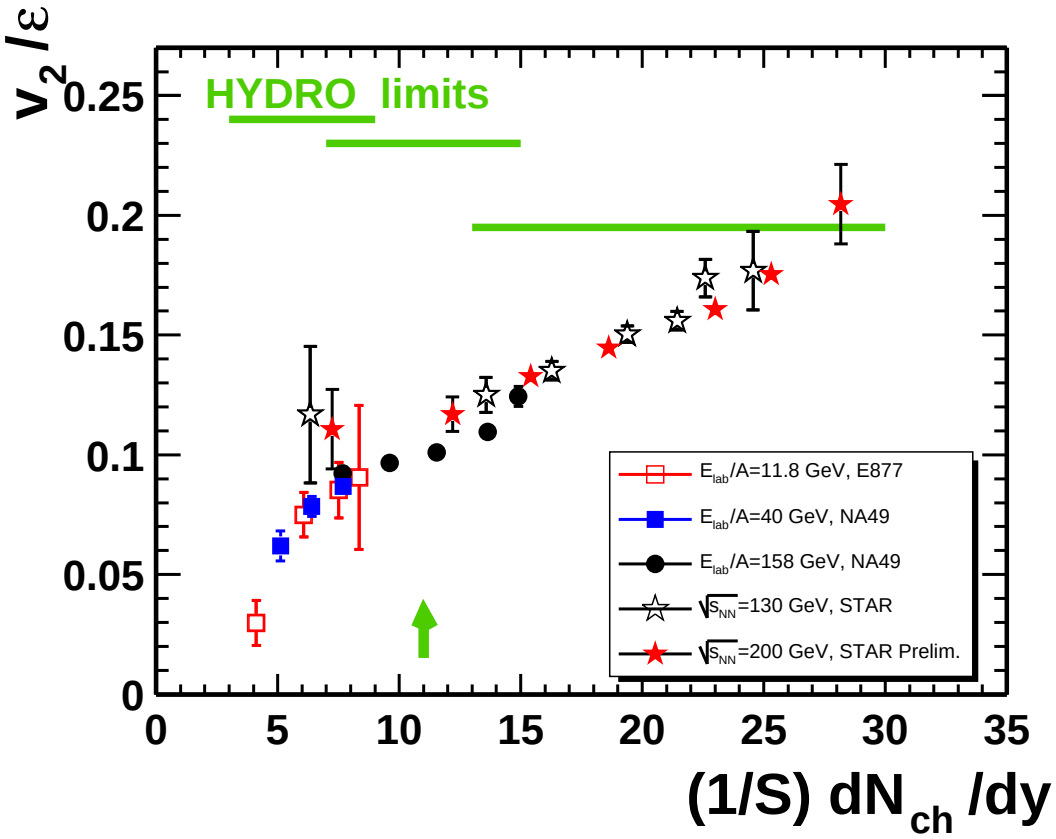
Why is v_2 such a good observable II?

Because it heavily depends on viscosity (D. Teaney, nucl-th/0301099)

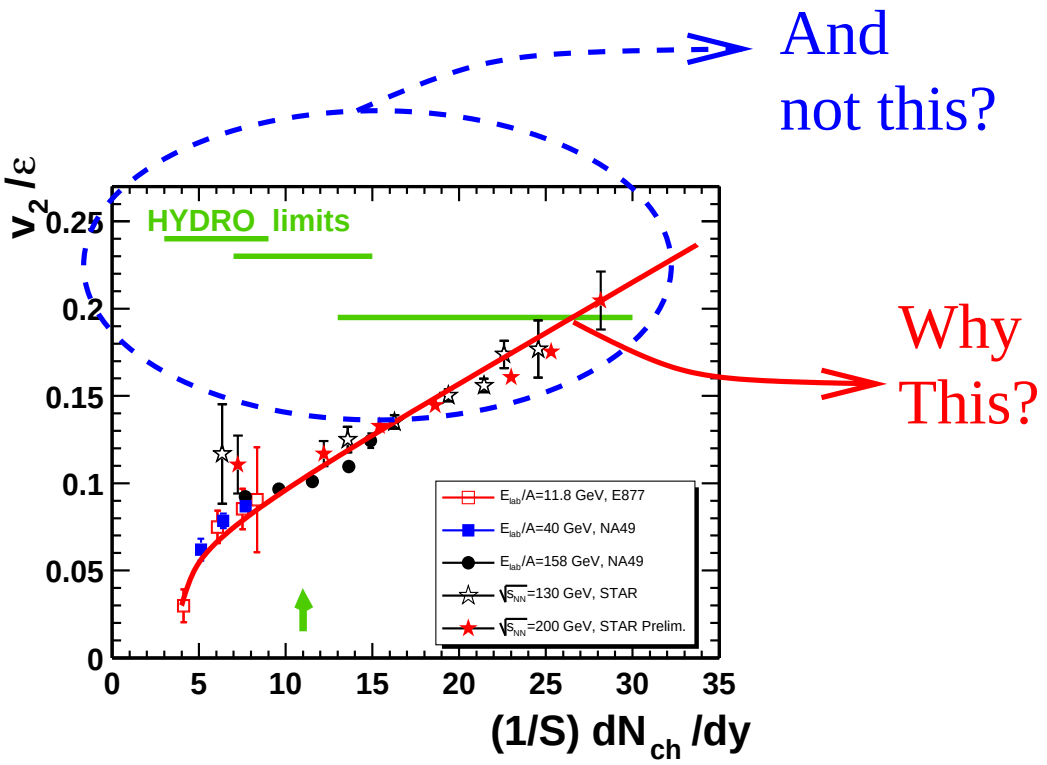


A small viscosity can quench it, so it's a test of fluidity.

RHIC has found the perfect fluid from v_2 measurements

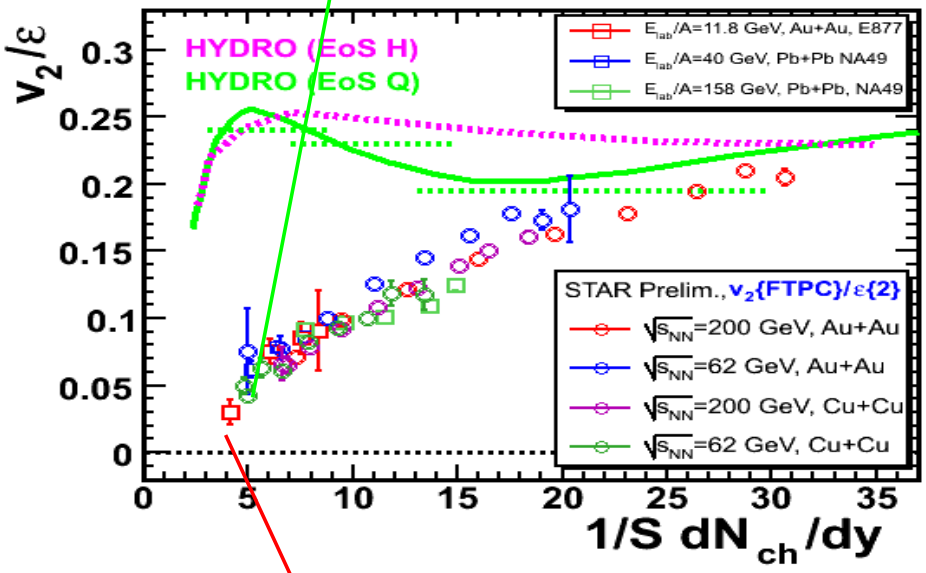


At lower energies than RHIC, data below the hydro limit $\Rightarrow$ imperfect fluid



Or is it?

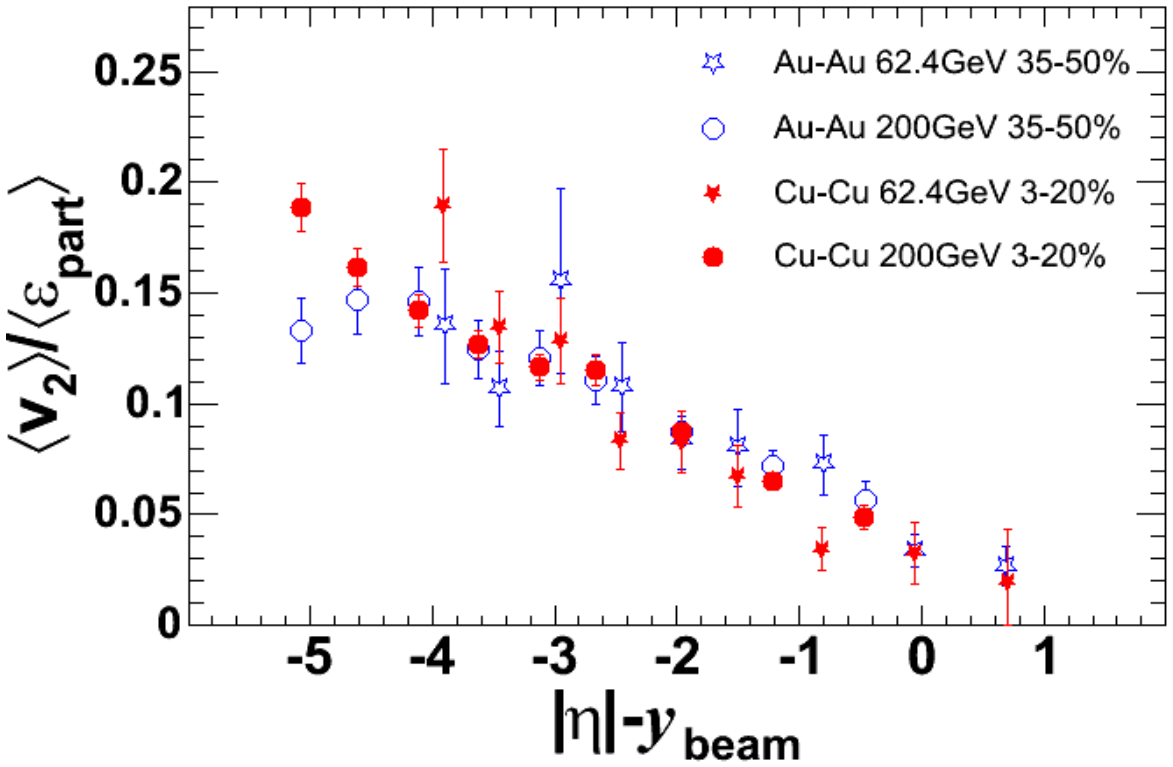
This is Cu−Cu@200 GeV



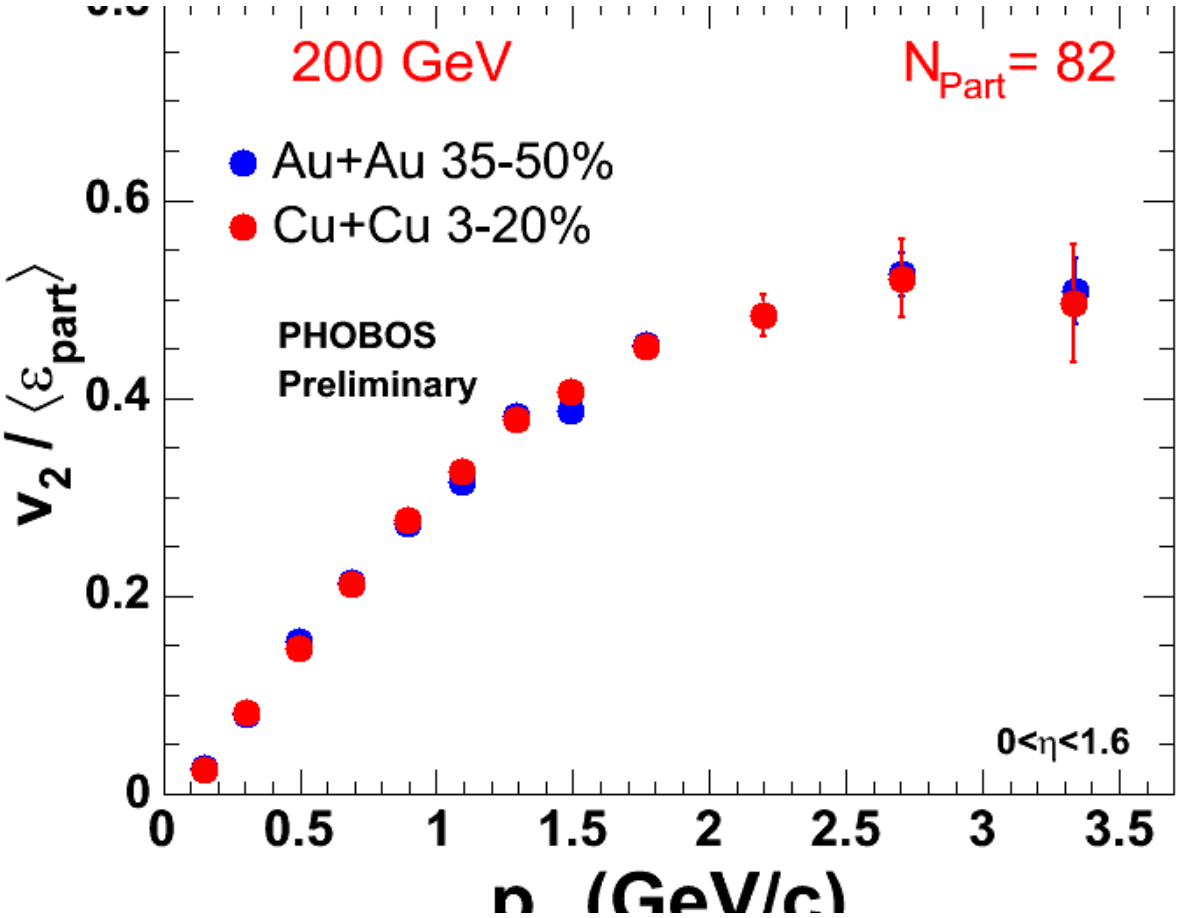
This is Au+Au@11.8 GeV

v_2/ϵ is the same for a given $\frac{1}{S}\frac{dN}{dy}$, even if the energy is very different!!!!

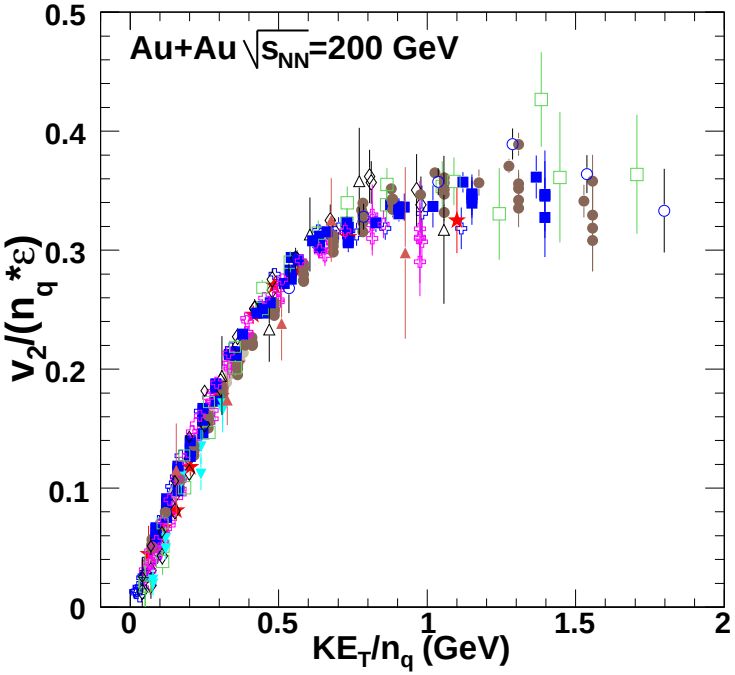
If same $\frac{1}{S}\frac{dN}{dy}$, agreement could extend to η bins (PHOBOS)



p_T bins (PHOBOS)



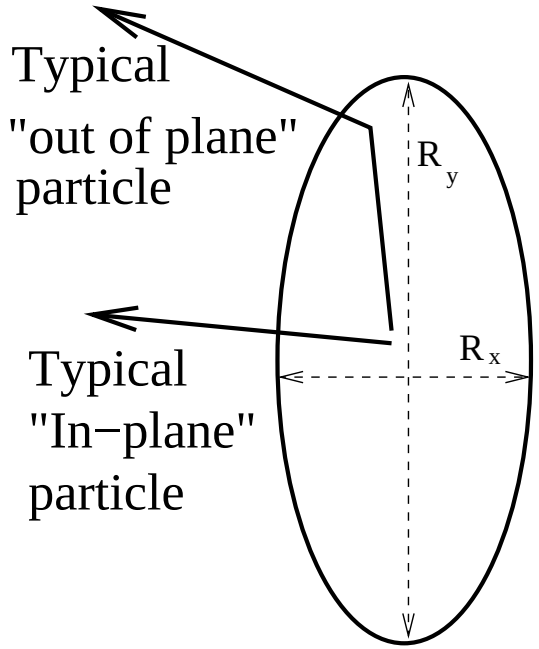
And, when combined with coalescence, particle species (Lacey,Taranenko)



(So this is a Partonic effect?)

Why plot $\frac{v_2}{\epsilon}$ vs $\frac{1}{S}\frac{dN}{dy}$?

H. Heiselberg and A. M. Levy,PRC **59**, 2716 (1999) nucl-th/9812034



$l_{mfp} \sim R \quad 1 \quad Interaction/particle/lifetime$

Boltzmann equation with these initial conditions...

$$\frac{v_2}{\epsilon} \propto \frac{\langle \sigma_{ij} v_{ij} \rangle}{R_x R_y} \frac{dN}{dy}$$

A break of this scaling would have signalled a sudden change in $\langle \sigma_{ij} v \rangle$, driven perhaps by the transition from a weakly coupled hadron gas to a weakly coupled quark-gluon gas.

...But what does this have to do with an ideal fluid, where $l_{mfp}/R \ll 1$

H.J. Drescher, Borghini, Dumitru, Gombeaud, Ollitraut

$$v_2 \sim \underbrace{v_2|_{ideal}}_{\sim\epsilon} \left(1 - a \underbrace{\frac{l_{mfp}}{R}}_{=Kn}\right)$$

So...

$$\frac{v_2}{\epsilon} \simeq \underbrace{\frac{v_2}{\epsilon}\Big|_{ideal}}_{constant} \frac{1 + O(Kn^2)}{1 + \frac{Kn}{K_0}}$$

K_0 is a parameter, ~ 1, dependant on the microscopic details of the DoFs.
Calculated from a kinetic model to be ~ 0.7

H.J. Drescher, Borghini, Dumitru, Gombeaud, Ollitraut

Particle density n in a Bjorken hydro goes as

$$n \sim \frac{1}{\tau S}\frac{dN}{dy}$$

The initial mean free path is related to the cross-sectional area and density

$$l_{mfp} = \frac{1}{\sigma n(\tau_0)}$$

while τ_0 is the time-scale for v_2 building up

$$\tau_0 \sim \frac{R}{c_s}$$

Thus

$$Kn^{-1} = \frac{\sigma}{S}\frac{dN}{dy}c_s \qquad \frac{v_2}{\epsilon} = \frac{Kn^{-1}}{Kn^{-1} + K_0^{-1}}$$

and we recovered the Heiselberg-Levy scaling within hydro

But...

- Does scaling at AGS/SPS/RHIC energies imply that l_{mfp}, c_s, σ is unchanged at all regimes? QGP@AGS?!?!

- There is no such thing as $\frac{v_2}{\epsilon}\Big|_{ideal}$. What about...
  - Initial c_s/EoS($\sim QGP$at some energies, $\sim HG$ at others, mixed at others)
  - Initial longitudinal structure? (Bjorken/Landau/...?)

- dependance of $Kn = \frac{l_{mfp}}{R}$on energy/system size is not trivial: l_{mfp} depends on intensive quantities such as initial energy density, R is of course extensive. Why is scaling so simple over such a wide range of energies/system sizes?

What can v_2 depend on?

- On the initial eccentricity $\epsilon = \frac{x+y}{x-y}$
- On the system size ($R, \epsilon R, \langle z \rangle \ (\sim fm)$) compared to the mean free path $\frac{l_{mfp}}{R}, \epsilon\frac{l_{mfp}}{R}, \frac{l_{mfp}}{z}$(NB: Mean free path is simply a scale separating microscopic and bulk systems)
- On the stiffness of the initial EoS, ie the initial speed of sound c_s
- Mean free path and c_s depend on the initial T, which in turn should $\sim \sigma = \sqrt{s}A^{1/3}$

NB: For a fluid, rather than a system of interacting particles, all the quantities in blue are < 1 , so we should be able to expand

This is all... just about

There is also the system lifetime τ_l , which in turn depends on $T_{freeze-out}/T_{initial} < 1$ Probably a secondary factor since v_2 is sensitive to initial dynamics, but don't discount it a priori: v_2 formation time could be longer for more Landau-like systems

So...

- Since hydro is deterministic, any dimensionless function, such as $v_2, \frac{dN}{dy}$ can be regarded as a (complicated, transcendental) function of the initial conditions, i.e. of initial $\epsilon, R, \langle z \rangle, l_{mfp}, c_s, T$

- If function is integrable (no turbulence) and continuus, we can expand around any parameter reliably < 1, i.e. $\epsilon, \frac{l_{mfp}}{\langle z \rangle}, \frac{l_{mfp}}{R}, c_s, \frac{T_{freeze}}{T_0}$

A "general" reason for plotting $\frac{v_2}{\epsilon}$ vs $\frac{1}{S}\frac{dN}{dy}$

$\epsilon < 1, \ll 1$ for more central collisions, so

$$v_2 \sim a_{100}\epsilon + a_{200}\epsilon^2 + a_{101}\epsilon c_s + \epsilon\left(a^R_{110}\frac{l_{mfp}}{R} + a^z_{110}\frac{l_{mfp}}{\langle z\rangle} + \right) + \epsilon c_s\left(a^R_{111}\frac{l_{mfp}}{R} + a^z_{111}\frac{l_{mfp}}{\langle z\rangle}\right) +$$

Where $T_i(\sigma)$is the initial temperature, $R(A)$is the nuclear radius ($\sim$ transverse initial size), $\langle z\rangle$ ($\mathcal{S}(\sqrt{s})$) is the longitudinal structure, and m, n are any powers. R and ϵ depends only on A and centrality,c_s and l_{mfp} on σ,$\langle z\rangle$ on $\sqrt{s}$. The coefficients a_{ijk} are transcendental functions of the boundary conditions ($\sim \langle z\rangle^n R^m T^{n+m}$.)

How about $\frac{1}{S}\frac{dN}{dy}$?

Isenthropically it is a measure of initial $n(T, \mu_B)$ and $\langle z \rangle$

$$\frac{1}{S}\frac{dN}{dy} \sim \langle z \rangle \, n$$

If $l_{mfp} \neq 0$ Additional entropy ($\sim$ multiplicity) will be created

$$\Delta S \sim \int dt \eta \left\langle \partial_\mu u_\nu \right\rangle^2 \underbrace{\frac{V(t)}{T}}_{\sim \langle z \rangle}$$

Viscosity is in turn proportional to l_{mfp} and, again, to n

$$\eta \sim l_{mfp} n(T, \mu_B) \left\langle p \right\rangle (T, \mu_B)$$

So viscous effect on $\frac{1}{S}\frac{dN}{dy} \sim \langle l_{mfp} \rangle \, n$

$$\frac{1}{S}\frac{dN}{dy} \sim \langle z \rangle \, n(T, \mu_B) \left(1 + b^R_{010} \frac{l_{mfp}}{R} + b^{\langle z \rangle}_{010} \frac{l_{mfp}}{\langle z \rangle} + \right.$$

$$\left. \epsilon \left(b^R_{110} \frac{l_{mfp}}{R} + b^{\langle z \rangle}_{110} \frac{l_{mfp}}{\langle z \rangle} \right) + \, c_s \left(b^R_{011} \frac{l_{mfp}}{R} + b^{\langle z \rangle}_{011} \frac{l_{mfp}}{\langle z \rangle} \right) + \ldots \right)$$

Similarly, $\frac{1}{S}\frac{dN}{dy}$ can be expanded around the same quantities, except for the dimensionful scale $\langle z \rangle \, n(T, \mu_B)$ in front of all prefactors.
(b_i can also be expanded around $\frac{l_{mfp}}{R}(t_0), \frac{l_{mfp}}{\langle z \rangle}(t_0), c_s(t_0)$)

Once again, b_{ijk} are transcendental functions of the boundary conditions.
A priori, b_{ijk} and a_{ijk} should not be strongly inter-dependent.

Impose boost-invariance, so

- $\frac{1}{S}\frac{dN}{dy} \sim \tau n$ (Bjorken formula)
- Get rid of the $\langle z \rangle$ dependence. $\langle z \rangle$ and $\mathcal{S}$ is eliminated, stopping is constant along rapidity.
- $T_{initial}$ is a function of the (known) transverse nuclear density, (known) σ and (constrainable) τ_0 (start of hydro)

Use your boost invariant code to find out that $a_2 \ll a_1$ at all energies, so $\frac{v_2}{\epsilon} \sim$ constant.
RHIC reaches this $\Rightarrow$ Perfect fluid!

Question: What happens at lower energies?

(a) l_{mfp}increase
(How close to "hydro" or "Heisenberg-Levy" limit? How sudden is the transition between the two?)

(b) Does the system go away from the Bjorken limit
(Transition between "Firestrak"/Landau,BGK/Bjorken,Color glass,...)
(Test of lifetime, since $\lim_{t\to\infty}$ Landau=Bjorken)

(c) Less life-time, so v_2 has less time to form?

(d) All of the above,+...

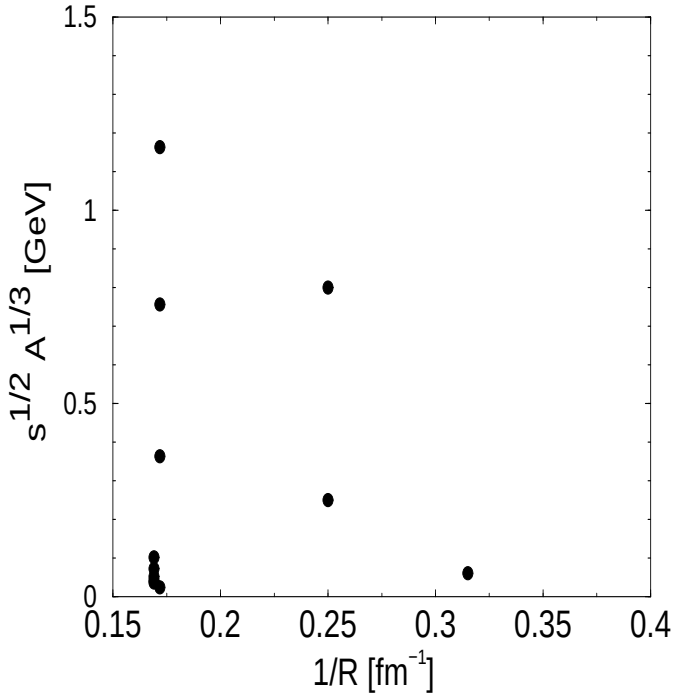
- As you move in centrality,system size you affect R, ϵ and more weakly ($\sim A^{1/3}$) intensive quantities (c_s, l_{mfp})

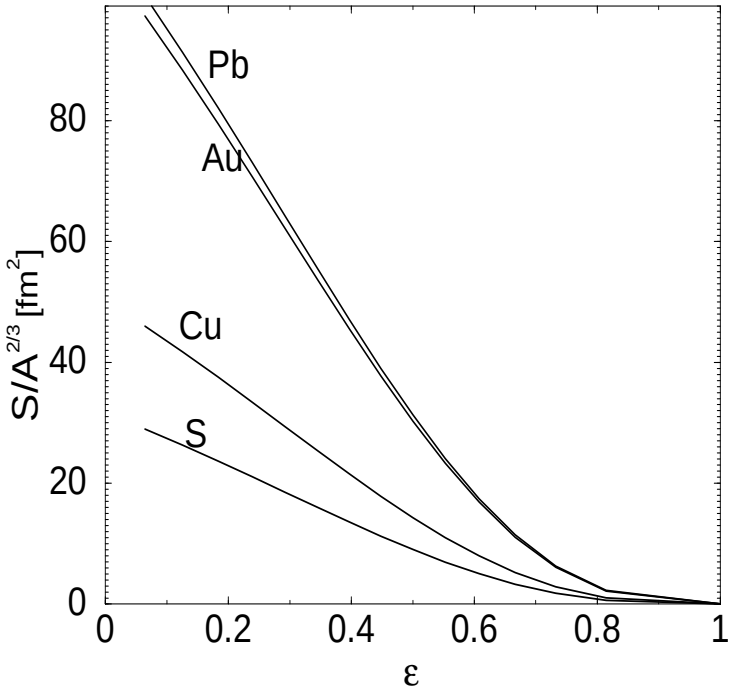
- As you move in $\sqrt{s}$, you affect $T, \langle z \rangle , l_{mfp}$, but not the initial system size

- Moving up in η probably (?) equivalent to moving down in R .

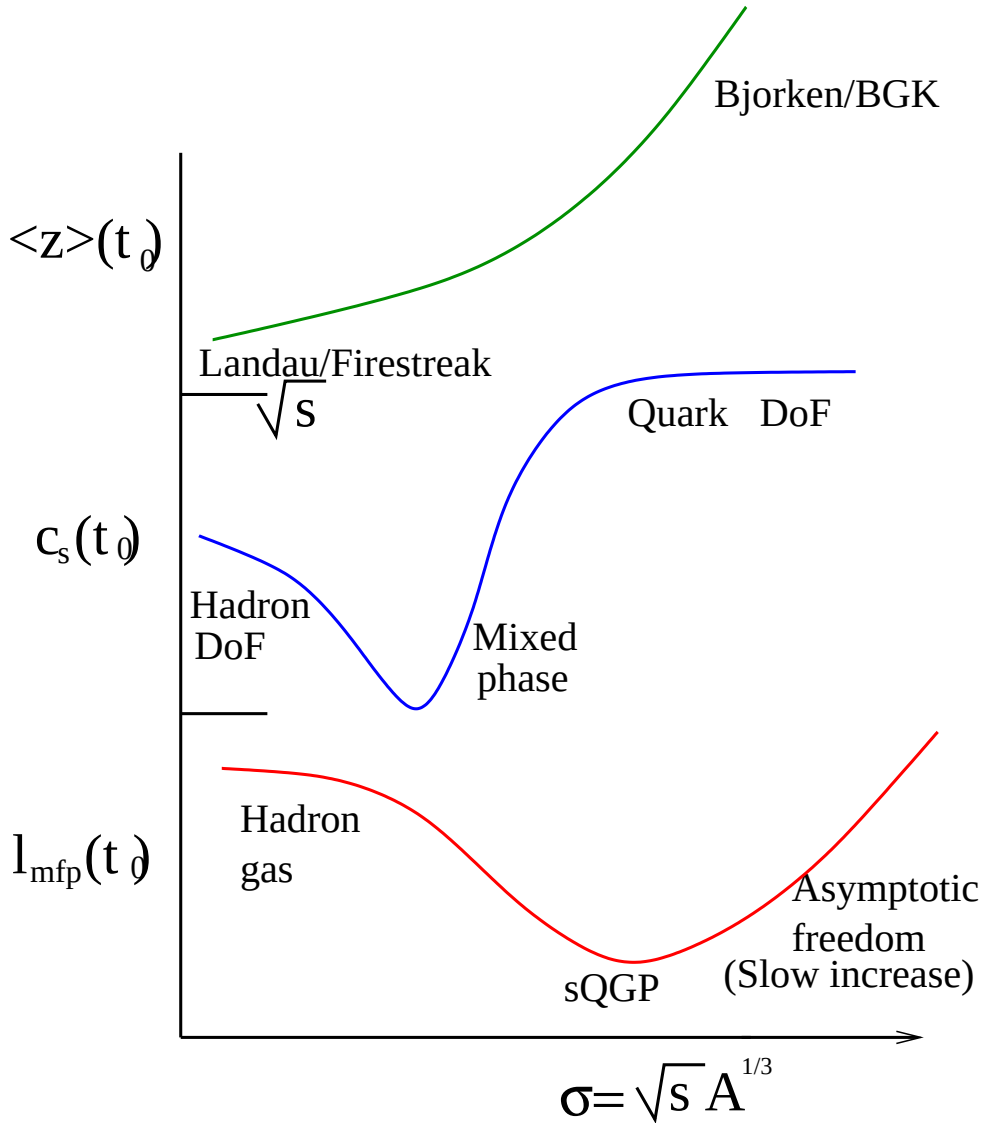
- Moving in p_T,particle species influenced by final state, so lifetime.

The range of parameter space explored by data so far





Scaling persists, through all explored $\sqrt{s}, A, b, \eta, p_T$, species parameter space! Is this "natural"?


Bjorken/BGK
<z>(t0)
Landau/Firestreak
√s
Quark DoF
cs(t0)
Hadron
DoF
Mixed
phase
Hadron
gas
lmfp(t0)
Asymptotic
freedom
(Slow increase)
sQGP
σ= √s A1/3

Qualitative effects of these changes
Increase of (everything else constant):

c_s v_2/ϵ increases (stiffer EOS), $\frac{1}{S}\frac{dN}{dy}$ unaffected

l_{mfp} v_2/ϵ decreases (viscosity), $\frac{1}{S}\frac{dN}{dy}$ increases (entropy production)

$\langle z \rangle$ Both v_2/ϵ and $\frac{1}{S}\frac{dN}{dy}$ go down (more diluited)

In general, scanning in available phase space affects v_2/ϵ,$\frac{1}{S}\frac{dN}{dy}$ in different ways

Lets test naturalness

- Lattice parametrization for c_s
  $c_s = \frac{0.1}{2}\left(1 + 0.9\tanh(T_c - T)\right) + \frac{c_s^{ideal}}{2}\left(1 + \tanh(T - T_c)\right)$

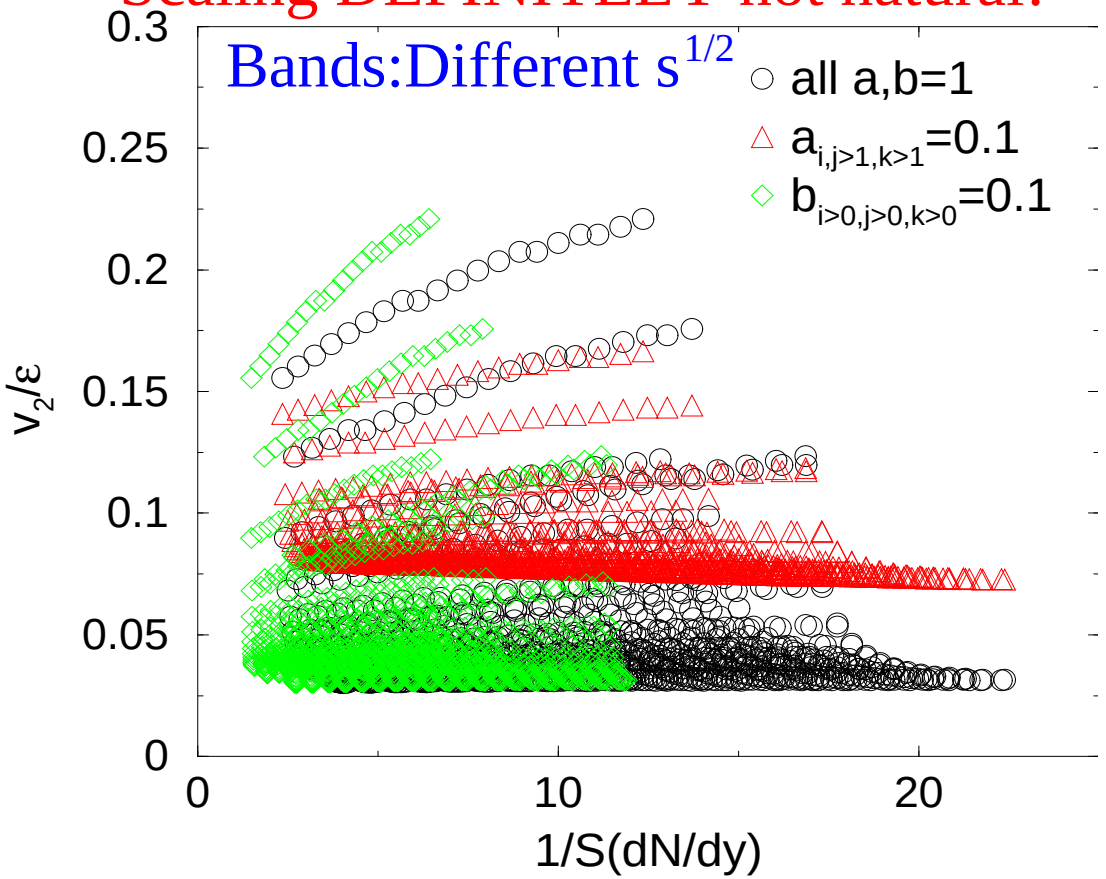
- "reasonable" parametrization for l_{mfp}(large at $T \ll, \gg T_c$, small at $\sim T_c$) $\frac{l_{mfp}}{fm} = \frac{5}{2}\left(1 + \tanh(T_c - T)\right) + \frac{0.1}{2}\left(1 + \tanh(T - T_c)\right) * \log\left(1 + \frac{T}{T_c}\right)$

- connect T and dN/dy by Bjorken formula
  $\frac{1}{S}\frac{dN}{dy} \sim \frac{1}{\pi A^2}\frac{dN}{dy} = \tau n(T, \mu_B)$

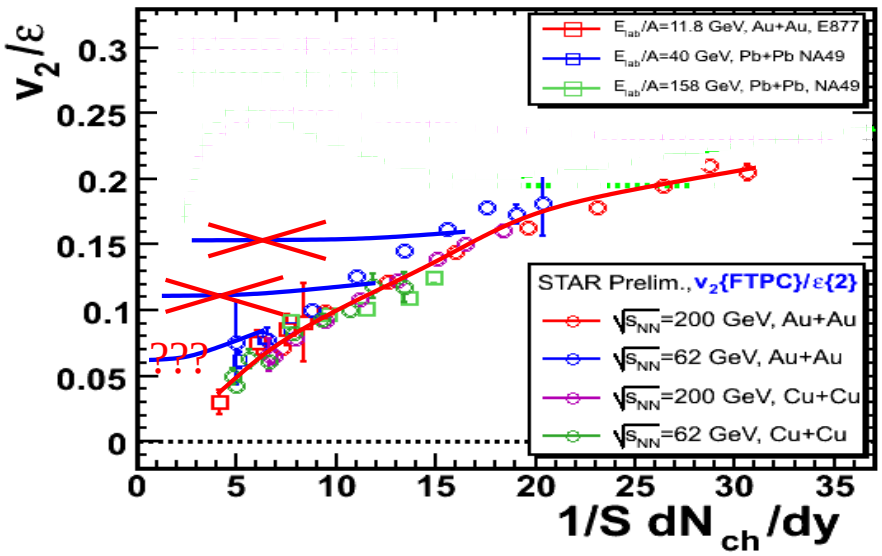
- use phenomenological formula to get dN/dy as a function of energy
  $\frac{dN}{dy} = \frac{N_{Participants}}{1.48}\ln\left(\frac{\sqrt{s}}{1.48\mathrm{GeV}}\right)$

- Plug into the previous expansion equations
- "Smallness" contained in parameters we expand around, so $a, b \sim 1$ (But put anything from 0.1 to 10)

Scaling DEFINITELY not natural!



3D viscous hydro should look like a dogs dinner



Except perhaps at small ($\sim$ horn?) energy, no hint of deviation

This,experimentally, says a lot!

Unless "miraculous" unnatural cancellations...

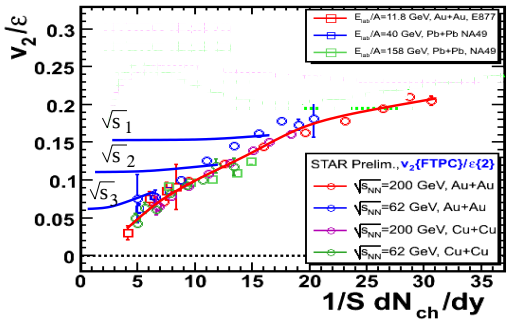
$$v_2 \sim a_1 \left(\underbrace{\frac{l_{mfp}}{R}}_{No\sim A}, \underbrace{\frac{l_{mfp}}{\langle z \rangle}}_{No\sim\sigma}, \underbrace{\frac{l_{mfp}}{\tau_l}}_{Must\sim\frac{1}{S}\frac{dN}{dy}}, , \underbrace{\sum_{lmn} \zeta_{lmn}}_{Must\sim\frac{1}{S}\frac{dN}{dy}}, \underbrace{c_s}_{No\sim\sigma} \right) \epsilon + ...$$

$$\frac{1}{S}\frac{dN}{dY} \sim \underbrace{\langle z \rangle \, n(T,\mu_B)}_{no\sim\sigma} \left(1 + b_1(\underbrace{\sum_{lmn} \zeta_{lmn}}_{Must\sim\langle z\rangle n} , c_s, ...) \underbrace{\frac{l_{mfp}}{z}}_{No\sim\sigma} + b_2(...) \underbrace{\langle z \rangle \frac{l_{mfp}}{R}}_{No\sim\sigma, A} + b_3(...) \underbrace{\frac{l_{mfp}}{\tau}}_{No\sim\sigma} \right) ...$$

BUT...

$\frac{v_2}{\varepsilon} = F_1(A, \sqrt{s})$

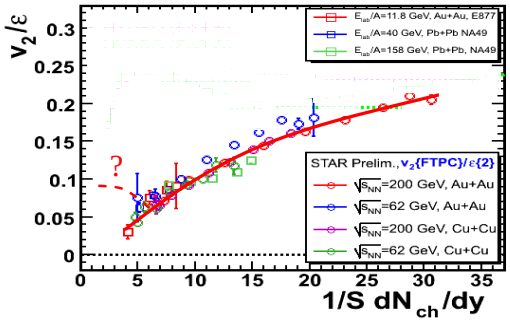
$\frac{dN}{S\,dy} = F_2(A, \sqrt{s})$



$\frac{v_2}{\varepsilon} = F_1(z)$

$\frac{dN}{S\,dy} = F_2(z)$

$z = F(A, \sqrt{s})$



The scaling of $\frac{v_2}{\epsilon}$ vs $\frac{1}{S}\frac{dN}{dy}$ means they are connected by one variable.

But "a priori" this should not be true: Extensive $\epsilon, \frac{1}{R}$ and intensive c_s, l_{mfp} enter the Taylor expanded equations independently

We knew (Becattini,...) of a "universal thermal $T = 160$ MeV" distribution in hadronic processes. Is there a "universal hydro" also?
$0th$ term in $\frac{1}{S}\frac{dN}{dy}$, $\langle z \rangle\, n(T, \mu_B)$ restricts a, b to be only $\sim \langle z \rangle\, n(T, \mu_B)$.

- There is a natural "unit of measure within hadronic systems Λ (Λ_{QCD}?)
- Dimensionful intensive quantities such as $l_{mfp} \sim$ this scale (Λ^{-1}?) indipendently from reaction size
- Increase of v_2/ϵ and $\frac{1}{S}\frac{dN}{dy}$ controlled by global "size" parameter (τ) in unites of this scale

  $$\tau\Lambda \sim RT$$

  Energy, system size and centrality "conspire" to produce one size!
  (Connected to $\frac{dN}{dy} \sim \ln\sqrt{s}$ scaling?)

Scaling constrains (Rules out?) Changes in intensive quantities (c_s, l_{mfp})

The expansion of v_2 and $\frac{1}{S}\frac{dN}{dy}$ considerably simplifies

$$v_2 \sim \epsilon\left(a_{100} + a_{101}c_s + a_{102}c_s^2 + ...\right) + \frac{\Lambda^{-1}}{\langle\tau\rangle}\epsilon\left(a_{110} + a_{111}c_s + ...\right) + \left(\frac{\Lambda^{-1}}{\langle\tau\rangle}\right)^2 \epsilon\left(a_{120} + a_{121}c_s + ...\right) + ...$$

$$\frac{1}{S}\frac{dN}{dy} \sim F(\langle\tau\rangle)\left(1 + b_{010}\frac{\Lambda^{-1}}{\langle\tau\rangle} + b_{011}\frac{c_s\Lambda^{-1}}{\langle\tau\rangle} + b_{200}\left(\frac{\Lambda^{-1}}{\langle\tau\rangle}\right)^2 + ...\right)$$

But getting a family of (3D viscous) hydro initial conditions, that to a good approximation spans different energies and system sizes, is non-trivial

Possibility I

(Dumitru, H.J. Drescher, Borghini, Ollitrault).

scaling variable is really $\frac{l_{mfp}}{R}$.

This means c_s, $\langle z \rangle$ really are about the same throughout the whole domain of validity of the scaling (Bjorken QGP at AGS, with comparable mean free path??).

Possibility II

(Gyulassy, Hirano)

Scaling parameter is fraction of system in "core" (presumably QGP) phase.

Problem:At no range of observed parameter space can this fraction approach 0, 1, otherwise breaking of scaling in that region likely.

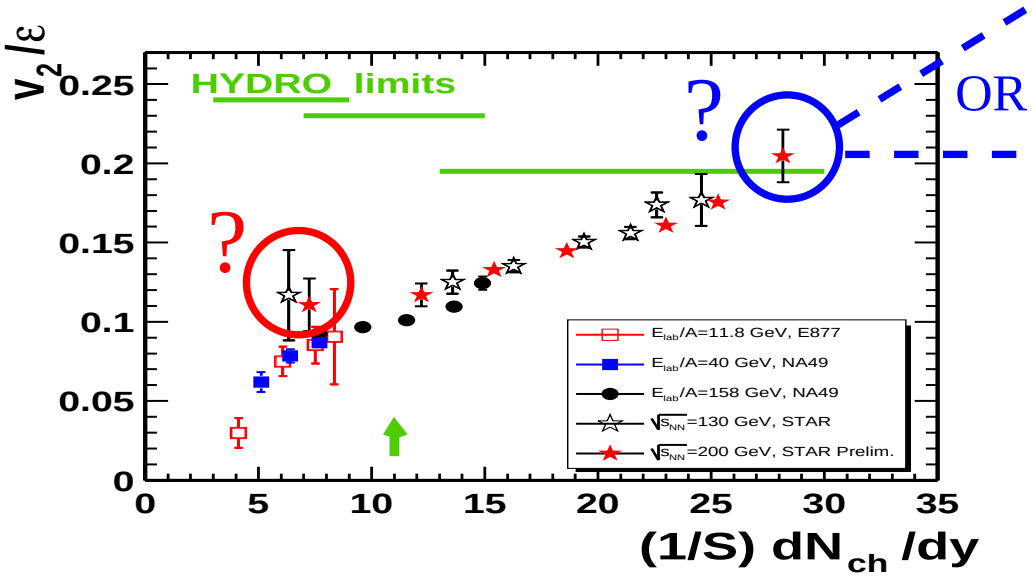
Possibilty III

We've got it all wrong and Heiselberg-Levy scaling holds at all energies. Unlikely because...

- Large value of v_2 at high energies unlikely
- Does not solve the question. Scaling holds as long as $\langle \sigma v \rangle$ between degrees of freedom approximately constant.
  Where's the phase transition/crossover?!

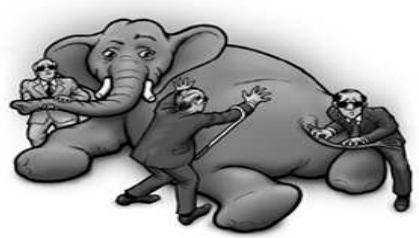
- Finding breaking of this scaling, at low energies, would be invaluable. Has it been found? (horn?)

- LHC,U-U will reveal if v_2/ϵ saturates at Ideal Boost-invariant value.



3-D viscous hydro needed to understand the coefficients $a_{ijk}, b_{ijk}(R,T)$

Scaling break~Horn/Kink/Step~HBT scaling change



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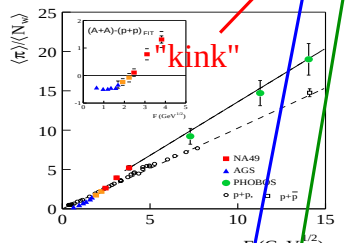
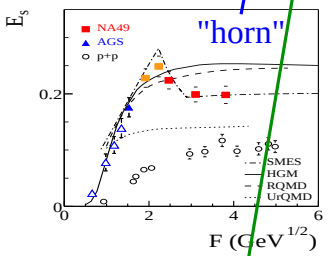
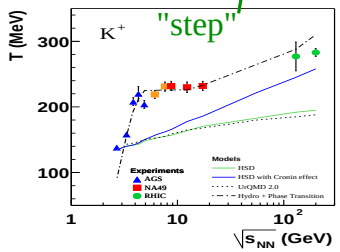
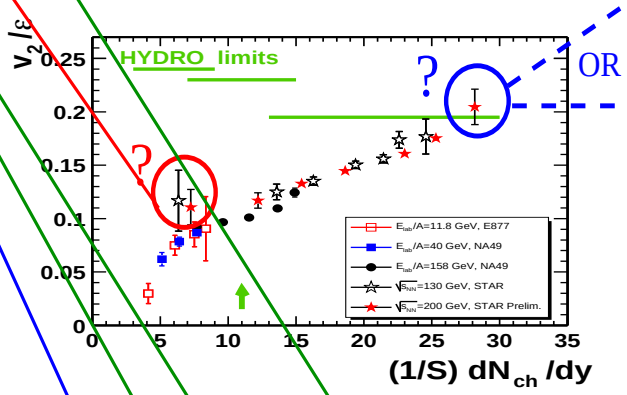
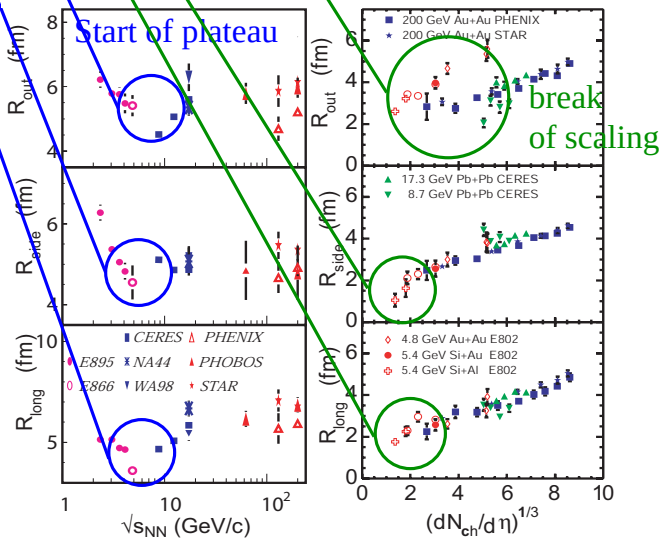










What next... some more "non-conformist" thoughts

- v_2 fluctuations (See M. Bleicher's talk)
- Correlations (see S. Gavin's talk)
- Polarization (B.Betz, M. Gyulassy, G. Torrieri, coming shortly)
  - reaction plane polarization probes longitudinal initial conditions $\langle z \rangle$
  - Ideal hydro locally isotropic... any local polarization (Reaction plane, Production plane, Jet plane...) proportional to the mean free path! l_{mfp}

$$P_q^i \sim \tanh\left[\frac{l_{mfp}}{T}\left(\epsilon_{ijk}\frac{d\langle \vec{p}_i \rangle}{d\vec{x}_j}\right)\right]$$

Many systematic uncertainites (initial state, vorticity propagation, freeze-out). But also many polarization directions.

(very few!!!) conclusions

- There is evidence of very good scaling between $\frac{v_2}{\epsilon}$ vs $\frac{1}{S}\frac{dN}{dy}$ over a wide range of $\sqrt{s}, A, N_{part}, \eta, p_T$

- This scaling holds for
  - A weakly interacting system
  - An ideal fluid coming from a uniform family of initial temperatures and longitudinal distributions

  A non-ideal 3D fluid would most likely break it!
  So if RHIC is an ideal fluid, what kind of fluid do we have at AGS, SPS?!

Conclusions II: A way out?

Experimentalists Should look for conditions, if any, where this scaling breaks.

Theorists , especially of the hydro variety, should explore the "naturalness" of the scaling further within a general family of EoS and initial conditions. Expansion technique presented here a suitable technique to do it.

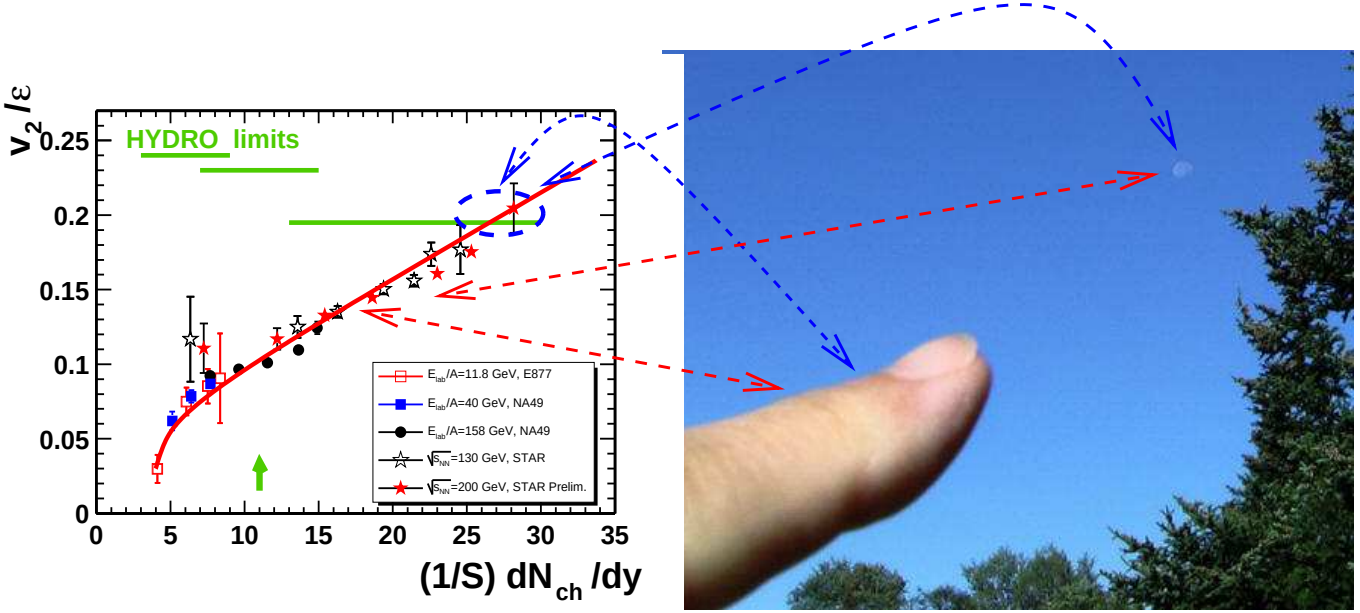
Other theorists Should suggest alternative ways to measure viscosity and/or initial conditions

Everyone Should look at the scaling, or lack of, of their favourite observable or explanation thereof.

Lest we risk becoming fools!

When the finger points to the moon, the fool looks at the finger.

Chinese proverb



Whats the moon here? Is it the meeting of the v_2/ϵ line with the predictions of ideal boost-invariant hydro? Or is it, rather, the existance of the line?