# How a “minimal” viscosity affects differential elliptic flow

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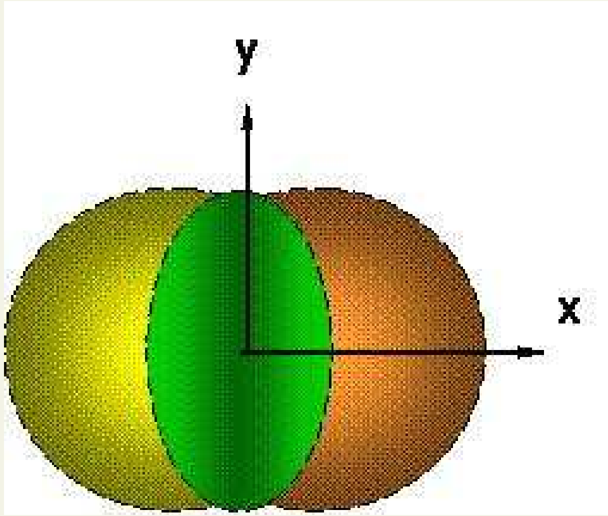
- Thermalization question
  - can pQCD rates do it at RHIC?
  - what if we have the highest possible rates?

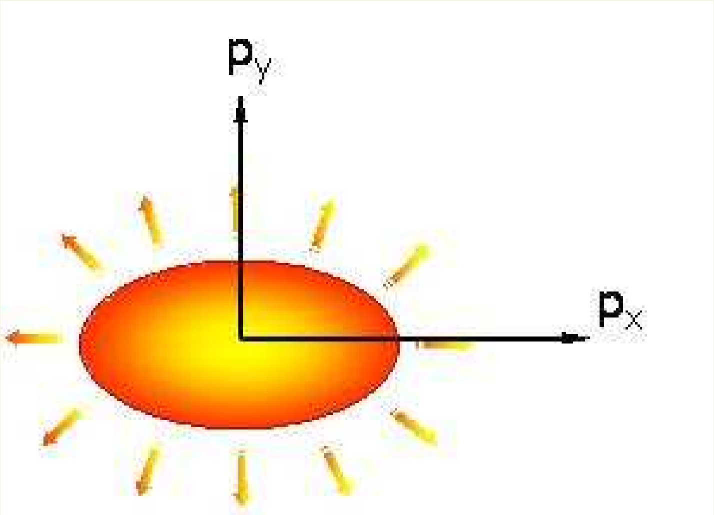
# to what degree QCD matter thermalizes in a RHIC collision?

**local equilibrium POSTULATE quite successful**
**but need to understand equilibration dynamics** Gyulassy, Pang, Zhang, DM...

- **one measure - "elliptic flow"** $(v_2)$




$\longrightarrow$

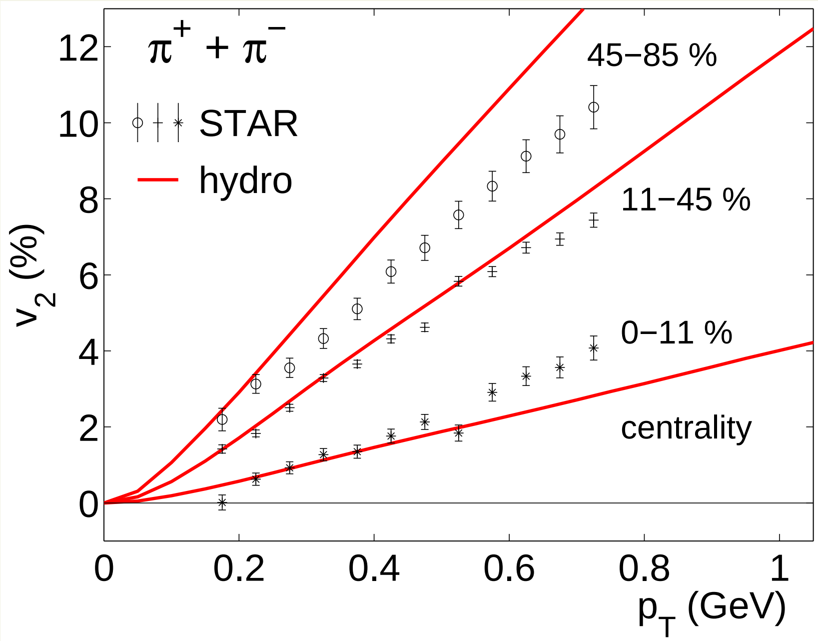



$$\varepsilon \equiv \frac{\langle x^2 - y^2 \rangle}{\langle x^2 + y^2 \rangle}$$

$$v_2 \equiv \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle}$$

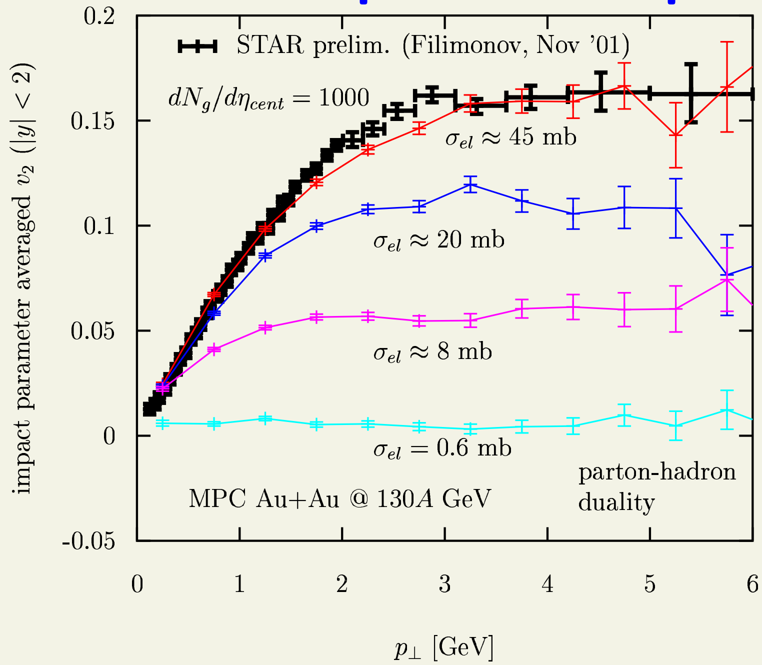
Kolb, Heinz et al

## ideal hydrodynamics



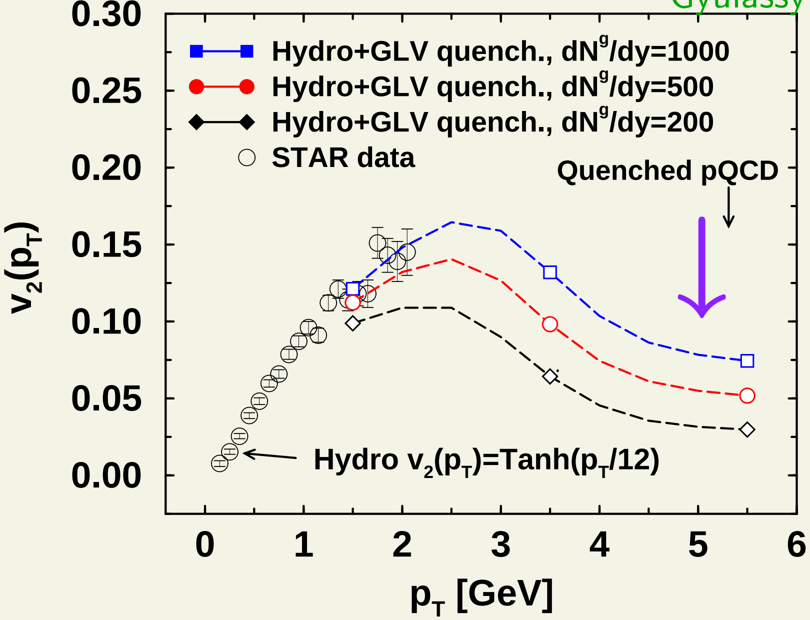

DM & Gyulassy

## covariant parton transport



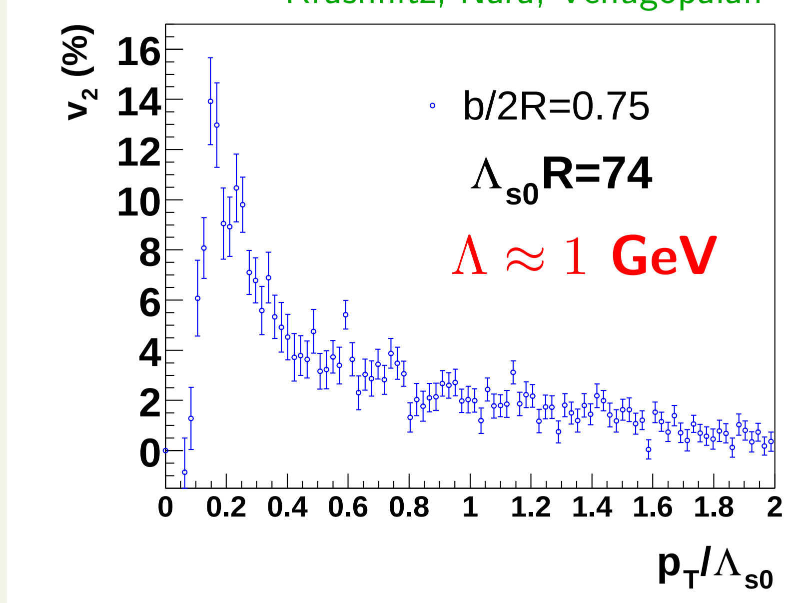

## parton energy loss...

Gyulassy & Vitev




## classical Yang-Mills ...

Krashnitz, Nara, Venugopalan



# Covariant transport

Boltzmann ..., Israel, Stewart, de Groot, ... Pang, Zhang, Gyulassy, DM, Vance, Csizmadia, Pratt, Cheng, Xu, Greiner ...

**Covariant, causal, nonequil. approach - formulated in terms of local rates.**

$$\Gamma_{2\to 2}(x) \equiv \frac{dN_{scattering}}{d^4x} = \sigma v_{rel} \frac{n^2(x)}{2}$$

**transport eqn.:** $f_i(\vec{x}, \vec{p}, t)$ - **phase space distributions**

$$p^\mu \partial_\mu f_i(\vec{x}, \vec{p}, t) = \overbrace{S_i(\vec{x}, \vec{p}, t)}^{\text{source}} + \overbrace{C_i^{el.}[f](\vec{x}, \vec{p}, t)}^{2 \to 2 \text{ (ZPC, GCP, ...)}} + \overbrace{C_i^{inel.}[f](\vec{x}, \vec{p}, t)}^{2 \leftrightarrow 3 \text{ (MPC, Xu}-\text{Greiner)}} + \ldots$$

**algorithms: OSCAR code repository @ http://nt3.phys.columbia.edu/OSCAR**

**HERE**: **utilize MPC algorithm** DM, NPA 697 ('02)

## rate is a local and manifestly covariant scalar

**for particles with momenta $p_1$ and $p_2$:**

$$\Gamma(x) = \sigma\, v_{rel}\, n_1(x) n_2(x) = \sigma\, \frac{\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}}{E_1 E_2}\, n_1(x) n_2(x)$$

**(n/E is a scalar)**

**an equivalent alternative form is** $v_{rel} = \sqrt{(\vec{v}_1 - \vec{v}_2)^2 - (\vec{v}_1 \times \vec{v}_2)^2}$

**[ in cascade algorithms, rate is evaluated in the pair c.o.m. frame, where $\vec{v}_1 || \vec{v}_2$ and thus $v_{rel} = |\vec{v}_1 - \vec{v}_2|$ ]**

# Example: Molnar's Parton Cascade

**Elementary processes:** elastic $2 \to 2$ processes + $gg \leftrightarrow q\bar{q}$, $q\bar{q} \to q'\bar{q}'$ + $ggg \leftrightarrow gg$

**Equation for** $f^i(x, \vec{p})$: $\quad i = \{g, d, \bar{d}, u, \bar{u}, \ldots\}$

$2 \to 2$

$$p_1^\mu \partial_\mu \tilde{f}^i(x, \vec{p}_1) = \frac{\pi^4}{2} \sum_{jkl} \int_2 \int_3 \int_4 \left(\tilde{f}_3^k \tilde{f}_4^l - \tilde{f}_1^i \tilde{f}_2^j\right) \left|\bar{\mathcal{M}}_{12\to 34}^{i+j\to k+l}\right|^2 \delta^4(12-34)$$

$2 \leftrightarrow 3$

$$+ \frac{\pi^4}{12} \int_2 \int_3 \int_4 \int_5 \left(\frac{\tilde{f}_3^i \tilde{f}_4^i \tilde{f}_5^i}{g_i} - \tilde{f}_1^i \tilde{f}_2^i\right) \left|\bar{\mathcal{M}}_{12\to 345}^{i+i\to i+i+i}\right|^2 \delta^4(12-345)$$

$3 \leftrightarrow 2$

$$+ \frac{\pi^4}{8} \int_2 \int_3 \int_4 \int_5 \left(\tilde{f}_4^i \tilde{f}_5^i - \frac{\tilde{f}_1^i \tilde{f}_2^i \tilde{f}_3^i}{g_i}\right) \left|\bar{\mathcal{M}}_{45\to 123}^{i+i\to i+i+i}\right|^2 \delta^4(123-45)$$

$$+ \tilde{S}^i(x, \vec{p}_1) \quad \leftarrow \textbf{initial conditions}$$

**with shorthands:**

$$\tilde{f}_i^q \equiv (2\pi)^3 f_q(x, \vec{p}_i), \quad \int_i \equiv \int \frac{d^3 p_i}{(2\pi)^3 E_i}, \quad \delta^4(p_1+p_2-p_3-p_4) \equiv \delta^4(12-34)$$

# Hydrodynamic limit

**mean free path:**

$$\lambda(x) \equiv \frac{1}{\text{cross section} \times \text{density(x)}}$$

- **Ideal fluid limit** $\lambda \to 0$: **local equilibrium**

$$T^{\mu\nu}_{id} = (e+p)u^\mu u^\nu - p\, g^{\mu\nu}$$

$$\partial_\mu S^\mu = 0 \quad \Rightarrow \text{entropy conserved}$$

- **Viscous hydro** $\lambda \ll \textit{length \& time scales}$: **near local equilibrium**

**dissipative dynamics in terms of transport coefficients and relaxation times**

$$e.g., \quad \text{shear viscosity } \eta \approx 0.8 \frac{T}{\sigma_{tr}} , \qquad \text{relaxation time } \tau_\pi \approx 1.2 \lambda_{tr}$$

Israel, Stewart ('79) ...

## two main frameworks for near-equilibrium evolution:

**causal viscous hydrodynamics** Israel, Stewart; ... Muronga, Rischke; Romatschke et al; Heinz et al...

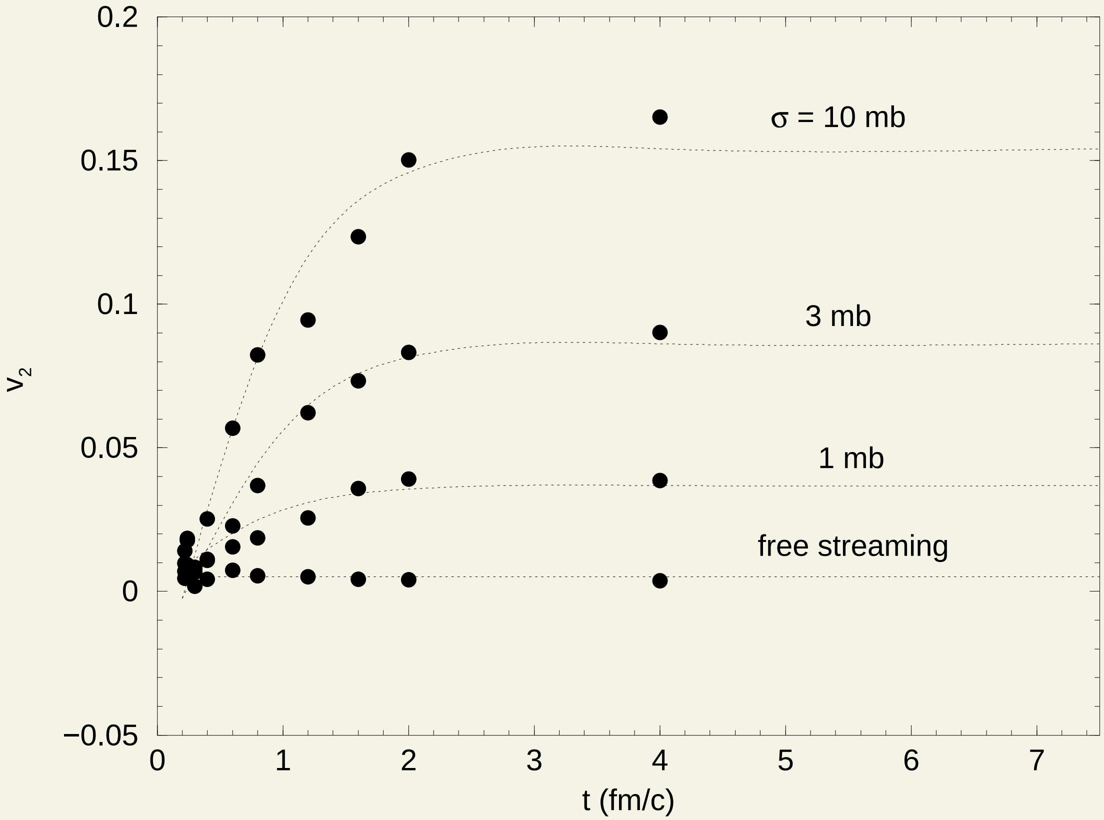
**main challenge - acausality and instability**

**covariant transport** DM

**much more difficult numerically but fully stable and causal**

## Which limit are we in at RHIC??

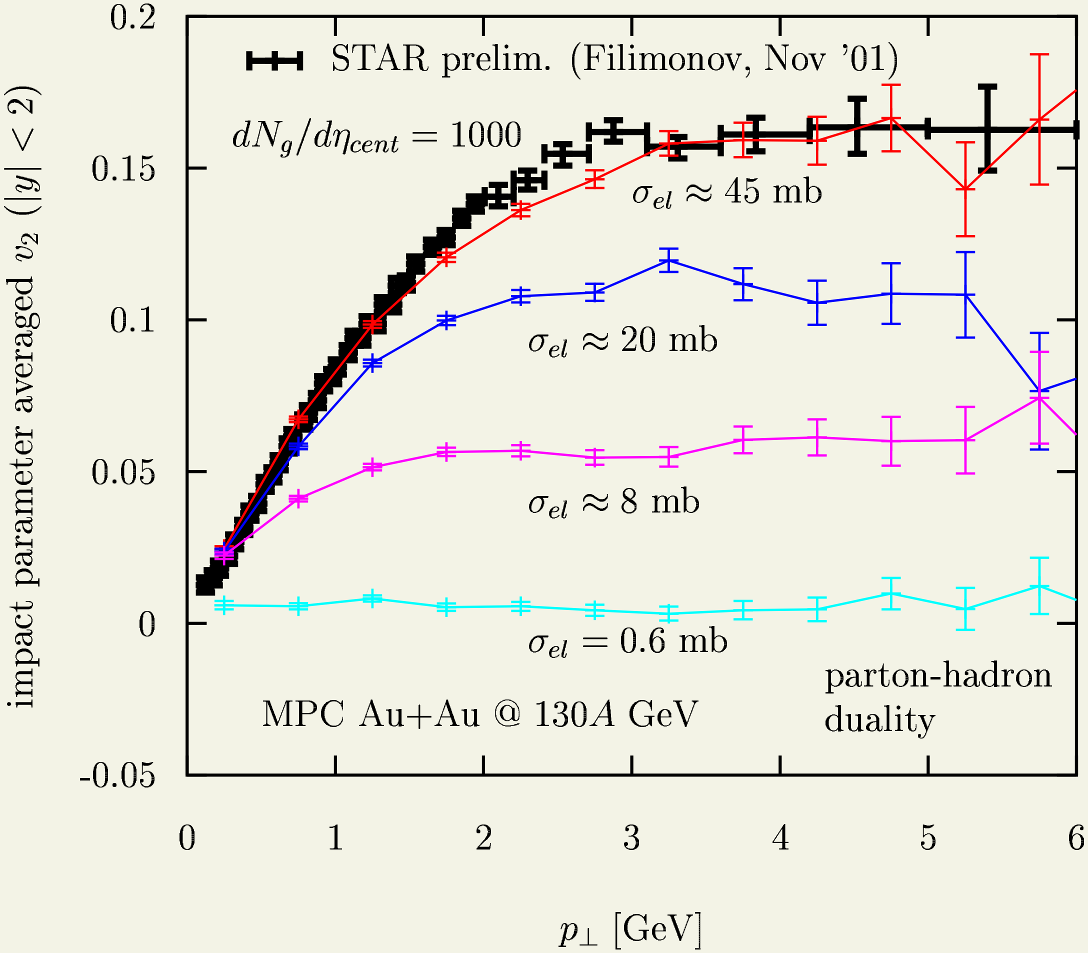
Zhang, Gyulassy & Ko, PLB455 ('99): **ZPC algorithm - proof of principle**




sharp cylinder $R = 5$ fm, $\tau_0 = 0.2$ fm$/c$, $b = 7.5$ fm, $dN^{cent}/dy = 300$

**anisotropy increases with cross section, and develops early ($\sim 1 - 2$ fm$/c$)**

DM & Gyulassy, NPA 697 ('02): **$v_2(p_T, \chi)$ at RHIC**




**parton transport model MPC**

**diffuse nuclear geometry**

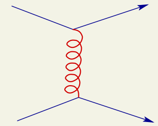
**$dN/d\eta$ based on EKRT saturation**

Au+Au @ 130 GeV, $b = 8$ fm

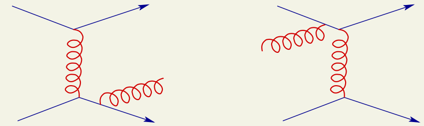
- HIJING (minijet+radiation) initconds
- binary transverse profile
- 1 parton $\longrightarrow$ 1 $\pi$ hadronization

**large RHIC $v_2$: perturbative $2 \to 2$ rates insufficient, need $15\times$ higher**

# radiative transport:

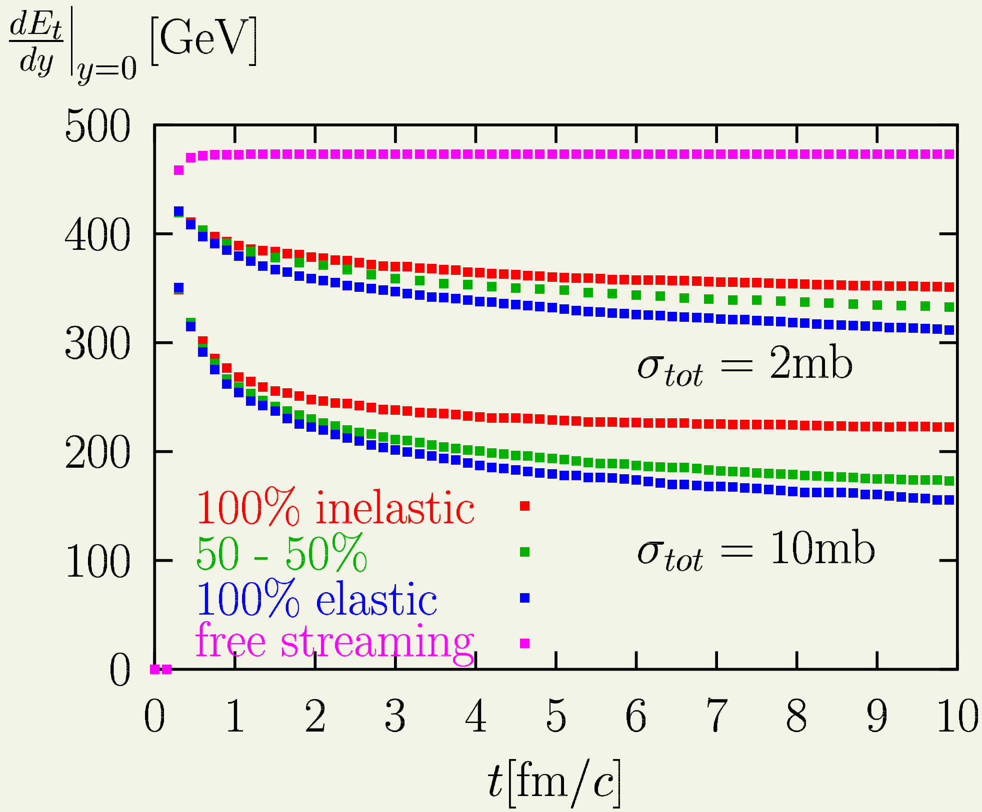


**2→2** **+**

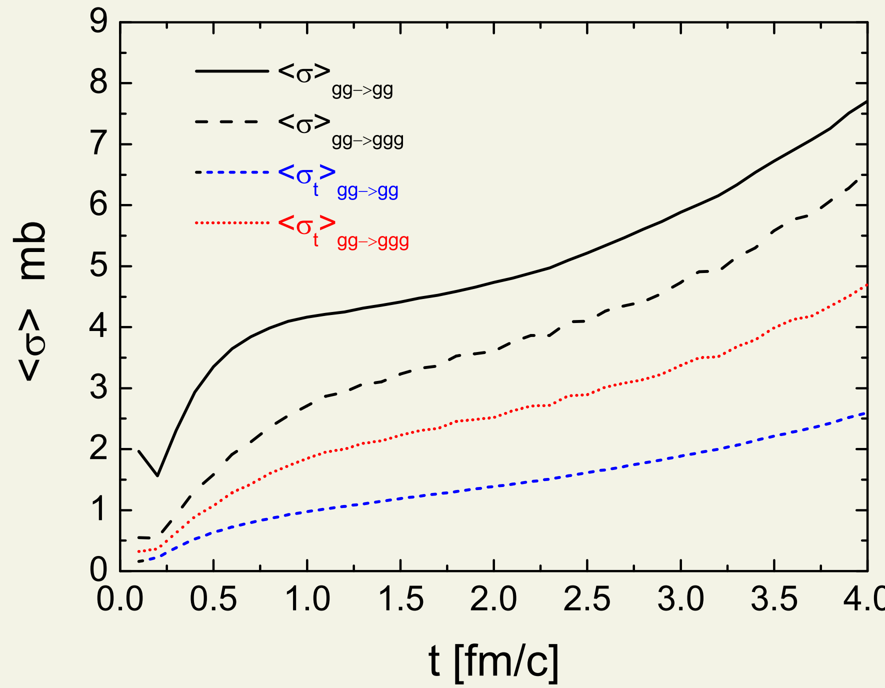


**3↔2 ...**

DM & Gyulassy, NPA 661 ('99): $p\,dV$ **cooling**




Greiner & Xu, PRC71 ('05): **transport xsec**



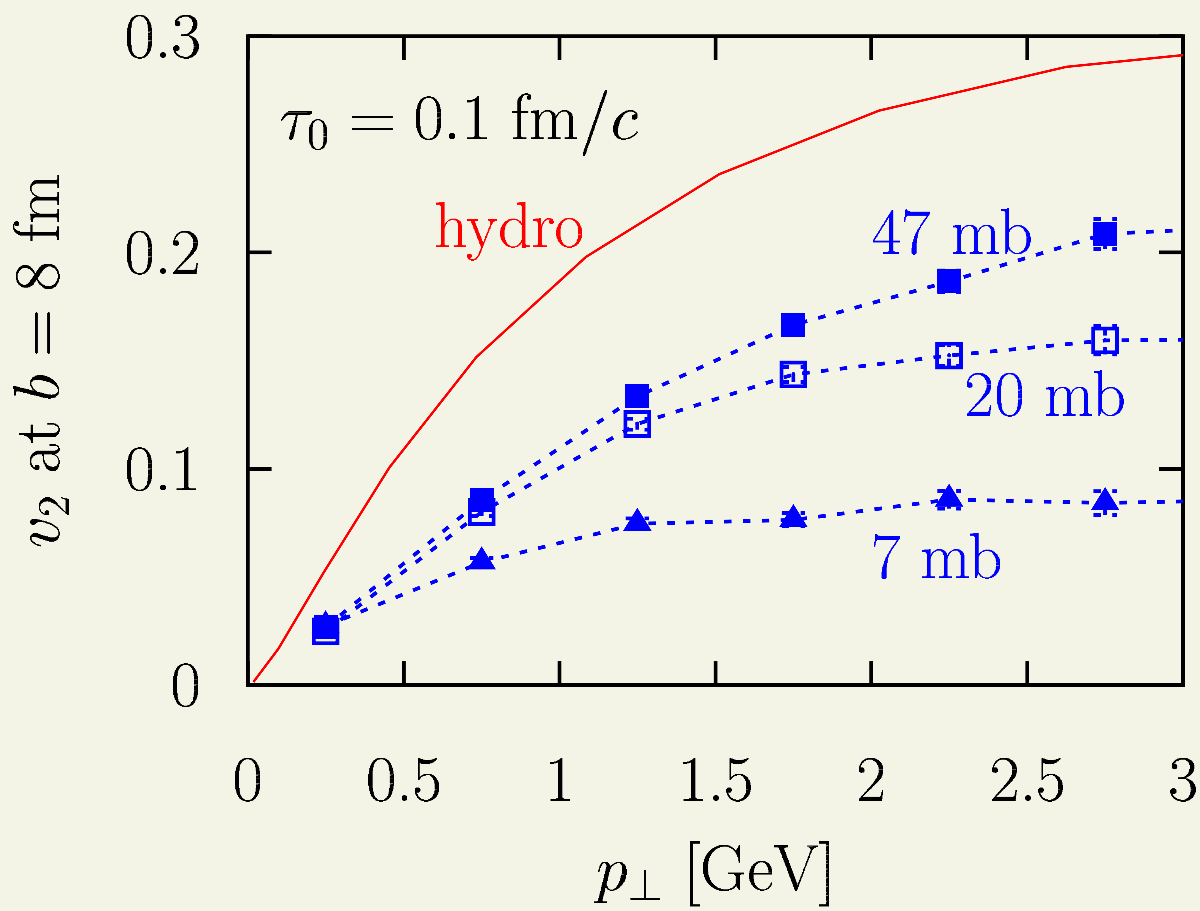

**mainly increase in $\sigma_{tr}$ matters**

**about $3\times$ larger with $3 \to 2$**

$\Rightarrow$ **big help but likely not enough** **(need $v_2(p_T)$ results)**

# No, still not ideal fluid

DM & Huovinen, PRL94 ('05): **final** $v_2(p_T)$




**large gradients**

$\Rightarrow$ **even a tiny viscosity matters**

[identical RHIC Au+Au initconds, $b = 8$ fm, binary profile, $T_0 = 0.7$ GeV, e=3p EOS]

# Classical transport rates get arbitrarily large as $\lambda_{MFP} \to 0$

**BUT, quantum mechanics:** $\Delta E \cdot \Delta t \geq \hbar/2$

**+ kinetic theory:** $T \cdot \lambda_{MFP} \geq \hbar/3$ Gyulassy & Danielewicz '85

$$\eta \approx 4/5 \cdot T/\sigma_{tr}$$

$$s \approx 4n$$

**gives minimal viscosity:** $\eta/s = \frac{\lambda_{tr} T}{5} \geq 1/15$

$\mathcal{N} = 4$ **SYM + gauge-gravity duality:** $\eta/s \geq 1/4\pi$

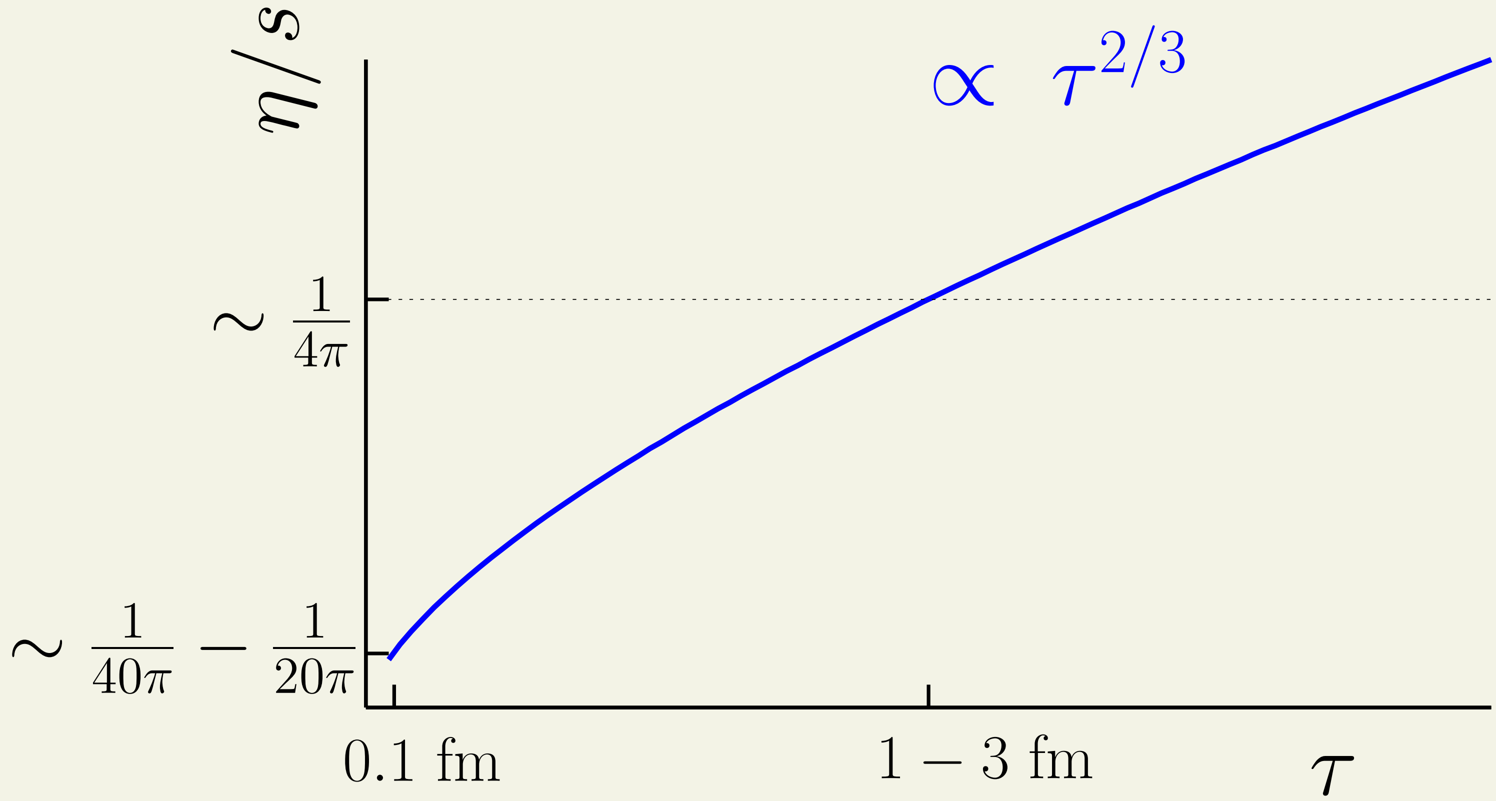
Policastro, Son, Starinets, PRL87 ('02)
Kovtun, Son, Starinets, PRL94 ('05)

**might be a universal lower bound - but general proof lacking**

$\Rightarrow$ **no ideal fluids - "most perfect" are those with minimal viscosity**

["most" is crucial - perfect $\equiv$ ideal already since '50s]

$\sigma \approx 47$ **mb dynamics corresponds to** $\eta/s \sim \lambda_{tr} T \sim 1/(\sigma T^2)$




**initially "better than perfect", after** $\tau \sim 1 - 3$ **fm "less than perfect"**

$\Rightarrow \eta/s = const$ **needs growing** $\sigma(\tau) \propto 1/T^2 \propto \tau^{2/3}$

# $\eta/s$ for transport

"minimal" viscosity - corresponds to $\lambda_{tr} \approx 1/(3T_{eff}) \approx 0.1$ fm at $\tau_0 = 0.1$ fm

**estimate from average density:** $\lambda_{tr} = \frac{1}{\langle n \rangle \sigma_{tr}}$

for $b = 8$ fm @ RHIC, transport with 47 mb gives

$\lambda_{tr}(\tau_0) = \frac{\tau_0 A_T}{\sigma_{tr} dN/d\eta} \sim 1 - 2 \times 10^{-2}$ **fm**

**estimate from transport opacity $\chi$:** assuming 1D Bjorken expansion

$\chi = \int dz\, \rho(z) \sigma_{tr} \sim \int d\tau \rho_0 \frac{\tau_0}{\tau} \sigma_{tr} = \frac{\tau_0}{\lambda_{tr}(\tau_0)} \ln \frac{L}{\tau_0}$

for $b = 8$ fm @ RHIC, transport with 47 mb gives $\chi \approx 20$

$\to \lambda_{tr}(\tau_0) \sim 1.5 - 2 \times 10^{-2}$ **fm (!)**

$\Rightarrow \sigma_{gg} \approx 50$ **mb is already better than best-case scenario**

**in fact, the perturbative QCD $\sigma_{TOT} \sim \alpha_s^2/\mu_D^2$ already has this built in, since $\mu_D = gT$!**

**although it is the transport cross section that matters,**

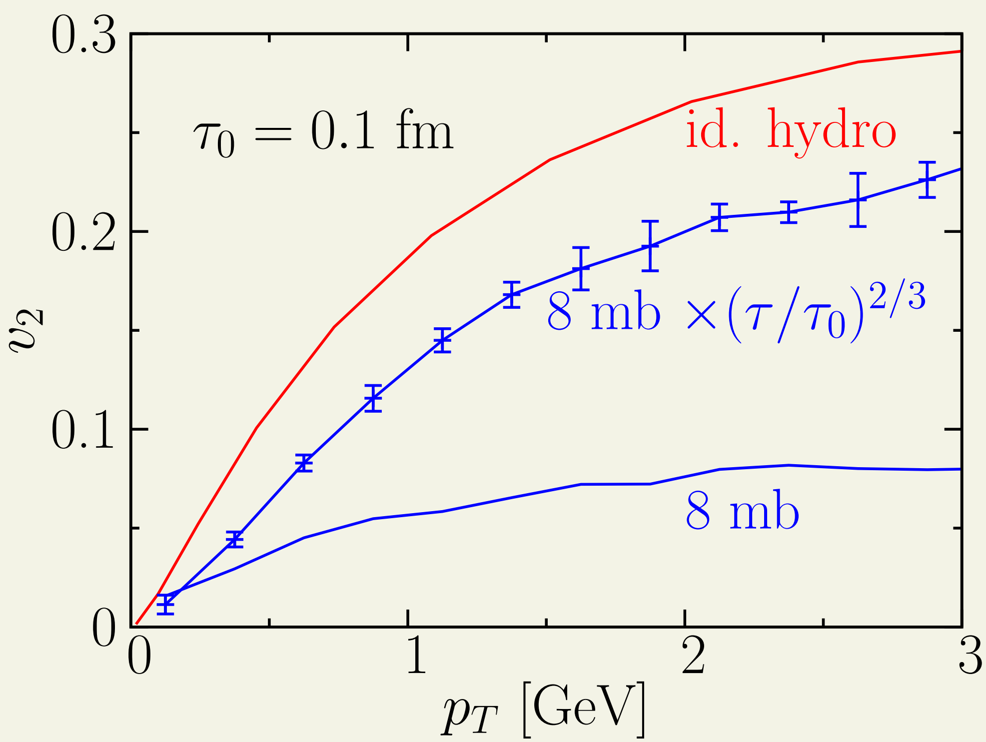
$$\sigma_{tr} \sim \frac{\alpha_s^2}{s} ln \frac{s}{\mu_D^2} \sim \frac{g^4}{T^2} ln \frac{1}{g^2}$$

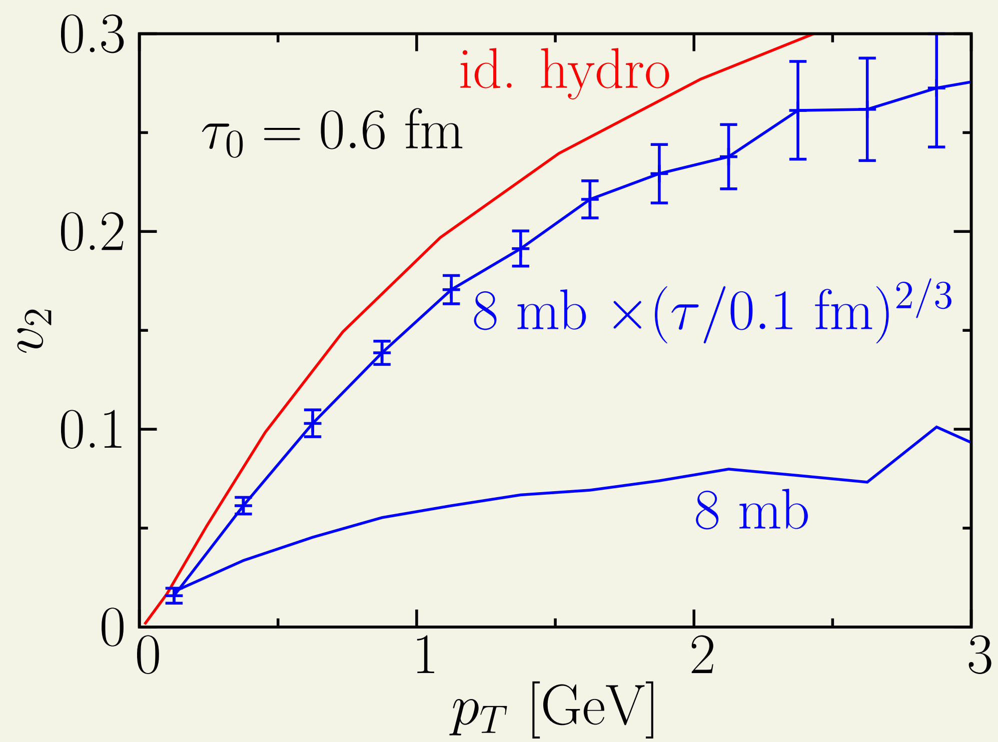
**is still proportional to $\sim 1/T^2$ for typical momenta.**

# hydro/transport RHIC comparison, now with "minimal viscosity"

$\Rightarrow \sigma_{gg}(\tau = 0.1 \text{ fm}) \sim 4 - 9$ **mb** [4 mb for center of collision zone]

DM '06: $b = 8$ **fm**




$\Rightarrow$ **still $20 - 30$% drop in $v_2$ due to dissipation, even at low $p_T$**

## Now apply this at LHC ... DM, arXiv:0707.1251

**and predict $v_2(p_T)$ for "minimum viscosity" system, i.e., maximal scattering rates**

**from a transport perspective, there are 3 relevant scales:**

$\sigma_{tr} \cdot dN/d\eta$, $T_{eff}$, **and** $\tau_0/R$

[DM & Gyulassy, NPA697 ('01)]

# RHIC vs LHC

**I. nuclear geometry identical (gold $\simeq$ lead)**

**II. larger $dN_{ch}/d\eta \sim 1200 - 2500$, highly uncertain but irrelevant(!)**

$\lambda_{tr} \propto \sigma_{tr} \cdot dN/d\eta$ **fixed by minimal viscosity requirement**

**III. higher typical momenta**

**for massless dynamics, momenta scale with initial $T_{eff}$ ($\langle p_T \rangle$, or for saturation model $Q_{sat}$ )**

**provided there are no other scales in the problem**

$\Rightarrow$ **universal $v_2(\frac{p_T}{Q_s})$, i.e.,**

$$v_2^{LHC}(p_T) \approx v_2^{RHIC}(p_T \frac{Q_s^{RHIC}}{Q_s^{LHC}})$$

**estimate $Q_s^{RHIC}/Q_s^{LHC}$ from saturation condition**

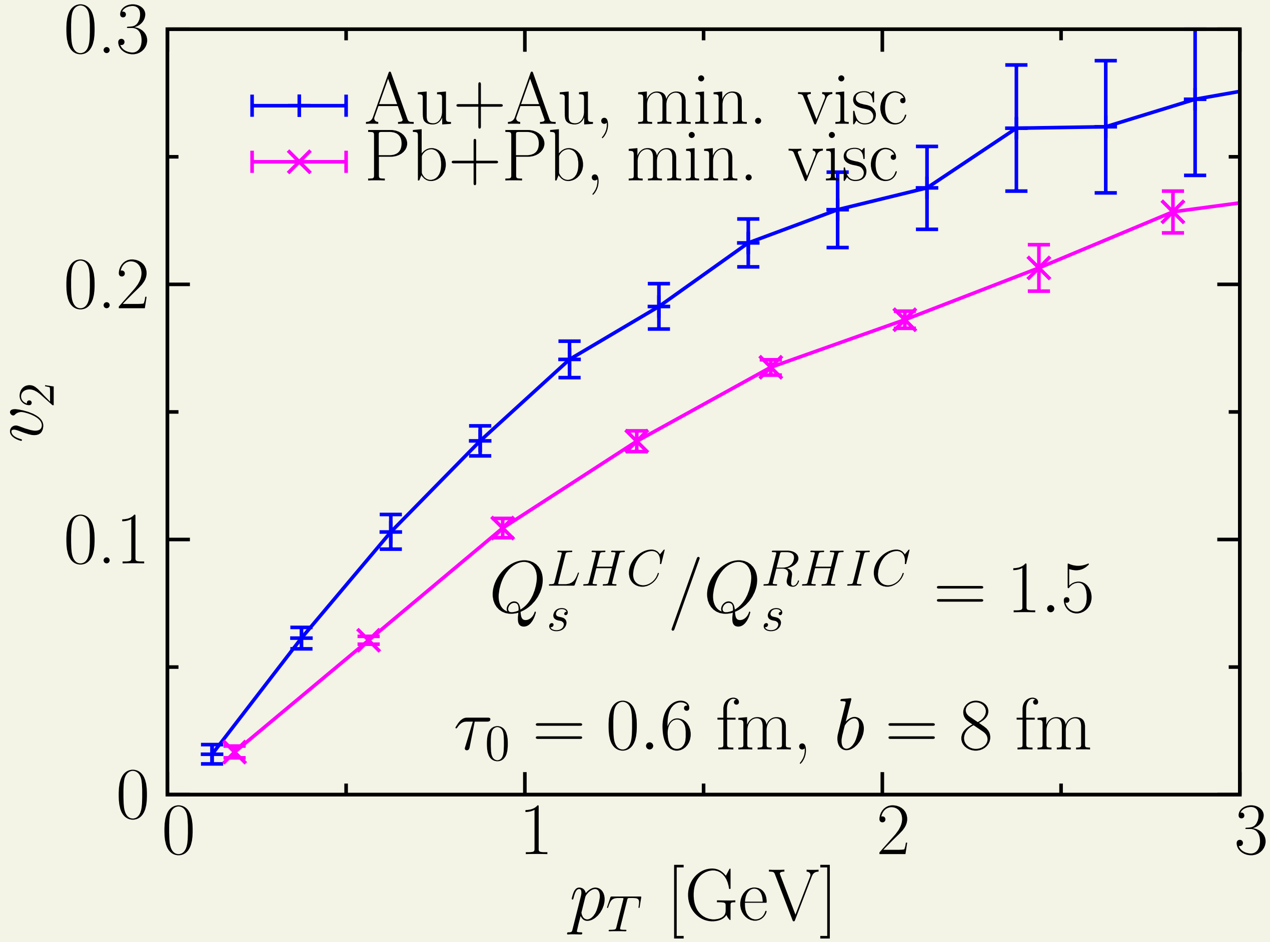
$$Q_s^2 = \frac{2\pi^2}{C_F}\alpha_S(Q_s^2)\ xG(x = \frac{Q_s}{\sqrt{s}}, Q_s^2)\ T_A$$

$\Rightarrow Q_s^{LHC}/Q_s^{RHIC} \approx 1.5$ (central collisions)

**refine for $b \neq 0$ with $\langle p_T^2 \rangle$ from $k_T$-factorized GLR** as in Adil et al, PRD73 ('06)

$$\frac{dN_g}{d^2x_T dp_T d\eta} = \frac{4\pi}{C_F}\frac{\alpha_s(p_T^2)}{p_T}\int d^2k_T\, \phi_A(x_1, \vec{p}_1, \vec{x}_T)\, \phi_B(x_2, \vec{p}_2, \vec{x}_T)$$

$\Rightarrow Q_s^{LHC}/Q_s^{RHIC} \sim \sqrt{\frac{\langle p_T^2\rangle^{LHC}}{\langle p_T^2\rangle^{RHIC}}} \approx\ 1.3 - 1.5$ **for $b = 8$ fm**




**at a given pT, $v_2$ at LHC will be smaller than at RHIC**

**in contrast, SPS → RHIC it stayed about same**

**IV. higher $T_{eff}$ also means higher $\sigma$, since $\lambda_{tr} \approx \frac{1}{3T_{eff}}$ quantum bound**

**i.e., need $v_2(p_T)$ for $1.3 - 1.5\times$ larger $\sigma$**

**$\Rightarrow$ would be small $5 - 10$% INCREASE in $v_2(p_T)$ relative to naive scaling**

**V. higher $Q_{set}$ also (likely) means faster thermalization** $\tau_0 \sim 1/Q_s$

**also increases initial density $n_0 \sim 1/\tau_0$, i.e., decreases $\eta/s$**

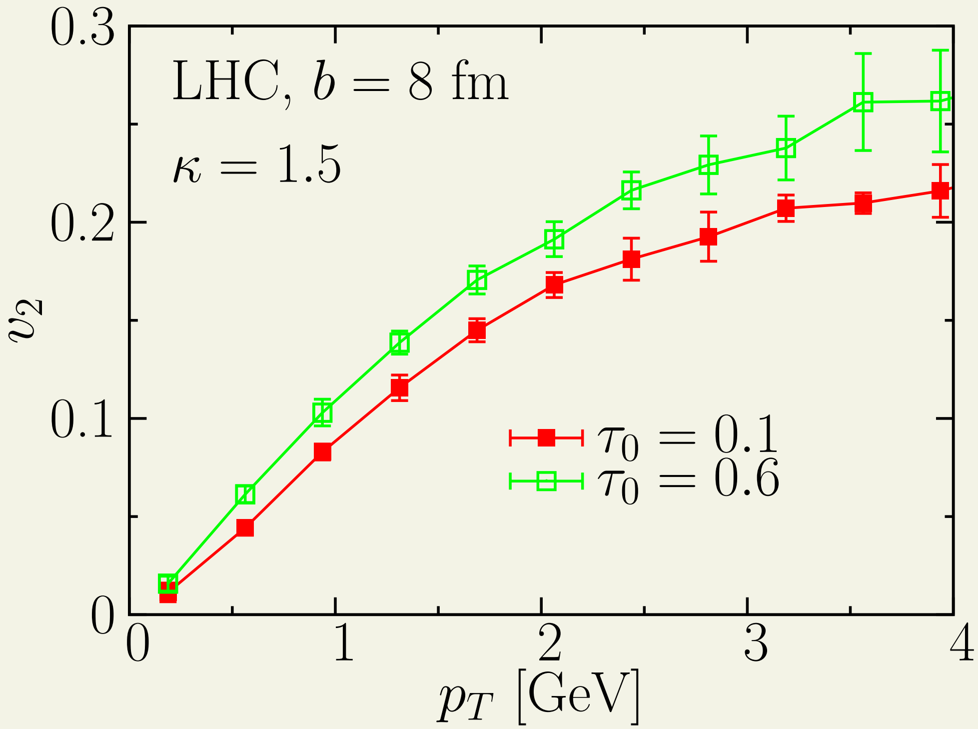
**$\Rightarrow$ IV + V = no need to adjust $\sigma$ at all**

**only change is in the last scale $\tau_0/R$ - controls interplay between longitudinal and transverse dynamics**

**starting earlier at LHC gives more Bjorken cooling** $T \sim 1/\tau^{1/3}$

**upon correction for cooling: factor 6 decrease in** $\tau_0$ **gives only 20% less** $v_2$

DM ('07):




**i.e.,** $Q_s^{LHC}/Q_s^{RHIC} \to \approx \ (Q_s^{LHC}/Q_s^{RHIC})^{2/3}$ **in scaling formula**

**needs to be studied in detail - but for 50% variation in** $\tau_0$ **corrections to the above rescaling should not be significant** $(< few\%)$

# Conclusions

**perturbative rates and large $v_2$ at RHIC: $2 \to 2$ is insufficient but $3 \leftrightarrow 2$ may work (still open)**

**there is a $20 - 30$% dissipative reduction of elliptic flow at RHIC even if scattering rates saturate their quantum bounds ("minimal viscosity" $\eta/s = 1/(4\pi)$)**

**<u>if</u> LHC and RHIC plasma are both "minimally viscous", expect**

$$v_2^{LHC,5500}(p_T) \approx v_2^{RHIC,200}(p_T \cdot k^{2/3})$$

**with $k \approx 1.3 - 1.5$ (GLR estimate for $b = 8$ fm).**

# Open issues

**initial geometry (eccentricity $\varepsilon$)**

- gluon saturation models can give $\sim 1.3\times$ larger $\varepsilon$ than for binary profile (depends on model details)

this mainly affects interpretation because $v_2 \sim \varepsilon$ (allows for larger $\eta/s$)

**missing $3 \leftrightarrow 2$ processes**

**not a big issue here because our viscosity is FIXED by the entropy.** Extra scattering channels decrease $\eta$ below the quantum bound, unless all cross sections are reduced at the same time.