Hubert P., Bendjoudi H., Schertzer D., Lovejoy S., 2001: Multifractal taming of extreme hydrometeorological events, The Extremes of the Extremes, Proceedings of a symposium on extraordinary floods held at Reyjavik, Iceland, July 2000. IAHS Publ. no 271.

Multifractal Taming of Extreme Hydrometeorological Events

Pierre HUBERT

UMR Sisyphe, Centre d'Informatique Géologique, Ecole des Mines de Paris, Fontainebleau

e-mail : hubert@cig.ensmp.fr

Hocine BENDJOUDI

UMR Sisyphe, Laboratoire de Géologie Appliquée, Université Pierre & Marie Curie, Paris

e-mail : bdj@biogeodis.jussieu.fr

Daniel SCHERTZER

Laboratoire de Modélisation en Mécanique, Université Pierre & Marie Curie, Paris

e-mail : schertze@ccr.jussieu.fr

Shaun LOVEJOY

Physics Department, Mc Gill University, Montréal

e-mail : lovejoy@physics.mcgill.ca

Abstract : For hydrologists, scaling is some kind of Philosopher's stone. There exists in the literature a lot of empirical works about extreme rains and floods, their mutual relationship and their dependence upon observation time-step and/or catchment size. Multifractal approaches, originating from turbulence studies and which are based upon the Navier and Stokes equation symmetries, have allowed to put such studies in a new perspective and would give them a rational ground. They permit to give prominence to spatial and temporal scale invariance of hydrometeorological processes and so to put in the same conceptual framework normal and extreme events. Furthermore, multifractal approaches suggest that extreme events, both rainfall and discharge ones, would be

ruled by algebraic statistical laws, rather than by exponential ones as it has been generally postulated up to now. The theoretical and practical consequences of such conjectures are highly significant, especially when taking into account hydrological risk for land use planning or water structure design, as they lead to assign to the so-called exceptional events a return period much more short than classical approaches.

I. INTRODUCTION

Even if men have experienced for a long time weather caprices and river overflowings, observation and measurement device developments and a greater vulnerability to hydrological risk as well as a willingness to tame it, have enabled mankind to get a better understanding of this variability. It appeared that this variability is a function of the observation scale and we would like to give two examples stemming from precipitation studies.

The first one concerns the 1986 rainfall pattern over Burkina Faso territory (figure 1). From the 113 precipitation stations of the national network it has been possible to draw by hand the national hyetograph of figure 1. Meanwhile, an experimental network (EPSAT86) with 120 precipitation stations has been set up in the square degree (about 100 km x 100 km) around the city of Ouagadougou, including a dozen of stations from the national network (Hubert et Carbonnel, 1988). This last set of data made it possible to draw a new map at the square degree scale which exhibits at this finer scale a non expected variability. The spatial variability of precipitations that one would have thought to subside, especially at the annual time scale, still goes on vividly at the kilometric scale. Does that mean that it is impossible to master this variability, even mathematically ? The following example about the maximum observed precipitations for different time scales (figure 2) should give us some reasons to keep hope ! This last figure has been published in numerous hydrological books and papers (Raudkivi, 1979 ; Réméniéras et Hubert, 1990). It shows that the recorded maximums depend upon the observation duration, which is not surprising, but that this dependence is not linear, which prove that rainfall is not homogeneous along time. It is noticeable that a real organisation of the data appears on a log-log diagram and that there exists a quantitative link between the empirical statistic and the corresponding time scale which is an empirical evidence of scale invariance.

II. SOME THEORY

When Mandelbrot created the Fractal Geometry (Mandelbrot, 1977) he recognized the fractal (non-integer) dimension as the link able to connect a measure and a measurement gauge for various geometrical objects. Some hydrological applications have been attempted, to describe hydrographical networks and to characterize the rainfall temporal domain (Hubert et Carbonnel, 1989 ; Lovejoy *et al*, 1987). Such applications should not make us forget that precipitation and runoff, cannot be summarized only by presence or absence and that it is necessary to take into account their intensity at the different time scales. Following Schertzer and Lovejoy (1984), it had been noticed (Hubert *et al*, 1995) that the fractal dimension of rainfall occurrence is a decreasing function of this threshold defining occurrence. This leads for such studies to overstep the fractal object concept in favor of that of multifractal field.

The objective of the multifractal approach is to link scale and intensity for cascade processes where matter and/or energy concentrate over thinner and thinner space-time domains (Schertzer and Lovejoy; 1986, 1987). At each step of a multiplicative cascade, each eddy subdivides into sub-eddies, redistributing all or part of his matter or energy, according to a random factor. For a given observation scale λ (λ is the ratio of the external scale to the observation scale) the probability that ε_λ exceeds a given threshold λ^γ is as (Schertzer and Lovejoy, 1991) :

$$\Pr ob\left[\varepsilon_\lambda > \lambda^\gamma\right] = A_\lambda \lambda^{-C(\gamma)} \qquad (1)$$

where γ is a singularity order and $C(\gamma)$ a codimension function characterizing the probability of occurrence of singularity of order greater than γ. A_λ is a cofactor slowly varying with λ. The most valuable results about multifractals concern the behavior of large order statistical moments. It can be derived (Schertzer and Lovejoy, 1991) that may exist a critical value q_D of the moment order q, such as the statistical moments diverge as son as $q > q_D$. Then, for large enough values of the singularity order γ, the probability distribution can be written as :

$$\Pr ob[X > x] \cup x^{-q_D} \qquad (2)$$

The consequences of such an algebraic behavior are extensive because algebraic laws decrease infinitely more slowly than exponential laws usually used to estimate the magnitude of events of given return time. This magnitude would then be considerably underestimated

III. APPLICATIONS TO HYDROMETEOROLOGICAL SERIES AND FIELDS

III.1 Applications to precipitations

A lot of multifractal applications have been devoted to studies and modeling of rainfall time series and fields (Olson, 1996 ; Ladoy *et al*, 1993). We will quote especially our study of the long rainfall time series gathered within the FRIEND-AMHY project (Bendjoudi et Hubert, 1998). This study was devoted to annual rainfall accumulations and we tried to see how convenient the previous formalism was to explain the behavior of extreme values. 87 annual rainfall series with a mean length of 116 years have been processed. We have plotted on a log-log diagram the empirical probabilities of exceeding a threshold against the corresponding thresholds. The curve resulting from the best fitting to a Gaussian law has been superposed on the same diagram. It is obvious that the points corresponding to large values depart notably from the Gaussian fitting and the fitting of a straight line to those points the probability of exceedance was letter than 0.05 seems suitable. This result encourages the hypothesis of an algebraic rather than exponential behavior of the statistical distribution of large annual rainfalls. Figure 3 exhibits four examples of such curves for the longuest series available in our data base : Padova, Marseilles, Rome and Gibraltar whose duration is greater than two centuries.

The practical consequences of these results are considerable. For example, about annual rainfall accumulations, what was the one thousand year event according to the Gauss law is roughly no more than the one hundred year event according to an algebraic law. The q_D values found are in good agreement with those found by others authors, slightly greater than 3, which would suggest that it is an intrinsic rainfall feature. Practically that means that, from a rainfall series, it would be vain to estimate the kurtosis coefficient (4^{th} moment) and that the estimate of the third order moment (skewness) should be quite poor.

III.2 Applications to runoff time series

The applications of scale invariance concepts to river discharges are more recent and consequently more limited. Turcotte and Greene (1993) studied the flood frequencies of 10 American rivers and characterized the scale invariance they had found by the ratio of the 100 years return period flood to the 10 years return period flood. The value of this ratio ranges from 2 to 8 and the authors attribute these differences to the various climatological conditions. Tessier *et al* (1996) studied rainfall and runoff series of 30 French catchments. They have pointed out a scale invariance for durations ranging from 1 day to 30 years. The critical divergence moment estimated from the whole set of data is about 3.2 for time scales greater than 30 days and close to 2.7 for time scales less than 16 days. A more recent study by Pandey *et al* (1998) is devoted to 19 American catchments the area of which ranges from 5 to 2 millions km^2 with a total of 700 year-stations. Their conclusion is that the behavior is multifractal for time scales ranging from 2^3 to 2^{16} days. Their estimates of the multifractal parameters, and especially those of q_D which is about 3.1 are close to those of Tessier *et al* (1996), but unlike Turcotte and Greene (1993) they attribute to chance the differences found for the different basins.

IV. CONCLUSIONS AND PERSPECTIVES

After this brief presentation of some applications of the multifractal approach in hydrometeorology we would like to set some perspectives. Starting from a phenomenological model of multiplicative cascade designed in turbulence studies which is likely to hold in hydrology and then to give to the multifractal approach some kind of physical basis. The applications attempted up to now are encouraging. A scale invariant behavior for time scale ranging roughly from one day to one century has been evidenced. In the future, this scale invariance should enable to overcome local and/or fragmented approaches, which are difficult or even impossible to generalize. Such approaches are still the rule today and we are facing, especially in the field of flood probability estimations, an unmanageable mushrooming of ad-hoc statistical laws. The most crucial point raised by multifractal studies is that the decrease of statistical laws ruling the floods would be algebraic rather than exponential. We do want to underline once more the practical significance of this point, as the discharge corresponding to a given return period estimated from an algebraic hypothesis will be much more large than that estimated from an exponential one. In front of theoretical and practical stakes it is necessary to carry on with multifractal studies of rainfalls and discharges, to strengthen their theoretical basis and to validate their relevance conclusively.

Acknowledgements : This study has been done within and with financial support of the "Institut National des Sciences de l'Univers (Programme National de Recherche en Hydrologie - Projet 99 PNRH 27)".

REFERENCES

Bendjoudi, H. et Hubert, P. (1998) A propos de la distribution statistique des cumuls pluviométriques annuels. Faut-il en finir avec la normalité ? *Rev. Sci. Eau* 4(1998), 617-630.

Hubert, P. et Carbonnel, J.P. (1988) Caractérisation fractale de la variabilité et de l'anisotropie des précipitations intertropicales, *C.R.Acad. Sci. Paris*, Série II, 307, 909-914.

Hubert, P. et Carbonnel, J.P. (1989) Dimensions fractales de l'occurrence de pluie en climat soudano-sahélien, *Hydrologie Continentale*, 4, 3-10.

Hubert, P., Friggitt, F. et Carbonnel, J.P. (1995) Multifractal structure of Rainfall Occurrence in West Africa*, New Uncertainty Concepts in Hydrology and Water Resources*, Z.W. Kundzewicz editor, Cambridge University Press, 109-113.

Ladoy, P., Schmitt, F., Schertzer, D. et Lovejoy, S. (1993) Analyse multifractale de la variabilité pluviométrique à Nîmes, *C.R.Acad. Sci. Paris*, Série II, 317, 775-782.

Lovejoy, S. and Schertzer, D. (1986) Scale invariance, symmetries, fractals and stochastic simulations of atmospheric phenomena, Bulletin of the AMS, 67, pp 21-32.

Lovejoy, S., Schertzer, D. and Tsonis, A.A. (1987) Functional box-counting and multiple elliptical dimensions in rain, *Science*, 235, 1036-1038.

Mandelbrot, B.B. (1977) *The fractal geometry of nature*, Freeman, San-Francisco, 461 p.

Olson, J. (1996) Validity and applicability of scale-independant, multifractal relationship for rainfall, *Journal of atmospheric research*, 42, 53-65.

Pandey, G., Lovejoy,S. and Schertzer. D. (1998) Multifractal analysis of daily river flows including extremes for basin of five to two million square kilometres, one day to 75 years, *Journal of Hydrology*, 208, 62-81.

Raudkivi, A.J. (1979) *Hydrology*, Pergamon Press,New York.

Réménieras, G. et Hubert, P. (1990) Hydrologie, *Encyclopaedia Universalis*, volume XI, 796-806.

Schertzer, D. and Lovejoy, S. (1987) Physical modeling and analysis of rain and clouds by anisotropic scaling and multiplicative processes, *J. Geophys. Res*., 92, D8, pp 9693-9714.

Schertzer, D. and Lovejoy, S. (1986) *On the dimension of atmospheric motion, Turbulence and chaotic phenomena in fluids*, T. Tatsumi editor, North Holland, 505-508.

Schertzer, D. and Lovejoy, S. (1991) Scaling nonlinear variability in Geodynamics : Multiple singularities, observables, universality classes, Nonlinear variability in *Geophysics, Scaling and Fractals*, D. Schertzer and S. Lovejoy editors, Kluwer, 41-82.

Tessier, Y., Lovejoy, S., Hubert, P., Schertzer, D. and Pecknold, S. (1996) Multifractal analysis and modeling of rainfall and river flows and scaling, causal transfer functions, *Journal of Geophysical Research*, 101, D21, 26427-26440.

Turcotte, D.L. and Greene, L. (1993) A scale-invariant approach to flood-frequency analysis, *Stochastic Hydrol. Hydraul*., 7, 33-40.

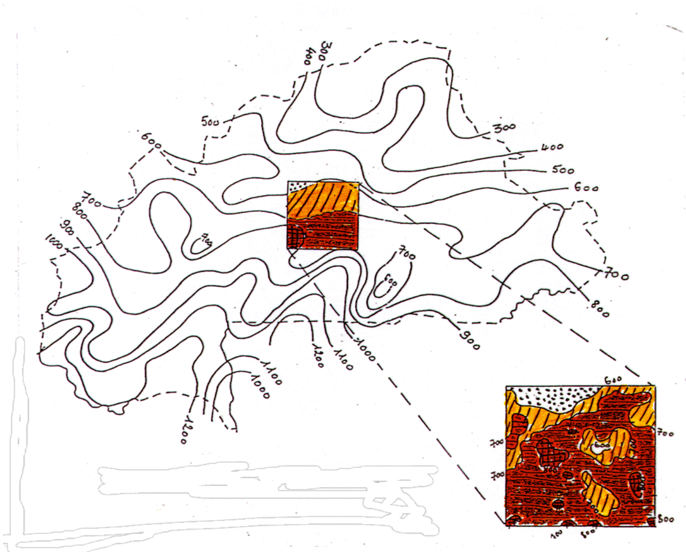
Figure captions

Figure 1 *1986 Hyetograph of Burkina Faso at the national scale (113 stations) and at the*

Ouagadougou square degree (120 stations including a dozen of the national network).

Figure 2 : *Maximum recorded precipitations for different time durations..*

Figure 3. *Empirical probabilities and Gaussian fitting of Padova, Marseilles, Rome and Gibraltar time series. Empirical values are represented by dots and the best Gaussian fitting by a*



solid curve. The linear fitting of the distribution tails is represented by a dotted line..

Figure 1

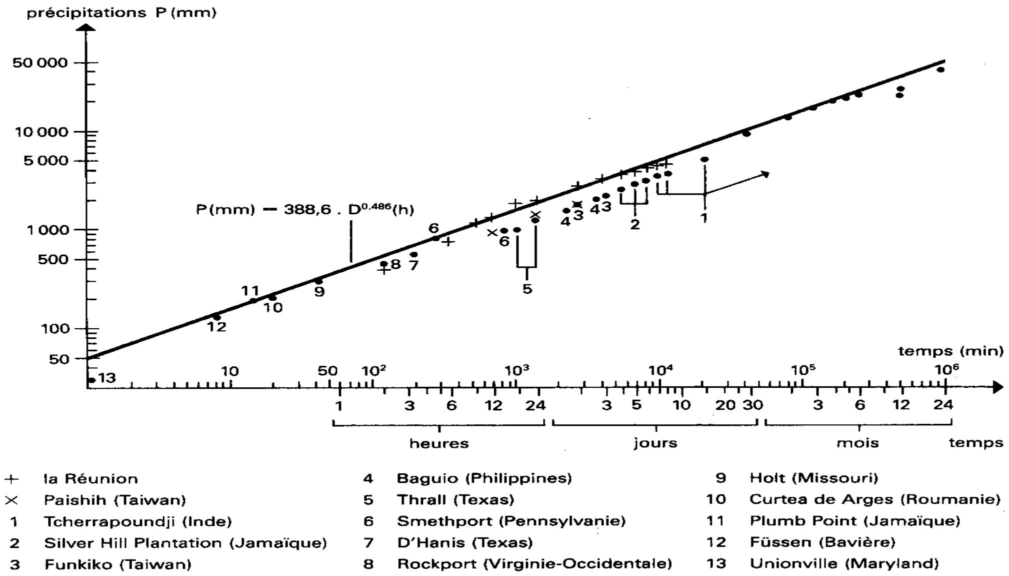


Fig2

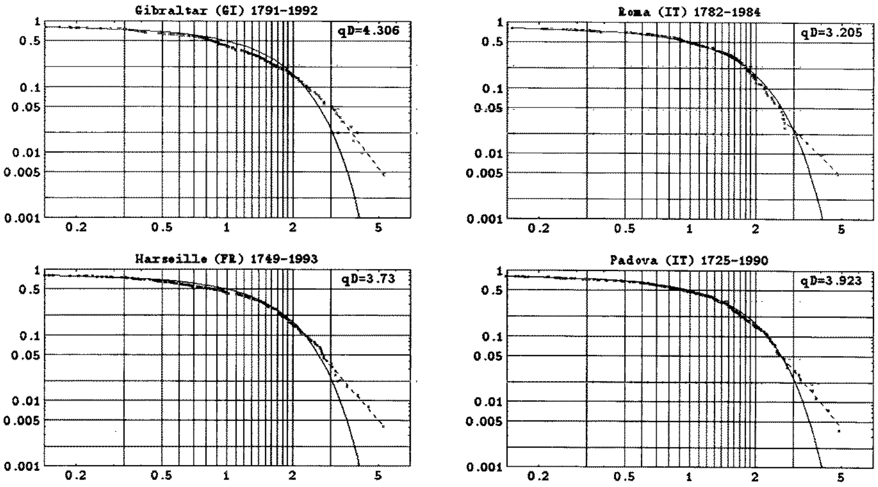

Gibraltar (GI) 1791-1992
qD=4.306
Roma (IT) 1782-1984
qD=3.205
Harseille (FR) 1749-1993
qD=3.73
Padova (IT) 1725-1990
qD=3.923
1
0.5
0.1
0.05
0.01
0.005
0.001
0.2
0.5
1
2
5

Figure 3