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UNIVERSAL MULTIFRACTAL INDICES FOR THE OCEAN SURFACE AT FAR RED WAVELENGTHS

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Abstract. For some time, ocean wave breaking has been conceptualized as a cascade process in which the large scale wind energy flux driving the system is dissipated by wave breaking at small scales, the two seperated by the "equilibrium" scaling range. Cascades are now known to generically lead to multifractals; with special "universal" multifractals theoretically predicted. In this paper we use far red (0.95μm) radiances at 1m resolution obtained from aircraft to test the multifractal behavior of the ocean surface and estimate the corresponding universal multifractal parameters of the radiance field.

Introduction

It has long been realized that over ranges spanning centimeters to at least several tens of meters (the "spectral peak"), that the ocean surface has scaling (power law) spectra $E(k) \sim k^{-\beta}$, where k is a wavenumber. Scaling or scale invariance indicates that some geometrical and statistical properties are preserved at different length scales. Simple illustrations of scaling are geometrical sets, such as the Cantor set, characterized by a unique scaling exponent, its (mono)fractal dimension (see Mandelbrot, 1983). Several (mono)fractal investigations of wave properties have been done (Glazman and Weichman, 1989; Glazman, 1991; Stiassnie et al, 1991; Huang et al, 1992; Kerman and Szeto, 1993). Following the development of cascade ideas in turbulence theory (Mandelbrot, 1974; Frisch et al., 1978; Schertzer and Lovejoy 1984, 1987, Parisi and Frisch, 1985; Meneveau and Sreenivasan, 1987) it would seem much more likely that the ocean surface is multifractal rather than monofractal: regions of the ocean surface exceeding a certain elevation threshold are expected to have fractal dimensions decreasing with increasing elevation (rather than remaining constant as implied in a monofractal model). Kerman (unpublished, 1993), Kerman and Bernier (1993) performed multifractal analyses of remotely sensed data of breaking waves on the sea surface following a statistical geometry approach used in strange attractor studies. This paper amplifies preliminary analyses of Lavallée et al (1991) showing with the help of a turbulent multifractal formalism that the surface radiances the are indeed multifractals of a specific "universal" form predicted by theory.

There are specific reasons connected with dynamical cascade processes why multifractals may be expected to arise in ocean waves. Cascades ideas have been used in the ocean wave "equilibrium range" spectrum (e.g., Zakarov and Filenko 1966, Zakarov and Zaslavskii 1982) which separates the large scales where waves are generated by wind and the small scales where they are dissipated by breaking. Since many cascade properties are now known to be generic, this extension to the ocean is natural. The basic physical ingredients for cascade models are:

-The dynamics are scaling over wide ranges in scale: there are significant ranges over which no strong scale breaking process occurs. This may apply over the ocean's "equilibrium" wave range from tens of meters down to the wave breaking scales of one centimeter or so.

-There are fluxes that are conserved by the nonlinear dynamics (such as the energy flux). For ocean waves, the energy flux is usually considered to be conserved (e.g. Zakarov and Filenko, 1966).

Although the dynamics of wave interactions and wave breaking are in many respects different from hydrodynamical turbulence (especially due to the nonlocalness of interactions), it plausibly respects the above properties and thus the general properties of cascade processes may be relevant to the ocean surface, especially the predictions of multifractal behavior.

In the following, we study two aircraft photos of the ocean surface at 0.95 μm wavelength, and with 1m resolution, over a zone 512 x 512m^2, obtained using a line scanner on board an aircraft operated by the Canadian Center of Remote Sensing. These photos were taken near the coast of Nova Scotia under differing dominant wind conditions. The mean wind speed was 12 m/sec for flight 5 and 17m/sec for flight 7. We estimated the basic universal multifractal parameters of the radiation field reflected from the surface. Because of the strong non-linear coupling of the far red radiance field with the dynamics (and whitecaps), multiscaling in the dynamical field will be reflected in the radiance which will also be multifractal (although with possibly different multifractal parameters).

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Multifractals and ocean surface radiances

In hydrodynamic turbulence, the energy flux density ε_λ from large to small scales is conserved: its ensemble average $<\varepsilon_\lambda>$ is independent of scale. Directly observable fields such as the velocity shear (Δv_λ) for two points separated by the adimensionalized distance λ^{-1} ($\lambda = l_0/l$, the ratio of the smallest scale of variability to the external/largest scale, $\lambda>1$) are related to the energy flux through dimensional arguments:

$$\Delta v_\lambda \approx \varepsilon_\lambda^{1/3}\lambda^{-1/3} \qquad (1)$$

This corresponds to the celebrated Kolmogorov (1941) law for homogeneous turbulence ($\varepsilon_\lambda = <\varepsilon>$), more genereraly this may correspond to fractionnal integration of order 1/3 over a highly fluctuating $\varepsilon_\lambda^{1/3}$ (Schertzer and Lovejoy, 1987), i.e. a $k^{-1/3}$ filter in Fourier space (k is the wavenumber).

The dynamical equations responsible for the distribution of the sea surface radiances are not so well known; we can only speculate on the appropriate quantity φ_λ analogous to ε_λ for the radiance fields, having the conservation property $<\varphi_\lambda> =$ constant (independent of scale). The observable (non conserved) surface infra red reflectivity fluctuations (ΔI_λ) are then given by:

$$\Delta I_\lambda \approx \varphi_\lambda^a \lambda^{-H} \qquad (2)$$

We lose no generality by taking in the following discussion a = 1. The scaling parameter H specifies the exponent of the power law filter (the order of fractional integration) required to obtain I from φ and how far the measured field I is from the conserved field φ: $<|\Delta I_\lambda|> \approx \lambda^{-H}$.

It is now known (Schertzer and Lovejoy 1984, 1987) that dynamical processes respecting the cascade properties will generically lead to multifractal measures with the following multiple scaling property with exponent K(q):

$$\langle \varphi_\lambda^q \rangle \approx \lambda^{K(q)} \qquad (3)$$

The relations between this turbulent multifractal formalism and the strange attractor formalism (Halsey et al, 1986) is discussed in Schertzer and Lovejoy (1992).

In actual dynamical systems involving nonlinear interactions over a continuum of scales (and/or involving multiplicative "mixing" of different processes) we generally obtain a considerable simplification. Schertzer and Lovejoy (1987, 1991) and Schertzer et al (1991) show that cascade processes admit stable (attractive) universal generators: irrespective of the details of the nonlinear dynamics. The behavior of a conserved process will be characterized by only two fundamental parameters, (α, C_1):

$$K(q) = \begin{cases} \dfrac{C_1(q^\alpha - q)}{\alpha - 1} & \alpha \neq 1 \\ C_1 q \mathrm{Log}(q) & \alpha = 1 \end{cases} \qquad (4)$$

α is the Lévy multifractal index ($0 \le \alpha \le 2$), it quantifies the distance of the process from monofractality; $\alpha = 0$ is the monofractal (dead/alive) β model of turbulence (Novikov and Stewart 1964, Mandelbrot 1974, Frisch et al 1978), $\alpha = 2$ is the lognormal model. The parameter C_1 is the codimension of the mean of the process; it quantifies the sparseness of the mean. The third parameter H determines the degree of nonconservation fro nonconserved fields.

Spectral Analysis

The limits of the scaling is conveniently determined by Fourier analysis. Fig. 1 shows E(k) for the two pictures analyzed; power laws correspond to scaling regimes. Phillips (1985) suggested that scaling for gravity waves holds for the range between the finest resolution ~0.1 and about ~15m. We observed a good power law fit ($E(k) \sim k^{-\beta}$) for the range between ~1 m and about 50 m which is roughly compatible with this result. The exponent β (1.29, 1.39 for flights 5 and 7 respectively) is related to the parameter H with the help of the following formula (e.g. Lavallée et al 1992):

$$H = \frac{\beta - 1 + K(2)}{2} = \frac{\beta - 1}{2} + \frac{C_1(2^\alpha - 2)}{2(\alpha - 1)} \qquad (5)$$

The Double trace Moment analysis technique

The Double Trace Moment (DTM) technique (Lavallée et al 1993) directly estimates the universality parameters by taking the η power of φ_Λ at the largest available scale ratio Λ. It generalize the single trace moment (Schertzer and Lovejoy 1987). The q, η double trace moment at resolutions λ and Λ is then defined as:

$$Tr_\lambda(\varphi_\Lambda^\eta)^q = \left\langle \sum_{B_\lambda} \left(\int_{B_\lambda} \varphi_\Lambda^\eta d^D x \right)^q \right\rangle \propto \lambda^{K(q,\eta) - (q-1)D} \qquad (6)$$

where the sum is over all the sets B_λ (scale λ, dimension D) required to cover the multifractal, and the (double) scaling exponent $K(q,\eta)$ satisfies the following relation:

$$K(q,\eta) = K(q,\eta) - qK(\eta,1)\ ; \quad K(q,1) = K(q) \qquad (7)$$

Applying eq. 6 to the field φ_Λ simply consists of taking various powers η of the field at its highest resolution (Λ), then degrading the result to a lower resolution (λ), finally averaging the q^{th} power of the result. If the

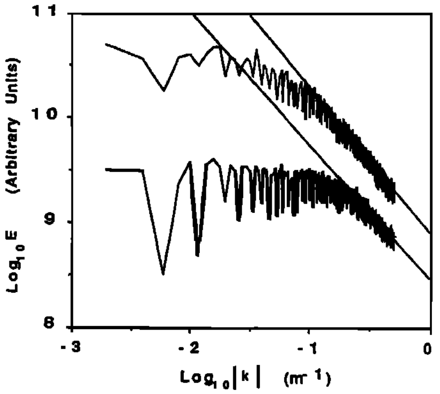

Fig. 1: Power spectrum for both scenes. The top curve is for flight 7 ($\log_{10}$ E +1) the straight line as slope β = 1.39, the bottom curve is for flight 5 ($\log_{10}$ E), the straight line as slope β = 1.29.

process is non conservative ($H \neq 0$) such as radiance field examined here, it suffices to power law filter it so that the spectrum is fairly flat. We use a simpler approach: we analyse the modulus of the gradient of the field which is roughly equivalent to filtering by a factor k (= wavenumber). As an example, in fig. 2, we show the above trace moments for q=0.5, and η=0.7, 1.2, 1.8 and 2.4. The lines are straight over the same range of scales λ for which the spectrum (fig. 1) is scaling. By replacing eq. 4 in eq. 7 we obtain the useful relation:

$$K(q,\eta) = \eta^{\alpha} K(q,1) \qquad (8)$$

showing that α and C_1 can be estimated from a linear regression of $\log |K(q, \eta)|$ vs. $\log \eta$ for fixed q. Fig. 3 shows the results of such an analysis (with q=0.5) comparing the two different scenes, showing that they have the nearly the same parameters (α=1.14, 0.99 and C_1=0.30, 0.22 respectively). A similar study for q = 2.0 give estimates of the statistical variability, we deduce: $\alpha \approx 1.1 \pm 0.1$, $C_1 \approx 0.25 \pm 0.05$ and $\beta \approx 1.35 \pm 0.05$ and due to eq.5 $H \approx 0.35 \pm 0.05$.

The breaks at both ends of fig.3 are not scaling breaks, they indicate that eqs. 4, 8 no longer hold. For low values of η, we are analyzing very low values of the field, and the effect of (space filling) noise will be dominant and leads to a flat $K(q,\eta)$. When η is large, undersampling leads to poor estimates of high order moments ($q>q_s$). There is also the possibility of divergence of high order statistical moments ($q>q_D$). These breaks correspond respectively to first order ($q_D<q_s$) and second order ($q_s<q_D$) multifractal phase transitions (Schertzer and Lovejoy, 1992, unpublished 1993) and $K(q,\eta)$ becomes constant for $\max(q\eta,\eta)>\min(q_s,q_D)$. The break in the $\log|K(q,\eta)|$ vs. $\log \eta$ curve for values of $\eta \approx 3$ seems to correspond to a first order multifractal phase transitions, since Kerman and Szeto (1993) find $q_D \approx 3$ using probability distributions on the same data sets; whereas we can estimate $q_s \approx 10$ by applying the procedure outlined in Schertzer and Lovejoy (1991) to the above estimates of C_1, α.

Our moderate value of α, (recall $0 \leq \alpha \leq 2$) shows that the radiance field is nearly halfway between a lognormal and a β-model. For

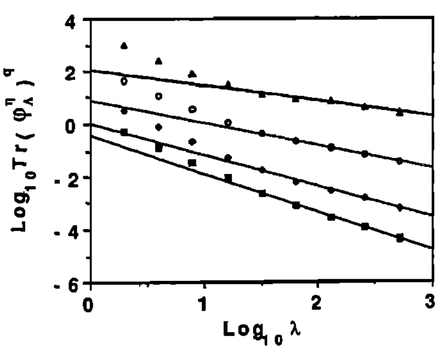

Fig. 2: This figure shows log of the trace moments v.s. log λ (λ is the resolution in pixels) with q=0.5, η=0.7, 1.2, 1.8 and 2.4 (top to bottom) for flight 5. The scaling is well respected in the range between 1 m and 50 m as indicated by the straight line fits.

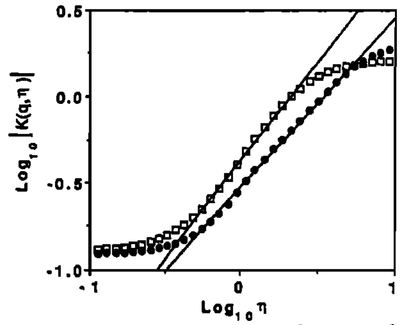

Fig. 3: This figure shows the results of the double trace moment technique for both photos using q = 0.5, (flight 5: filled circles, flight 7: empty squares). It shows that except for q greater than $\approx$3, or less than $\approx$0.3, that universal (straight line) behavior is observed. The fitted straight lines of slope α=1.14 and α=0.99 are shown for reference.

comparison, we have recently estimated $\alpha \approx 1.3$, 1.5 for wind tunel and atmospheric turbulence respectively (Schmitt et al 1992a,b), $\alpha \approx 1.8$ for the earth's surface topography (Lavallée et al 1993), α varies between 1.1 and 1.4 depending on the wavelength of the sensor for cloud radiances (Tessier et al., 1993), $\alpha \approx 1.4$ for the spatial distribution of rainfall (Tessier et al 1993), $\alpha \approx 0.5$ for rainfall time series (Hubert et al 1993).

Conclusion

In recent years there has been a growing literature on the dynamics of fractal surfaces. Surfaces are fields not geometrical sets of points, the natural and general framework is thus multifractals. We found that ocean surface far red rdaiances have multifractal properties over the range of 1m to 50m with universal parameters $\alpha \approx 1.1 \pm 0.1$, $C_1 \approx 0.25 \pm 0.05$ and $H \approx 0.35 \pm 0.05$. Presumably this scaling should hold down to centimeter scales. Up until now such strong intermittency has, been impossible to deal with using closures techniques (which typically assume non-intermittent quasi-gaussian statistics). It also implies that standard numerical modelling techniques must involve a wide range of scales otherwise the cascade process will be badly truncated.

The multifractal nature of the ocean surface may have important implications for studying the ocean. For example, (Glazman, 1990) suggested a reinterpretation of radar altimeter returns due to the fractal nature of the ocean surface. Universal multifractal formalism will allow to extend this reinterpretation to multifractal surfaces. Other oceanic fields such as the surface temperature and the salinity could also benefit from such a characterization. It might also help in linking the wind and wave fields.

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References

Frisch, U., P.L. Sulem, and M. Nelkin, A simple dynamical model of intermittency in fully developed turbulence. J.Fluid Mech., 87, 719-724, 1978.

Glazman, R. E., P. B. Weichman Statistical geometry of a small surface patch in a fully developed sea. J. Geophys. Res., 94, 4998-5010, 1989.

Glazman, R.E., Near-Nadir radar backscatter from a well developed sea, Radio Sci., 25, 1211-1219, 1990.

Glazman, R.E., Fractal Nature of Surface Geometry in a Developed Sea, in Non-Linear Variability in geophysics: Scaling and Fractals, edited by D. Schertzer and S. Lovejoy, pp. 217-226, Kluwer, Dordrecht, The Netherlands, 1991.

Halsey, T.C., M. H. Jensen L. P. Kadanoff, I. Procaccia and B. Shraiman, Fractal measures and their singularities: the characterization of strange sets, Phys. Rev. A, 33, 1141-1151, 1986.

Huang, N. E., S. R. Long, C.-C. Tung, M. A. Donelan, Y. Yuan, R. J. Lai, The local properties of ocean surface waves by the phase-time method. Geophys. Res. Lett., 19, 685-688, 1992.

Hubert, P., Y. Tessier, S. Lovejoy, D. Schertzer, P. Ladoy, J. P. Carbonnel, S. Violette, Multifractals and Extreme Rainfall events., Geophys. Res. Lett. , (in press), 1993.

Kerman, B. and L. Bernier, Breaking waves are fractal singularities, J. Geophys. Res. (in press), 1993.

Kerman, B., K. Szeto, Fractal properties of Whitecaps, Atmos. Ocean, (in press), 1993.

Kolmogorov, A. N., Local structure of turbulence in an incompressible liquid for very large Raynodls numbers, Proc. Acad. Sci. URSS. Geochem. Sect., 30, 299-303, 1941.

Lavallée, D., S. Lovejoy, D. Schertzer, Universal multifractal theory and observations of land and ocean surfaces, and of clouds, Proc. of the Society of Photo-optical Instrumentation Engineers (SPIE). 1558, 60- 75, 1991

Lavallée, D., S. Lovejoy, D. Schertzer, F. Schmnitt, On the determination of universal multifractal parameters in turbulence, in Topological aspects of the dynamics of fluids and plasmas. Edited by H.K. Moffat, G.M. Zaslavsky, M Tabov and P. Comte, Kluwer Academic Publisher, pp. 463-478, 1992.

Lavallée, D., S. Lovejoy, D. Schertzer, P. Ladoy, Nonlinear variability of Landscape topography: multifractal analysis and simulation., in Fractals in Geography, edited by L. De Cola and N. Lam, 171-205, 1993.

Mandelbrot, B., Intermittent turbulence in self-similar cascades: Divergence of high moments and dimension of the carrier., J. Fluid. Mech., 62, 331-350, 1974.

Mandelbrot, B., The fractal Geometry of Nature, Freeman Press, San Francisco, 1983.

Meneveau, C. and K. R. Sreenevasan, Simple multifractal cascade model for fully develop turbulence, Phys. Rev. Lett., 59(3), 1424-1427, 1987.

Novikov, E. A., and R. Stewart, Intermittency of turbulence and spectrum of fluctuations in energy-dissipation, Izv. Akad. Nauk. SSSR. Ser. Geofiz. 3, 408-412, 1964.

Parisi G. and U. Frish, A multifractal model of intermittency, in Turbulence and predictability in geophysical fluid dynamics, edited by Ghil M., R. Benzi and G. Parisi, North Holland, pp. 84-88, 1985.

Phillips, O.M., Spectral and statistical properties of the equilibrium range in wind-generated gravity waves, J. Fluid. Mech., 156, 505-531, 1985.

Schertzer, D. and S. Lovejoy, On the dimension of atmospheric motions. In , Turbulent and Chaotic phenomena in fluids. edited by T. Tatsumi, North-Holland, New-york, pp. 505-508, 1984.

Schertzer, D.and S. Lovejoy, Physically based rain and cloud modeling by anisotropic, multiplicative turbulent cascades. J. Geophys. Res., 92, 9693-9714, 1987.

Schertzer, D., S. Lovejoy, Scaling nonlinear variability in geodynamics: Multiple singularities, observables, universality classes, in Scaling, fractals and non-linear variability in geophysics, edited by D. Schertzer and S. Lovejoy , pp. 41-82, Kluwer, Dordrecht, The Netherlands. 1991.

Schertzer, D., S. Lovejoy, D. Lavallée, F. Scmitt, Universal Hard multifractal turbulence: Theory and observations, in Nonlinear Dynamics of Structures, Edited by R.Z. Sagdeev, U. Frisch, F. Hussain, S.S. Moiseev and N.S. Erokin, World Scientific, pp. 213-235, 1991.

Schmitt, F., D. Lavallée, S. Lovejoy, D. Schertzer, C. Hooge, Turbulence and Universal Multifractals III: First estimates of multifractal indices for velocity and temperature fields. C. R. Acad. Sci. de Paris , 314 Serie II, 749-754, 1992a.

Schmitt, F., D. Lavallée, D. Schertzer, S. Lovejoy, Empirical Determination of Universal Multifractal Exponents in Turbulent Velocity Fields. Phys. Rev. Lett., 68, 305-308, 1992b.

Stiassnie, M., Y. Agnon and L. Shemer, Fractal Dimensions of Random Water Surfaces, Physica D, 47, 341-352, 1991.

Tessier, Y., S. Lovejoy, D. Schertzer, Universal Multifractals: Theory and Observations for rain and clouds, J. of Appl. Meteor., 32, 223-250, 1993.

Zakharov, V.E. and N.N. Filonenko, 1966, The energy spectrum for stochastic oscillation of a fluid's surface. Doklady Academii Nauk S.S.S.R., 170(6), 1292-1295.

Zakharov, V.E. and M.M. Zaslavskii, 1982. The kinetic equation and Kolmogorov spectra in the weak turbulence theory of wind waves. Izvestiya, Atmospheric and Oceanic Physics , 18(9), 747-753.

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