PHYSICS 198-620B

Experimental Techniques in Sub-Atomic Physics

CALORIMETRY ASSIGNMENT SOLUTIONS

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1. *(50%)* For an incoming electron of energy E_0, **develop a simple shower model** where, on an average path χ_0 (the radiation length), every electron of the cascade always radiates half of its energy to bremsstrahlung and every photon of the cascade always makes pair production until the particles reach the known critical energy E_c. Only at this point would all energies be released into ionization processes and/or be dissipated.

   (a) *(20%)* Find the depth at which this occurs.

   t in radiation lengths: $N(t) = 2^t$, $E(t) = \frac{E_0}{2^t}, = E_c$
   hence $t_{max} = \frac{\ln(E_0/E_c)}{\ln 2}$

   (b) *(30%)* Find the number of particles involved.

   $N_{max} = 2^{t_{max}} = \frac{E_0}{E_c}$ *at the end, but*
   $N_{tot} = 2^{t_{max}+1} - 1 \approx 2\frac{E_0}{E_c}$ *cumulative*

   (c) *(20%)* Find the total track length for charged particles.

   Total track $= \frac{2}{3} \cdot 2\frac{E_0}{E_c} \cdot \chi_0$

   (d) *(30%)* Estimate what would be the duration time of a 1 or 10 GeV shower in lead.

   Pb: $Z = 82, E_c = 7\text{–}10\ MeV, \chi_0 = 0.56\ cm$
   $time = t_{max} \cdot \chi_0/c = etc..$
   $= 0.27 \times 10^{-10} \ln(E_0/10MeV)\ sec$
   $= 186(124)\ ps\ for\ E_0 = 10(1)\ GeV$

2. *(50%)* Imagine a **wall made of a large assembly of BGO crystals**. Their front faces are squares with half-sides equal to a Moliere radius. Given "d" as the perpendicular distance to a source of energetic π^0's,

   (a) *(50%)* Define your own rough criterium to resolve with high probability (e.g. better than 68%) the two most probable decay products, i.e. identify the π^0's.

   θ = half of opening angle between photons towards BGO plane.
   kinematics: $\sqrt{p_\pi{}^2 + m_\pi{}^2} = \gamma m_\pi = 2E_\gamma$,
   and: $p_\pi = 2E_\gamma cos\theta$, *hence:* $\theta = sin^{-1}(\frac{1}{\gamma}) = cos^{-1}\beta$
   geometry: $\beta = cos\theta = \frac{p_\pi}{E_\pi}$
   criterium/condition: one empty tower is needed in between,
   i.e. $tan\theta \geq \frac{r}{d}$, *where* r = *radius* (R_M=*27 mm*) *or* $d \geq \gamma\beta r$

   (b) *(50%)* For a wide range of π^0 energies (one value per order of magnitude, e.g. 0.01 GeV, 0.1 GeV, 1 GeV, etc.. as long as a detector size might make sense):

      i. Calculate the average decay length in the laboratory.
      ii. Calculate the minimum distance "d" where the wall detector should be to perform π^0 identification according to your criterium. Can the decay lengths be neglected?

   $length = vt = (\beta c)(\gamma\tau) = \beta\gamma c\tau$ (tiny, can be neglected)
   $distance = \gamma\beta r$

p_π *[GeV]*	*length [cm]*	*distance [cm]*
0.01	*1.9*$\times 10^{-7}$	*0.20*
0.1	*1.9*$\times 10^{-6}$	*2.0*
1.0	*1.9*$\times 10^{-5}$	*20.0*
10.0	*1.9*$\times 10^{-4}$	*200.0*
100.0	*1.9*$\times 10^{-3}$	*2019.1*