GEOPHYSICAL RESEARCH LETTERS, VOL. 30, NO. 11, 1559, doi:10.1029/2002GL016022, 2003

Percolating magmas and explosive volcanism

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Received 1 August 2002; revised 21 November 2002; accepted 28 January 2003; published 5 June 2003.

[1] Magma under pressure rises in conduits, depressurizes, forms bubbles by the exsolution of gas and – at void fractions (**P**) typically of the order of 0.7 – can fragment and explode. The study of overlapping geometrical units – percolation theory – predicts that at a critical volume fraction $\mathbf{P}_c$ the size of the largest simply connected region becomes infinite. We apply percolation theory to overlapping bubbles arguing that this geometric singularity at $\mathbf{P}_c$ implies a physical singularity in the magma rheology. This would imply that if the magma is under stress, - whether it is ductile or brittle - this rapid development of a network of infinitely long "bubbles" triggers fragmentation and explosion. Classical monodisperse (equal size) continuum percolation theory predicts $\mathbf{P}_c = 0.2985 \pm 0.005$ which is far from the observed values. However, it has recently been shown that the bubble distribution is a power law associated with a huge range of bubble sizes. Using Monte Carlo percolation simulations, we show that distributions exhibiting the empirical exponents are very efficient at "packing" the bubbles, drastically raising $\mathbf{P}_c$ to the value = 0.70 ± 0.05. Explosive volcanism is thus explained by singular rheology at $\mathbf{P}_c$. *INDEX TERMS:* 3220 Mathematical Geophysics: Nonlinear dynamics; 8414 Volcanology: Eruption mechanisms; 8429 Volcanology: Lava rheology and morphology. Citation: Gaonac'h, H., S. Lovejoy, and D. Schertzer, Percolating magmas and explosive volcanism, *Geophys. Res. Lett.*, *30*(11), 1559, doi:10.1029/2002GL016022, 2003.

1. Introduction

[2] Due to it's role in triggering volcanic eruptions, the problem of vesiculation and fragmentation of magmas has long challenged geologists [e.g., *Sparks*, 1978; *Gardner et al.*, 1996; *Kaminski and Jaupart*, 1998; *Sahagian*, 1999; *Papale*, 1999]. As the magma rises toward the surface it reaches a level where bubble nucleation starts. Small bubbles grow by diffusion and decompressive expansion; however for large enough vesicles [*Gaonac'h et al.*, 1996a] coalescence is more rapid and can dominate diffusion. At around 10 MPa these and other processes [*Papale*, 1999] trigger an explosive Plinian eruption. Although pure coalescence does not increase the vesicularity, even binary coalescence can generate bubbles far larger than those possible by diffusion and expansion alone [*Gaonac'h et al.*, 2003; *Lovejoy et al.*, 2003]. Its exact role has recently been hotly debated [*Klug and Cashman*, 1994, 1996; *Gaonac'h et al.*, 1996a, 1996b; *Herd and Pinkerton*, 1997; *Gaonac'h et al.*, 2003; *Blower et al.*, 2001; *Lovejoy et al.*, 2003]. In this paper, we consider multibubble coalescence which we argue can - via the percolation mechanism discussed below - create infinite simply connected regions near the observed critical **P** (the volume fraction of the gas phase). We propose that the resulting networks of infinite "bubbles" must critically weaken the magma so that if under stress it will fragment provoking an explosion.

[3] The classical explosive eruptive models fall into two classes: the first assumes that fragmentation is the consequence of brittle magmas exceeding a critical stress [e.g. *Zhang*, 1999] or critical strain [e.g. *Papale*, 1999]; in the second class the magma is modeled as a foam whose ductile fragmentation is controlled by bubble surface tension [e.g., *Mader et al.*, 1996; *Proussevitch et al.*, 1993]. Both classes require high densities of nonoverlapping (noncoalescing) bubbles, and the critical properties are modeled by purely local (single bubble scale) stresses and strains. In both classes, fragmentation is a consequence rather than a cause of stress induced rheological changes.

[4] Explosive fragmentation often occurs around vesicularity $\mathbf{P} \approx 70\%$, (e.g. *Sparks* [1978] suggests 75%, *Gardner et al.* [1996] 64%). However, when the bubble fraction gets that high very extensive bubble overlap is inevitable. In statistical physics, the general problem of connected geometric elements has been intensively studied; a highly developed "percolation" theory is now available [see e.g. *Stauffer and Aharony*, 1992]. We expect – with the modifications discussed below – at least near the critical "percolation threshold" $\mathbf{P}_c$, that the basic results of "continuum percolation" are directly applicable to magmas. If the shapes are monodisperse spherical bubbles, percolation theory shows that under very general circumstances (independent of many of the details) that as **P** approaches $\mathbf{P}_c$ there is a mathematical singularity, the size of the largest simply connected region diverges: this "bubble" will span the entire system no matter how large (Figure 1a). Under stress, the corresponding ductile or brittle magma will rapidly disintegrate, triggering an explosion.

[5] Percolation is considered here as a critical state resulting from a many-body coalescence process, with its singular onset and potentially drastic rheological consequences. The principle problem in applying it to real magmas is that in classical (monodisperse) percolation $\mathbf{P}_c$ is much too low; the most recent estimates for spheres in three dimensions (3-D) gives $\mathbf{P}_{3,c} = 0.2895 \pm 0.0005$ [*Rintoul and Torquato*, 1997]; in 2-D, $\mathbf{P}_{2,c} = 0.312 \pm 0.005$ [*Vicsek and Kertesz*, 1981]. How can we postpone percolation to much larger **P** values? The key is scaling.

2. Scaling Bubble Distributions

[6] One key prediction of binary coalescence models is that the number-size distribution of bubbles is a power law [*Gaonac'h et al.*, 1996a; *Gaonac'h et al.*, 2003; *Lovejoy et*

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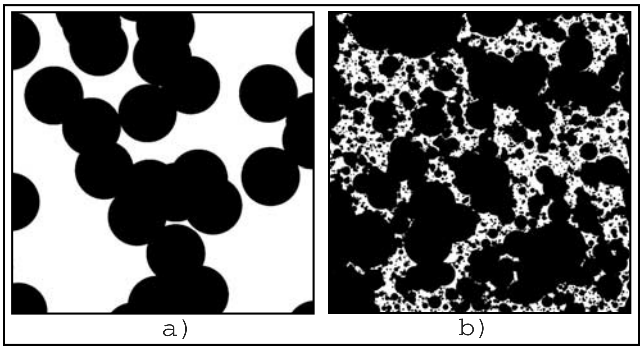

Figure 1. (a) A 1024 × 1024 simulation of continuum percolation with (monodisperse) circles (radius 100 pixels, placed in a homogeneous random way; **P** = 0.49); (b) similar to (a) except that a power law size distribution with B_2 = 0.75 was used with the same external scale ($A^* = \pi 100^2$ pixels). Here, we are slightly above the percolation threshold, **P** = 0.78.

al., 2003]. Such statistics have recently been verified by several independent groups [*Simakin et al.*, 1999; *Klug et al.*, 2002]. In a such scaling regime, the number density n(V) of the bubbles follows:

$$n(V) = \left|\frac{dN_3}{dV}\right| = \frac{1}{V}\left|\frac{dP}{dV}\right| \propto \left(\frac{V}{V^*}\right)^{-B_3-1} \quad (1)$$

where V is the volume of the gas vesicle, N_3 is the 3-D cumulative number distribution, B_3 the volumetric scale invariant exponent. For $B_3 < 1$ finite vesicularity requires a largest vesicle volume V* [*Gaonac'h et al.*, 1996a, 1996b]. Here and elsewhere the number subscripts indicate the dimension of space.

[7] Since empirically, it is easier to study vesicle area distributions from sample cross-sections rather than from volumes it is convenient to use the "naïve" formula $B_3 = \frac{2}{3}B_2 + \frac{1}{3}$ for the dimensional conversion [*Gaonac'h et al.*, 1996a]. Strictly speaking, this relationship is only valid for convex bubbles. This restriction is necessary to guarantee that when 2-D cross-sections are taken, that a single simply connected 3-D bubble gets mapped onto a single simply connected cross-section. The more sophisticated methods outlined in *Sahagian and Proussevitch* [1998] still require the convexity assumption and yield the same exponent relation. Near the percolation threshold, when bubbles are frequently very far from convex, two apparently distinct bubbles in a 2-D cross-section have a reasonable probability of ultimately proving to be part of the same complex 3-D bubble - the above relationship breaks down. However, even for pure percolation, this effect is not so large: below we see that the naïve exponent is in error by 15%.

[8] Pumice samples commonly have bubble volumes ranging over a factor of 10^6 in size; this single fact virtually mandates the use of power laws since standard exponential tailed distributions can only account for a narrow range. In (explosive) Plinian pumice, *Gaonac'h et al.* [2003] find B_2 = 0.75; extrapolating this to 3-D gives (to the nearest 0.05), $B_3 \approx 0.85$ - indistinguishable from the nonexplosive basaltic lava value *Gaonac'h et al.* [1996a] - and also that of the pumice distributions in *Simakin et al.* [1999] (see *Gaonac'h et al.* [2003] for discussion). The universality of the exponents between very different volcanic products can be explained by the binary coalescence-expansion equation results of *Lovejoy et al.* [2003]. However, in a recent paper on explosive volcanic products *Klug et al.* [2002] also finds power law distributions, but with a slightly larger exponent (B_3 = 1.1 ± 0.1 compared to our B_3 estimate of 0.85). Below, we show how this too can be explained; here via multibody coalescence. Hence independent of the characteristics of the magma and eruption style, decompression-coalescence may be the dominant process of vesicle size increase through scaling processes identified by $B_3 \approx 0.85$ from a few microns to at least a few centimeters in diameter.

3. Percolation

[9] The bulk of the percolation literature considers regular grids with connectedness defined by nearest neighbors. Also classical is "continuum percolation" where the centers of circles/spheres are uniformly randomly distributed and connectedness is defined by overlap (Figure 1a). Theory shows that, as **P** increases, the volume V^*_{per} of the largest connected region ("bubble") diverges as $V^*_{per} \propto |P - P_{3,c}|^{-3\nu_3}$ with $\nu_3 = 0.88 \pm 0.02$ [3-D; *Grassberger*, 1983]; in 2-D the largest area diverges as $A^*_{per} \propto |P - P_{2,c}|^{-2\nu_2}$ with $\nu_2 = \frac{4}{3}$ [*Den Nijs*, 1979]. At the percolation threshold $\mathbf{P}_c$ a single continuous bubble crosses an infinite system; beyond $\mathbf{P}_c$, the probability of a given bubble being connected to the infinite bubble increases exponentially fast. For the rheology, the 2-D value $\mathbf{P}_{2,c}$ may be more relevant than the slightly smaller $P_{3,c}$ since for an isotropic process at $\mathbf{P}_{3,c}$ there will almost surely exist an infinitely long bubble while at $\mathbf{P}_{2,c}$ bubble networks will be so extensive that an infinite bubble will almost surely cleave every planar cross-section.

[10] For **P** approaching $\mathbf{P}_c$ from below the number density of bubbles is a truncated power law with exponents $B_{3,per}$ = 1.186 ± 0.002 [*Jan and Stauffer*, 1998], $B_{2,per} = \frac{96}{91} = 1.055$ [*Den Nijs*, 1979]. In addition, if the bubble connectivity is empirically established on 2-D sections (i.e. assuming convexity) and if the distribution is extrapolated to 3-D using the equations above, we obtain the extrapolated 3-D exponent $B_{2-3,per} = 1 + \frac{2}{3}\left(B_{2,per} - 1\right) = \frac{283}{273} = 1.037$; the difference between $B_{2\text{-}3,per}$ and $B_{3,per}$ is purely due to the difference in the definition of bubbles.

[11] The basic features of percolation theory are extremely insensitive to the details; indeed, the exponents (ν, B_{per}, and others) - although for us crucially not the value of $\mathbf{P}_c$ - are believed to be universal; they depend only on the dimension of space. We therefore anticipate that changing the basic distribution from the usual monodisperse to power law will only change $\mathbf{P}_c$. Since a distribution of sizes allows small bubbles to fit in the interstices of larger ones, using a wide distribution of sizes necessarily raises $\mathbf{P}_c$; it can more efficiently "pack" spheres.

[12] However, it is not so easy to pack spheres to the required density while avoiding percolation. To our knowledge the main relevant studies have been with hard spheres (i.e. without overlap). For example, using hard spheres with various volume distributions (including gaussians - and even using the long tailed lognormal distribution), *Soppe* [1990], *Konakawa and Ishizaki* [1990], were unable to raise **P** above 0.65.

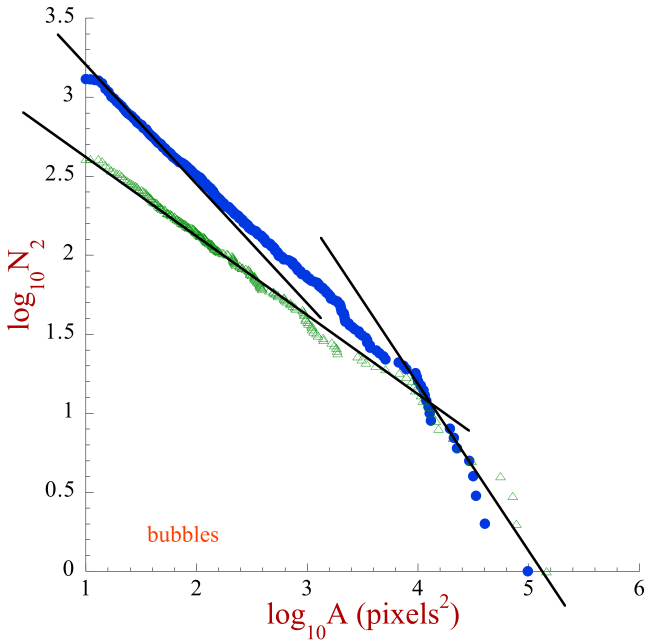

Figure 2. $N_2(A' > A)$ for bubbles from two simulations just below the percolation threshold $\mathbf{P}_c$ for B_2 (0.75 and 0.5 top to bottom, respectively; slopes shown for reference). For larger bubbles the distribution is governed by the percolation exponent $B_{2,per} = 1.05$.

[13] Once we consider power laws, the situation is quite different: a fractal construction - dubbed the "Apollonian gasket" by *Mandelbrot* [1982] - in which the interstices between 4 neighboring hard spheres (3 neighboring circles in 2-D) are iteratively filled with the largest possible sphere can in principle attain P = 1, i.e. packing with 100% efficiency. In 3D, *Anishchik and Medvedev* [1995] show how to use this to produce packings with $\mathbf{P} > 0.9$ with a range of scales of sphere diameters $\approx 10^2$. They also find that the corresponding sphere volume distribution has $B_{3,appol} = 0.82$ close to our empirical $B_3 = 0.85$. This means that we expect our distribution to be very efficient in raising the percolation vesicularity threshold $\mathbf{P}_c$. The 2-D problem has been theoretically and numerically investigated by *Hermann et al.* [1990]; they find a family of efficient packing algorithms with $B_{2,apoll}$ in the range 0.65–0.75 (close to the empirical $B_2 = 0.75$). The numerical proximity of these values was noticed by *Blower et al.* [2001] who even suggested that this Apollonian hard sphere packing ultimately *explained* the observed value (with the help of a nucleation-diffusion mechanism). Since bubbles are hardly rigid spheres, and this mechanism is fundamentally a *four* body one, it is likely to be inefficient compared to *binary* coalescence; as an explanation for the power law exponent we find this unconvincing.

4. Monte Carlo Simulations

[14] In order to study the effect of power law distributions on the percolation problem (especially to see whether or not we could reach $\mathbf{P}_c$'s of the order of 0.7), we performed 2-D Monte Carlo continuum percolation simulations of a vesiculated magma (Figure 1b). These simulations are nonclassical since they use a power law bubble size distribution. Simulations showed that $\mathbf{P}_c$ was a systematic function of B; for $B_2 = 0.75$, we obtained $\mathbf{P}_c = 0.70 \pm 0.05$. Note that there is considerable sample to sample variability and that estimates are biased (in a fairly well understood way) by finite size effects; the error cited in this estimate is the standard error estimated from many simulations, attempting to take these factors into account. This is for $\mathbf{P}_c$ in 2-D: every cross-section will almost surely be cleaved in half. The 3-D value of $\mathbf{P}_c$ must be slightly lower and it is possible that 3-D percolation could be enough to trigger explosion; this is an interesting topic for future research. In any case the precise value of the vesicularity at which explosion occurs clearly depends on the applied stresses.

[15] It is of interest to study the size distributions of simply connected bubbles near $\mathbf{P}_c$ (see Figure 2). For most of the size range, the imposed pre-percolation distribution with exponent $B_2 = 0.75$ dominates the percolation induced power law with $B_{2,per}$; this is possible because $B_2 < B_{2,per}$. However, for large enough bubbles the latter is dominant; this could explain the recent results of *Klug et al.* [2002] who found a mean exponent $B_3 = 1.1 \pm 0.1$; close to the theoretical value $B_{2-3,per} = 1.037$.

[16] If percolation is indeed the trigger for fragmentation, then the simulations can be used to determine the distribution of primary fragments (Figure 3). It is found to be a power

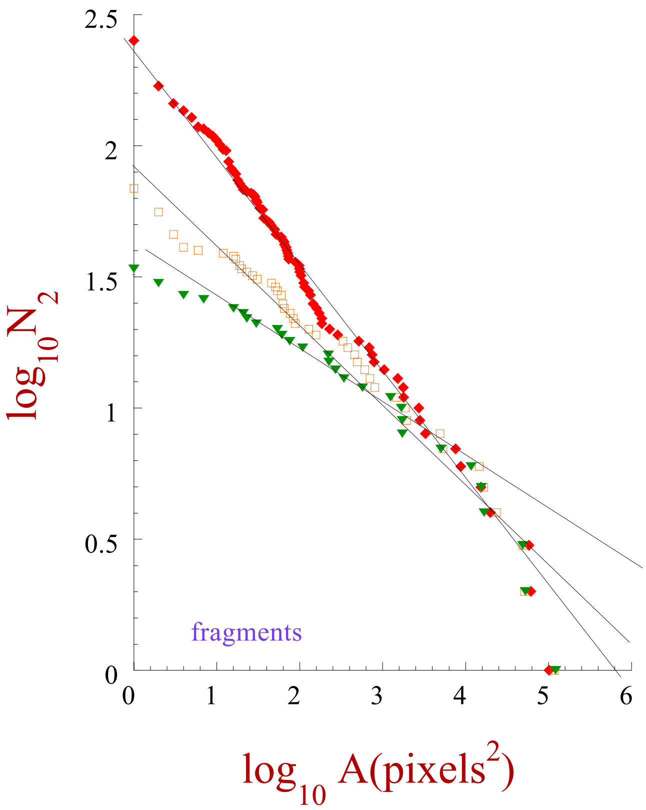

Figure 3. $N_2(A' > A)$ of the fragments (their total area including their *internal* vesicles) that would result if the magma in the simulations (such as Figure 1b) were ruptured. Three different B_2 values were chosen (0.8, 0.6, 0.4 top to bottom respectively); the fragments apparently have $B_{2,frag} = B_2/2$ as indicated by the reference lines. All three simulations were made at $\mathbf{P}$ = 0.78 (near the corresponding percolation thresholds).

law; simulations show the following relation between the exponents of the fragments and of the bubbles: $B_{2,frag} \approx \frac{B_2}{2}$; the ''naïve'' transformation from 2-D to 3-D yields $B_{3,frag} = \frac{2B_{2,frag}}{3} = \frac{B_2}{3}$. The implied values for $B_{3,frag}$ are unrealistically small: in the same Plinian deposits that *Gaonac'h et al.* [2003] found $B_3 = 0.85$ – *Kaminski and Jaupart* [1998] obtained $B_{3,frag}$ = 1 to 1.2. However, the deposits are presumably not of primary fragments. Some – if not most - fragments continue to expand, coalesce and refragment with the whole cycle repeating until either the fragments are quenched or the internal pressure is low enough. However, due to their scaling nature, it is unlikely that by themselves the latter processes will modify the B_{frag} exponents; our results give indirect support to the suggestion from *Kaminski and Jaupart* [1998] that collisions between fragments are necessary (collisions are known to be capable of yielding fragment distributions with $B_{3,frag} > 1$).

5. Conclusions

[17] At the large vesicularities prevailing near the eruption point, binary and then multibody coalescence is likely to be a dominant growth mechanism providing the key to the last phases of eruption dynamics. This is the justification for our application of percolation theory to highly vesicular magmas. At the percolation threshold many singular changes occur in the geometry of the magma; they imply a catastrophic drop in the tensile strength. In highly viscous or brittle magmas under stress they provide the trigger for an explosive release of pressure. In low enough viscosity basaltic magmas the stress will most frequently be "relaxed", hence the rheological consequences of percolation would be minor and would not affect their effusive character. However in some exceptional cases there may still be enough stress to cause basaltic Plinian explosive eruptions.

[18] Contrary to prevailing models in which fragmentation is the consequence of stress induced rheological changes, in the present paper the percolation induced fragmentation is responsible for catastrophic changes in the rheology. To be realistic such a model requires a $\mathbf{P}_c$ of the order 0.7. We showed that the observed power law bubble distributions are particularly efficient at bubble packing. Monte Carlo bubble simulations with $B_2 = 0.75$ numerically yielded $\mathbf{P}_c = 0.70 \pm 0.05$. Near $\mathbf{P}_c$, we also found evidence for the many-body (percolation) exponent $B_{2,per} = 1.05$, very close to the value claimed in *Klug et al.*'s [2002]. If this unified picture is correct, then a power law bubble distribution provides: a) an explosive fragmentation mechanism for stressed highly viscous or brittle magmas; b) an effusive mechanism in most low viscosity magmas; c) an explanation of the origin of the critical eruption vesicularity $\mathbf{P}_c$ value near 0.7; d) an explanation of the vesicle size distributions, - both those dominated by binary collisions and those directly influenced by the percolation process; e) an explanation for basaltic Plinian eruptions.

[19] **Acknowledgments.** This research has been supported by the National Sciences and Engineering Research Council of Canada. We thanks anonymous reviewers for their critical comments on the previous manuscript.

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