Scale invariance in climatological temperatures and the local spectral plateau

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ABSTRACT. Instrumental temperatures averaged both locally (over a small region); and hemispherically, as well as paleotemperatures from both ice and ocean cores are analysed statistically on time scales (Δt) of minutes to 9×10^5 years.

a) *Hemispheric temperatures* : For $5 \lesssim \Delta t \lesssim 40000$ years, characteristic hemispheric temperature changes (Δt) follow the scale invariant law : $\Delta T \sim 0.077\ \Delta t^{0.4}$ K which corresponds to a spectrum $f^{-1.8}$ where f is the frequency. For $\Delta t \gtrsim 8 \times 10^4$ years, ΔT is roughly a white noise with standard deviation $\pm$ 2.7 K.

b) *Local temperatures* : For 1 month $\lesssim \Delta t \lesssim 450$ years, local temperature spectra exhibit a flat « plateau » corresponding to a white noise magnitude $\pm$ 0.44 K (ignoring the annual cycle). For $\Delta t \lesssim$ 1 month (the « synoptic maximum »), and ignoring the diurnal cycle and its harmonics, the spectrum again follows the $f^{-1.8}$ form down to periods of the order of minutes or less.

Key words : climate, scaling, paleotemperatures, spectra, temperature, scale invariance, fractals.

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1. INTRODUCTION

Geochemically inferred paleotemperatures have conclusively established the existence of climatological temperature fluctuations at all observed time scales. Instrumental records and climatological proxy records of temperature are used in this paper to show that the energy spectrum of climatological temperature fluctuations with frequency (f) between at least (5 years)$^{-1}$ to about (40000 years)$^{-1}$ are of the scale invariant form $f^{-\beta}$ with $\beta \sim 1.8$.

We also spectrally analyse local temperature series. If we ignore the diurnal peak and its harmonics we find that approximately the same value of β is relevant on scales of roughly 1 month (the « synoptic maximum ») down to periods of the order of minutes or less. This « red noise » spectrum is also very close to the $f^{-5/3}$ spectrum predicted for the spectrum in a turbulent fluid (see section 2.1). The relevance of turbulence fluctuations up to planetary sizes, in the strongly anisotropic atmosphere, is discussed in Schertzer and Lovejoy, 1985*a*. The following paper is a considerable elaboration of the results discussed in Lovejoy and Schertzer (1983).

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The temperature $T(t)$ has fluctuations (ΔT) that are scale invariant, parameter H if they follow the scale invariant relationship :

$$\Delta T(\lambda\, \Delta t) \underset{=}{d}\, \lambda^{H}\, \Delta T(\Delta t) \qquad (1)$$

where

$$\Delta T(\Delta t) = T(t_1) - T(t_0)\,, \quad \Delta t = t_1 - t_0$$

and

$$\Delta T(\lambda\, \Delta t) = T(t_2) - T(t_0)\,, \quad t_2 = t_0 + \lambda(t_1 - t_0)$$

for arbitrary scale ratios λ (and Δt). The « $\underset{=}{d}$ » sign indicates that equality is understood in the sense of probability distributions. The random variables u, v are equal in this sense when $\Pr(u > q) = \Pr(v > q)$ for any threshold q (« Pr » indicates « probability »). The parameter $0 \leqslant H \leqslant 1$ is related to the spectral exponent β by the formula $\beta = 2H + 1$ (when the variance is finite). When $\lambda > 1$, this equation relates fluctuations at small scales (Δt) to those at large time scales ($\lambda\, \Delta t$). Note that an immediate consequence of this type of scale invariance is that moments of all orders (as well as cross correlations) vary as powers of λ. Another type of scale invariance also of geophysical interest, involves various temporal (or spatial) averages rather than the fluctuations. This type of scale invariance is associated with cascade processes and is now called « multiplicative chaos » — (Kahane, 1985; Schertzer and Lovejoy, 1985*b*, 1986). In this case only the extreme fluctuations — not all the fluctuations follow equation (1) (see section 5 for details).

In the spirit of modern physics, we may regard scale invariance (abbrievated « scaling ») as a basic symmetry principle which is expected to hold in the absence of symmetry breaking mechanisms. This approach shows promise in meteorology (Schertzer and Lovejoy, 1984; Lovejoy and Schertzer, 1986*a*) and is closely related to the approaches used in both renormalisation and fractal modelling. A problem is « symmetric » when it is invariant under a class of transformations (e.g. rotations). In the simplest case of scaling fluctuations in one dimension (Δt is a scalar), the fluctuations are « symmetric » because they are invariant under the mathematical operation of magnifying the time scale by λ and rescaling the fluctuations by λ^H. In the more general case of interest for example when considering fluctuations of temperature in space as well as in time, it is shown that fluctuations may be scale invariant when they are « symmetric » under the action of a more general class of transformations involving not only magnification but also differential stretching and rotation.

If over a certain range in time, fluctuations occur at all scales, with no characteristic period, then over this interval, the fluctuations are scaling. Intuitively, scaling is the simplest way that fluctuations at different scales may be related to each other. An investigation of the limits of scaling is therefore necessary if the basic time scales of a system are to be understood. Strictly speaking, the scale invariance of the fluctuations should be established by directly evaluating the probability distributions of ΔT over a wide range of time lags Δt. However, this has the disadvantage of requiring very long individual series (see Lovejoy, 1981; Schertzer and Lovejoy, 1985*a* for meteorological examples), and is postponed until section 5. In sections 2, 3, and 4 we rather investigate various second order statistics — the structure function, spectrum, and rescaled range — all of which show clear evidence of scaling behaviour.

To our knowledge, until recently the only investigations of scaling in series with climatological significance were Hurst (1951), and Mandelbrot and Wallis (1969). Using the range and rescaled range statistics, they were able to substantiate the scaling hypothesis for river discharges, tree rings, varves and precipitation series over various ranges of time scales. The findings of Lovejoy and Schertzer (1983) and the following do not wholly confirm their analyses in that a significant difference is found between locally and globally averaged temperature series with the former displaying a long « spectral plateau » apparently absent in the latter. Both the onset of the plateau at $\Delta t \sim 1$ month, and its end at $\Delta t \sim 450$ years mark breaks in the scaling of *local* (but not necessarily global) temperatures. Another relevant reference is Berger and Pestiaux (1984) and Pestiaux (1984), where spectral analyses of ocean core paleotemperatures show that the spectral form $f^{-1.8}$ does not extend to frequencies much less than about $(10^4 \text{ years})^{-1}$.

The scaling spectrum $f^{-\beta}$ has an « excess » of energy at low frequencies and thus (in the sense of Gilman *et al.*, 1963) is a « red noise ». In climatological temperatures, the importance of this excess has long been recognized (e.g. Lamb *et al.*, 1966). It has also been known for some time that the « background » of empirical spectra often have broadly $f^{-\beta}$ forms (e.g. Hays *et al.*, 1976). Furthermore, several authors (e.g. Hasselmann, 1976; Bhattacharya *et al.*, 1982; Le Treut and Ghil, 1983) have proposed theoretical models with $f^{-\beta}$ background spectra with $\beta = 2$ (the latter model can also allow for $\beta < 2$ — Ghil, private communication). The present paper builds on these results in two directions. First, we show that the scaling extends much further than previously believed, and second, that the exponent β is significantly less than 2 ($= 1.82 \pm 0.04$) and is very close to that determined for local temperature fluctuations on a scale of minutes (and perhaps seconds) up to several weeks. This raises the intriguing possibility that the scaling of the global mean temperature — if it could be measured at these scales — may continue down to periods as short as seconds. In that case, for periods less that ~ 40000 years, climatological and meteorological temperatures would differ fundamentally only in the time and space scales over which they are averaged.

2. ANALYSIS OF INSTRUMENTALLY DETERMINED TEMPERATURE SERIES

2.1. Local temperatures

Temperature is of necessity a spatially averaged quantity. As temperatures are averaged over progressively larger regions, eventually covering the entire globe, the amplitude of ΔT for a fixed Δt will decrease because averaging « smooths out » any large local variations. It is therefore important to carefully distinguish the globally averaged fluctuations (referred to as « global »), from those averaged over only a small part of the earth's surface (referred to as « local »).

Figure 1*a* shows the energy spectrum of temperature fluctuations from the climatological recording station in Macon, France from 1951-1980. At the high frequency end $(2 \text{ days})^{-1}$ to about $(1 \text{ month})^{-1}$, the spectrum follows the straight line on a log-log plot corresponding to $E(f) \propto f^{-1.8}$. Over the range $(2 \text{ days})^{-1}$ to (6 h), the spectrum is dominated by the diurnal peak and various subharmonics. At higher frequencies, figure 1*b* shows that the $f^{-1.8}$ form continues down to at least $(10 \text{ mn})^{-1}$. Other evidence for $\beta \sim 1.8$ in this high frequency region can be obtained from spatial (wavenumber) spectra by the commonly used technique of transforming between spatial and temporal statistics with an appropriate velocity factor (Taylor's hypothesis of « frozen turbulence »). If this transformation is valid it implies that the values of β for both frequency and wavenumber spectra should be the same. Relevant empirical results on temperature wavenumber spectra are Tatarski (1956) ($k^{-1.82}$) and Deschamps *et al.*, (1981) ($k^{-1.8 \pm 0.1}$). These spectra are very close to the $f^{-5/3}$ form predicted by Corssin (1951) and Obukhov (1949) for isotropic, homogeneous turbulence (for a

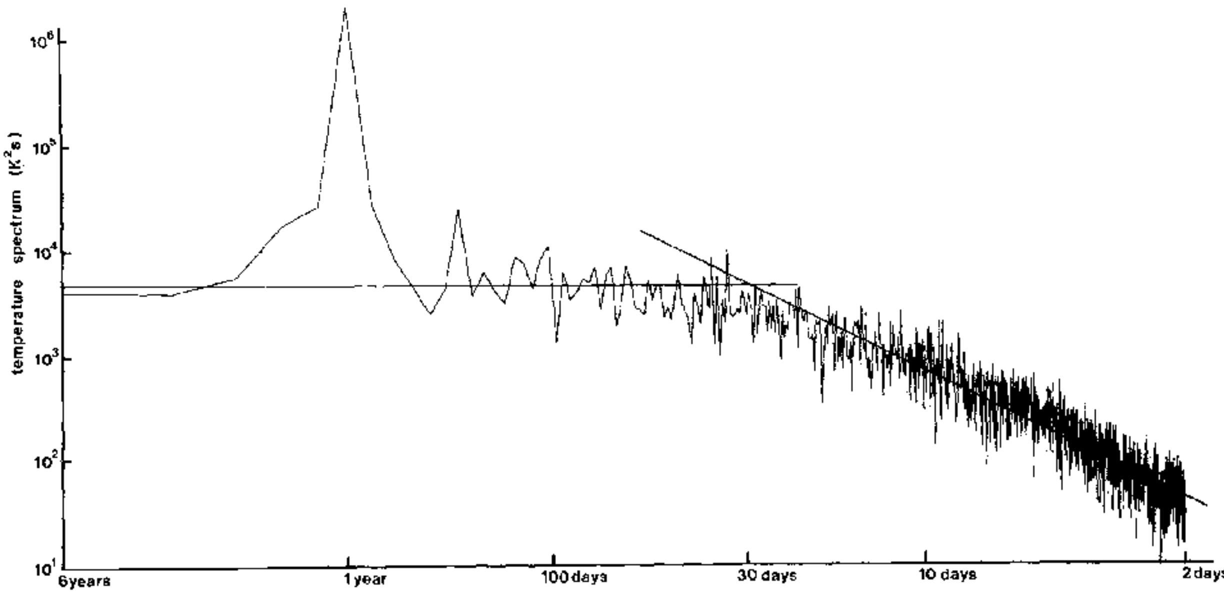

Figure 1*a*

The average of 5 temperature spectra from consecutive 2160 *days* ($\sim$ 6 *years*) *periods* (*the 5 independent spectra were averaged in order to permit an accurate estimate of the slope*). *The temperatures used were mean daily temperatures from Macon, France, collected during the period* 1951-1981. *The daily means were calculated by averaging 8 three hourly measurements. The shortest resolvable period is therefore 2 days. The sloping straight-line on this* log-log *plot is a spectrum of the form* $f^{-1.8}$. *The local spectral plateau is indicated by the flat straight line.*

contemporary discussion, see Herring *et al.* (1982) and, for extensions to the anisotropic turbulence in the atmosphere, see Schertzer and Lovejoy, 1985*a*). An interesting possibility is that this « background » follows $f^{-1.8}$ up to frequencies of $(1\ \mathrm{s})^{-1}$.

The high frequency scaling regime clearly breaks down for $f \lesssim (1\ \mathrm{month})^{-1}$ and the spectrum becomes fairly flat — the « spectral plateau » referred to earlier. This plateau, « white noise » spectral region shows that $E(f)$ is almost constant at these scales. If we had plotted $fE(f)$ *vs* $\log f$ (rather than $\log E(f)$ *vs* $\log f$) as was done in Kolesnikova and Monin (1965), we would have obtained a peak instead of a smooth transition at $f \sim (20\ \mathrm{days})^{-1}$ — the « synoptic maximum ». These authors estimated the period of the maximum to be between 1 and 3 weeks. It is generally agreed that this is the minimum time scale for fluctuations of planetary size.

With the exception of the sharp peak at $f = (1\ \mathrm{year})^{-1}$, the spectral plateau continues down below the lowest frequencies available at Macon $(30\ \mathrm{years})^{-1}$ to at least $(300\ \mathrm{years})^{-1}$, which is the frequency associated with the longest instrumental temperature series (see Mason's (1976) analysis of Manley's (1974) central England data). A way of exhibiting the spectral plateau which will be useful in later discussions is to consider the structure function $[S(\Delta t)]$. This is defined as the square root of the mean square fluctuation :

$$S(\Delta t) = \langle [\Delta T(\Delta t)]^2 \rangle^{1/2} .$$

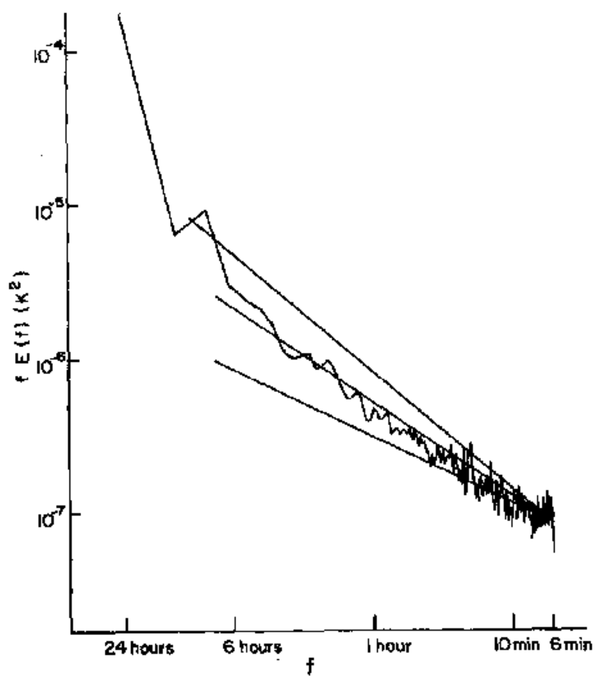

Figure 1*b*

A plot of $\log f E(f)$ vs $\log f$ *for the average of* 36 *spectra taken over consecutive* 24 *h periods sampling every* 3 mn *from a meteorological shelter taken from Belzanne,* 1985, *figure* 5. *The straight lines correspond to* $E(f) \sim f^{-2}, f^{-1.8}$, *and* $f^{-1.6}$ *spectra* (*top to bottom respectively*). *For frequencies lower than* $(6\ \mathrm{h})^{-1}$ *the spectrum is dominated by the diurnal peak and its harmonics, whereas for the higher frequencies the spectrum is clearly near the* $f^{-1.8}$ *form.*

$S(\Delta t)$ gives us direct information about the likely magnitude of temperature differences for time intervals Δt. Figure 2 shows the $S(\Delta t)$ function for the Manley series, showing that over this period, $S(\Delta t)$ is almost constant — the amplitude 0.88 K corresponds to a white noise of amplitude $\pm$ 0.44 K. This is the structure function counterpart of the local spectral plateau.

At the very low frequencies corresponding to the period of the interglacials, we know that $S(\Delta t)$ must be significantly larger. For example, Emiliani and Shackleton (1974) estimate that there were strong oscillations in the temperature of the entire earth in the last 4-6 $\times$ 10^5 years of amplitude $\pm$ 3 K. Thus, during a half-period (Δt) of 3-5 $\times$ 10^4 years, $S(\Delta t)$ has the value 3 K (there is some agreement that this amplitude is between 2 and 4 K). Therefore, at these scales, the entire temperature of the earth varies considerably more than that of local regions at scales of hundreds of years. A convenient way of visualizing this is to plot the « interglacial window » (i.e. the region of the $S(\Delta t) - \Delta t$ plane where the global $S(\Delta t)$ function must cross), as shown as the square box in figure 2. At some point, apparently for $\Delta t > 300$ years, the spectral plateau must end and $S(\Delta t)$ will rise and pass through the « window ». Below, we argue, that the end of the spectral plateau occurs at $\Delta t \sim 450$ years.

2.2. Hemispheric temperatures

In the previous sub-section, we have argued that over short-time scales, global average temperatures have considerably smaller amplitude variations than local ones whereas at long enough time scales, the amplitude of the local fluctuations are likely to be primarily due to the overall temperature variation of the entire globe. This may be expressed by saying that the global variations eventually become large enough to dominate the local ones. To illustrate this idea, we analyse the Budyko (1969) and Jones *et al.* (1982) northern hemisphere temperature series for the last century (unfortunately, data from the southern hemisphere is too sparse to warrant the estimation of global averages). Figure 2 shows the $S(\Delta t)$ function for these two series. The Budyko (1969) series is not shown for $\Delta t < 5$ years since a moving 5 years average was used. Note that although the Jones *et al.* (1982) (« optimum grid ») data extend down to periods of 1 month (see fig. 3), the average annual cycle was removed.

Estimating the mean hemispheric temperature is not a simple task because measuring stations are distributed highly inhomogeneously over the surface of the earth. Lovejoy *et al.*, 1986 show that in geophysical measuring networks the inhomogeneities can occur over a wide range of scales, reflecting the fact that the fractal dimension of the stations (considered as a set of points on a sphere) is less than 2 (the value corresponding to a uniform distribution). Interpolating onto a higher dimensional space (e.g. a uniform grid) is therefore a difficult task and generally involves biases in the statistical properties of the measured fields (see Lovejoy *et al.*, 1986 ; Ladoy, 1986 ; Ladoy *et al.*, 1986 ; Lovejoy and Schertzer, 1986*b* for more details). The difference between the Budyko and Jones *et al.* curves in figure 2 is due both to the different estimation techniques and data bases used. Budyko subjectively selected « representative » measurements from data rich regions in an attempt to obtain a uniform sampling of temperatures. Jones *et al.*, (1982) chose a particular objective technique which linearly interpolates temperatures onto a regular grid, before the averages are taken.

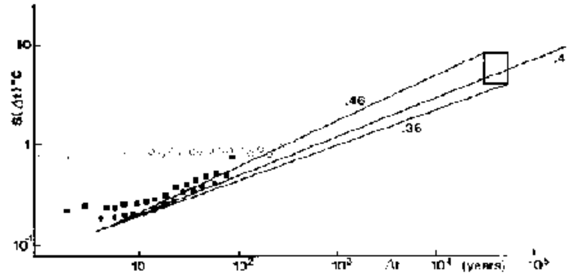

Figure 2

The structure function $[S(\Delta t)]$ for the instrumental temperatures analysed in the text. The closed circles are calculated from the Budyko (1969) series of northern hemisphere temperatures, and the squares, from the Jones et al. *(1982) northern hemisphere series. Note these two are approximately parallel. The straight line, slope 0.4 is the function $S(\Delta t) \sim 0.077\,\Delta t^{0.4}$ which has been fit to the large Δt part of the Budyko (1969) $S(\Delta t)$ function. The lines slope 0.36, 0.46 are the shallowest and steepest respectively that are compatible with ; i) $S(5) = 0.15$ K ii) scaling from 5 years out to the « interglacial window » shown as rectangle.*

Although the sampling methods differ, we would not expect different averaging or sampling methods to alter either the overall form of the $S(\Delta t)$ function or of the spectrum. In spite of these differences, figure 2 shows that the *ratio* of the $S(\Delta t)$ functions is roughly a constant equal to 1.20. This important point requires some discussion. The situation would be quite different if the two series differred primarily by a random measurement error. In that case, the difference between the variances $[S^2(\Delta t)]$ would be constant (equal to the sum of the error variances), and would imply that log-log plots of the Budyko and Jones *et al.* curves would approach each other for large Δt. In particular, if we distinguish the Budyko and Jones *et al.* $S(\Delta t)$ functions by the subscripts « B », « J » respectively, then the plot of $\log (S_B - S_J)^{1/2}$ *vs* $\log \Delta t$ will have a slope ~ 0.4 if the fluctuations have a constant ratio, but will have slope 0 if the *difference* between the two is constant. Such an analysis does indeed support the constant ratio hypothesis.

This constant ratio may be explained if we consider that neither series has sufficient sampling density to measure the true northern hemisphere temperature only to allow for more or less spatially averaged estimates. If, over the scales considered (here $\Delta t > 5$ years), the sampling and averaging do not introduce a characteristic time then they will not change the shape of the $S(\Delta t)$ function, hence, difference will be a constant ratio.

We therefore interpret the fact that $S_B < S_J$ to indicate that the Budyko series has a higher degree of spatial averaging. It is therefore probably closer to the true $S(\Delta t)$ function, and will be used below. The most important point to note about these empirical $S(\Delta t)$ functions is that unlike the Manley (local) temperature $S(\Delta t)$ function, when Δt is sufficiently large, these

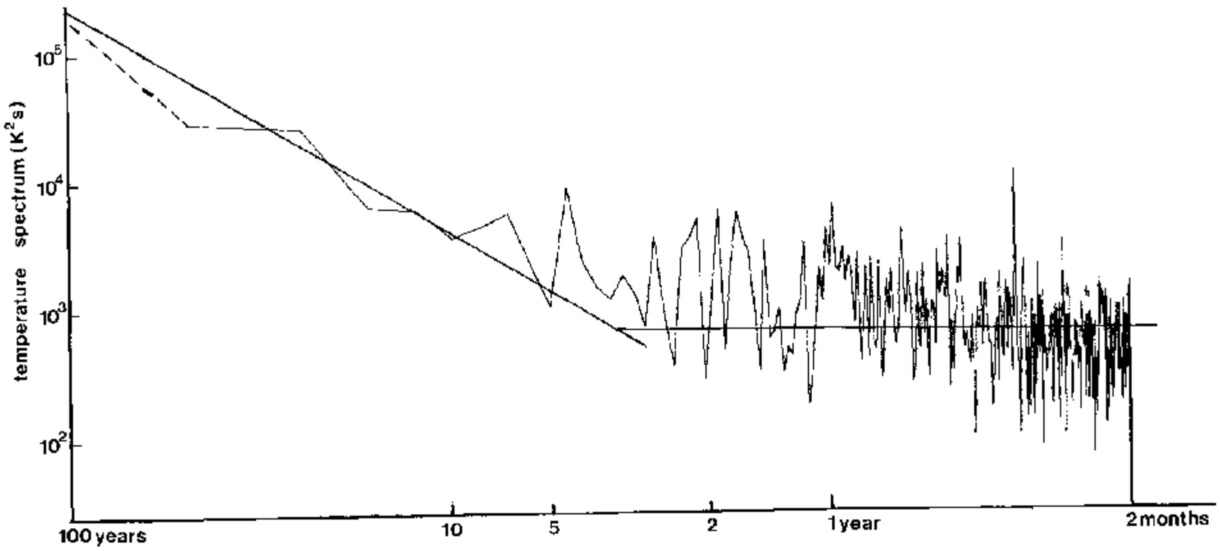

Figure 3

The temperature variance spectrum of the northern hemisphere estimated from the Jones et al. (1982) *series (note that the annual cycle has been removed). Two spectra from consecutive 50 years periods were averaged (to reduce the fluctuations), and the single point corresponding to a period of 100 years (obtained from the spectrum of the entire 100 years series), was added separately (shown as dashed line). The straight line on this log-log plot is a spectrum of the form $f^{-1.8}$. The series had a resolution of 1 month so that the smallest period resolvable is 2 months.*

hemispheric $S(\Delta t)$ follow the scaling function :

$$S(\Delta t) = 0.077\ \Delta t^H\ , \quad H \sim 0.4 \quad \text{(Budyko, 1969)}$$

$$= 0.092\ \Delta t^H\ , \quad H \sim 0.4 \quad \text{(Jones } et\ al., \text{ 1982)}$$

the former is indicated as the straight line in figure 2. Note that the exponent $H \sim 0.4$ corresponds to a spectral exponent of $\beta = 2H + 1 = 1.8$. Figure 3 shows a log-log plot of the spectrum which confirms that for low frequencies $\beta \sim 1.8$. At frequencies higher than (5 years)$^{-1}$, there is apparently a flat plateau. The exact point of transition is somewhat difficult to judge : the $S(\Delta t)$ curve suggests it occurs at $\Delta t \sim 8$ years, while the spectrum, $\Delta t \sim 3$ years. We choose the geometric mean 5 years although this is obviously a crude estimate. It is not clear whether, as in the local temperature spectrum, this is true spectral plateau, or whether it is an artifact resulting from the difficulty of accurately measuring such small temperature fluctuations. To put this problem in perspective, the variance associated with this high frequency noise is (0.21 K)^2. In contrast, if we assume that the scaling (straight-line) in figure 2 continues down to 1 month, then a measurement precision of $0.092\,(1/12)^{0.4} = 0.034$ K would be required to clearly show it. For comparison, Jones *et al.* (1982) cite a difference of between 0.16 K and − 0.08 K for the Budyko (1969) series, and that of another series compiled by Vinnikov *et al.* (1980). Existing series may therefore not be adequate for resolving this problem.

If we extrapolate the low frequency scaling using the equation $S(\Delta t) \doteq 0.077\ \Delta t^{0.4}$ for Δt up to 40000 years, we find that it passes right through the « window » defined by the interglacials. Indeed, if we assume that between $\Delta t = 5$ years (where $S(\Delta t) = 0.15$ K for the Budyko series) and the limits of the interglacial « window » in figure 2, that there is no fundamental time scale (i.e. $S(\Delta t)$ is of the form Δt^H) then H is constrained to lie in the range $0.36 \lesssim H \lesssim 0.46$. Since the empirical H lies in this interval, a simple extrapolation from the current northern hemisphere $S(\Delta t)$ function therefore is consistent with the frequency and magnitude of the interglacials. In the next section, we show that this behaviour, including the value of H, is also fully consistent with many of the existing paleotemperature series.

Note that at $\Delta t = 5$ years, the average northern hemisphere temperature fluctuations ($\pm$ 0.074 K) are ~ 6.0 times less than the Manley (local) fluctuations of $\pm$ 0.44 K. The local temperature fluctuations therefore dominate the hemispheric ones until $0.077\ \Delta t^{0.4} > 0.88$ or until $\Delta t \gtrsim 450$ years — a result which is consistent with both the Manley series, with the Bryson and Kutzbach (1974) estimated spectra (from botanical records) and, with the spectral inflection point at 400 years shown in Mitchell (1976).

3. PALEOTEMPERATURE SERIES ANALYSIS

3.1. Ice cores

The investigation of the scaling hypothesis at scales of the order of centuries or longer, requires the use of paleotemperature (proxy) data. Probably the most reliable for this purpose are the oxygen isotope ratios (O^{18}/O^{16}) determined from Arctic and Antarctic ice cores. Ice cores have a deficit of O^{18} because the lighter (H_2O^{16}) evaporates more readily from the oceans than the heavier H_2O^{18}. This deficit is highly correlated with the temperature at the time of evaporation and deposition (see Dupplessy (1978) for a discussion). If this is true, then we expect O^{18}/O^{16} ratios to provide an estimate of local temperature because the local environment over the ice cap is expected to strongly influence these processes. It would seem reasonable to assume that the O^{18}/O^{16} ratios in these cores are proxies for the mean temperature of a region greater than Manley's « central England » area, but nonetheless, considerably smaller than the entire northern hemisphere. We therefore expect that at sufficiently short time scales (where the local fluctuations dominate the hemispheric ones), the $S(\Delta t)$ function will exhibit a flat plateau. However due to the larger effective area of spatial averaging, it should have a somewhat smaller value of $S(\Delta t)$ than in the corresponding plateau region for central England. Again, for $\Delta t \gtrsim 450$ years, the hemispheric fluctuations are likely to dominate yielding $S(\Delta t) \sim 0.077\,\Delta t^{0.4}$ behaviour.

Figure 4 shows the $S(\Delta t)$ functions for both arctic and antarctic series, taken from Johnsen *et al.*, (1972). We used their dating and the linear temperature calibration constant was adjusted so that these $S(\Delta t)$ functions would lie on the lines $S(\Delta t) = 0.077\,\Delta t^{0.4}$ on the log-log plot shown in figure 4 (i.e. the curves were moved up and down on the log-log plot until the straight-line sections matched this function). Note that when this is done, the $S(\Delta t)$ curves pass through the interglacial « window », reaching a maximum at about $6\text{-}8 \times 10^4$ years (a behaviour characteristic of $S(\Delta t)$ functions of oscillatory series). For the largest Δt, the $S(\Delta t)$ function corresponds to the plausible temperature fluctuation of $\pm$ 2.7 K. The $S(\Delta t)$ values were obtained from both high resolution (every ~ 100 years for 2×10^4 years) as well as low resolution (every ~ 700 years for the last 1.2×10^5 and 9×10^4 years for arctic and antarctic series respectively).

In this analysis we do not wish to minimize the calibration problems inherent in the use of paleotemperatures. For example, we have ignored the possibility of a small non-linear correction to the O^{18}/O^{16} temperature relationship (see Dupplessy, 1978). We have also assumed that the sample dating was correct although as long as this contained only a linear error (corresponding to a left-right shift of the $S(\Delta t)$ points in figure 4), it could only be distinguished from a linear temperature calibration error (an up-down displacement) at the curved (small and large Δt) ends of the $S(\Delta t)$ function. Although non-linear calibration problems undoubtedly influence the shape of the $S(\Delta t)$ function, as long as they are of second order they should not alter the scaling exponent H found in the scaling regime.

It is of considerable interest to clarify the nature of the maximum in $S(\Delta t)$ at scales of $\Delta t \gtrsim 40000$ years, which are only poorly resolved by these series. In order to do this, we turn to the examination of ocean cores, which make it possible to investigate $S(\Delta t)$ up to $\Delta t \sim 9 \times 10^5$ years.

3.2. Ocean cores

The interpretation of deep-sea cores is not as straightforward as that of ice cores. For example, O^{18}/O^{16} ratios may be obtained from the calcium carbonate of fossil plankton. In this case, the ratios depend not only on the

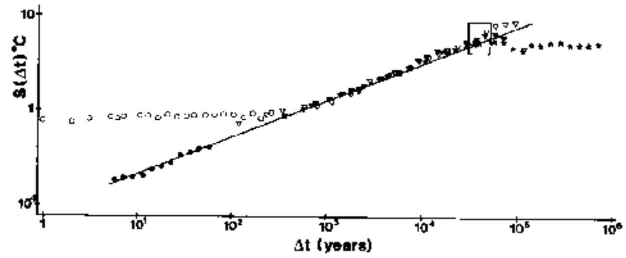

Figure 4

The composite structure function $S(\Delta t)$ showing the Manley series local temperatures (open circles), Budyko northern hemisphere (closed circles), antarctic ice core (closed triangles), arctic ice core (open triangles), ocean core (asterixes). The rectangle is the « interglacial window » according to Emiliani and Shackleton (1974), and the straight line is the scaling function $S(\Delta t) \sim 0.077\,\Delta t^{0.4}$.

temperature of the surface layer where the plankton once lived; but also on the O^{18}/O^{16} ratio in their surrounding water, which in turn depends on the volume of water in the ice caps. It is now generally agreed that the latter is the dominant effect (see Dupplessy, 1978), implying that the ocean core ratios are best interpreted in terms of the volume of water in the ice caps. However, on time scales of $\sim 10^4$ to 10^6 years, fluctuations in the size of the ice caps are correlated with the mean global temperature (although probably with an associated time lag — Bhattacharya *et al.*, 1982, Le Treut and Ghil, 1983). Other complications include bioturbation effects in the sediments from which the cores are taken (e.g. Goreau, 1980). Indeed, Dalfes *et al.* (1984) has even suggested that at least for scales $\Delta t \lesssim 10^4$ years, the latter might in fact be a scaling, first order effect and could therefore modify H for shorter time scales. In spite of all these complications, ocean cores may still be expected to permit us to examine the extent of the scaling regime and at least to get a rough idea of the fluctuations at very long periods. Let us stress that the basic conclusions of this paper (i.e. the existence of a spectral plateau, and global scaling up to ~ 40000 years) are based entirely on the instrumental and ice core series discussed earlier. The ocean cores are only used to get a feel for the temperature variations over very long time periods — especially for $\Delta t \gtrsim 10^5$ years.

In figure 4, we have plotted data analysed from Shackleton and Opdyke (1973). The $S(\Delta t)$ curve obtained has a scaling behaviour, with $H \sim 0.4$ for roughly an order of magnitude in Δt up to $\Delta t \sim 4 \times 10^4$ years, after which it is fairly constant. The (linear) calibration was performed by aligning the $H \sim 0.4$ part to the ice core data using the dating specified in Shackleton and Opdyke (1973). This curve fits the ice data fairly well except for the region near the peak (where the latter has less statistical significance anyhow). If this peak is real, then it is an indication of oscillatory behaviour of ΔT on a scale of $2 \times 4 \times 10^4 = 8 \times 10^4$ years (the factor 2 is necessary because 4×10^4 corresponds to a half-period).

Finally, it should be noted that the recent spectral analysis of ocean cores in Berger and Pestiaux, 1984; Pestiaux, 1984 tends to confirm that the $H \sim 0.4$ behaviour ($f^{-1.8}$ spectrum) cannot continue to frequencies much below $(10^4 \text{ years})^{-1}$.

4. THE ACCURACY OF THE SCALING EXPONENT H

It is important to establish the accuracy of our estimate of H since the amplitude of ΔT for $\Delta t \gtrsim 40000$ years is very sensitive to its exact value (as indicated in figure 2). For example, $H = 1/2$ corresponding to the $\beta = 2$ « background spectrum » yields a temperature change of $13.4 = \pm 6.7$ K which is probably too large to be compatible with other estimates of the magnitude of ΔT, and $H = 1/3$ (the value corresponding to the $f^{-5/3}$ Kolmogorov spectrum) yields $\Delta T = 3.0 = \pm 1.5$ K, which is probably too small.

In order to accurately measure the value of H, a robust measure of long-run dependence known as R/S analysis was used (see Mandelbrot, 1972). This statistic is robust, because it is insensitive to large fluctuations around the long-term trend. It has the disadvantage that it is biased for short time series. When it was applied to the Arctic and Antarctic ice core series in the range $4 \times 10^3 \lesssim \Delta t \lesssim 4 \times 10^4$ years, this method yielded $H = 0.420$, and 0.404 respectively (with correlation coefficient of 0.995, 0.985 respectively). We therefore estimate $H = 0.41 \pm 0.02$. However, a formula more precise than $S(\Delta t) \sim 0.077\ \Delta t^{0.4}$ is not warranted because of the uncertainties in the values of $S(1)$. Note that care has been taken to apply R/S analysis only over scales not affected by the spectral plateau.

5. THE SCALING OF EXTREME FLUCTUATIONS

In the preceding sections we have examined various second order statistics of the temperature fluctuations. If equation (1) holds exactly then the probability distribution of the fluctuations scale, and hence not only the second order statistics but those of all other orders also scale. However, if scale invariance is associated with averages and not fluctuations (as it does in multiplicative chaos), equation (1) still holds but now only for the extremes (large ΔT) (e.g. Schertzer and Lovejoy, 1985*b*). In any case the scaling of the extremes is important in its own right because it is a fundamental characteristic of the climate's intermittency.

To investigate the scaling of the probability distributions determine Pr $(\Delta T'(\Delta t) > \Delta T)$ for various time lags Δt. This is the probability that a random fluctuation $\Delta T'(\Delta t)$ exceeds a fixed ΔT (in the following the argument Δt will be understood implicitely). If equation (1) holds, then the distributions are scaling with parameter H. Increasing time scales by a factor λ ($\Delta t \to \lambda \Delta t$) increases fluctuations by λ^H hence on a plot of $\log(\Pr(\Delta T' > \Delta T))$ against $\log \Delta T$ this yields a linear shift of $H \log \lambda$. If only the extremes scale, then the shift is constant only for the probability tail (large ΔT). Figure 5 shows the probability distributions when $\lambda = 4, 16, 64$ for both the antarctic ice core and for the Jones *et al.*, 1982 northern hemisphere data. Although the distributions are quite « noisy » the tails if not the rest of the distributions are fairly similar in shape for the different lags. Furthermore for Δt within the previously

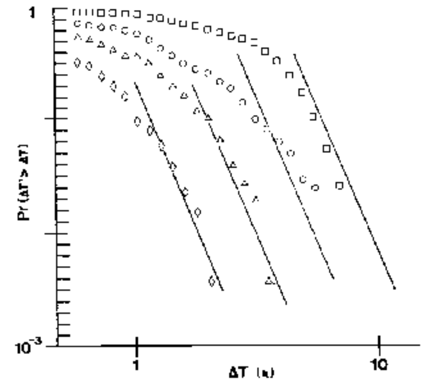

Figure 5*a*

A log-log plot of $\Pr(\Delta T' > \Delta T)$ *which is the probability of a random temperature fluctuation* $\Delta T'$ *exceeding a fixed threshold* ΔT. *The data used are from the antarctic series with the linear temperature calibration obtained from figure 4. The curve to the far left is for fluctuations over intervals* (Δt) *of* 350 *years. The remaining curves, from left to right are obtained by increasing* Δt, *by factors of* 4 (*i.e.* 1400, 5600, 22400 *years respectively*). *The straight lines indicate the functions* $\Pr(\Delta T' > \Delta T) = (\Delta T/\Delta T^*)^{-\alpha}$ *with* $\alpha = 5$ *and the amplitude of the fluctuations* (ΔT^*) *varying as* $\Delta T^* \sim \Delta t^H$ *(as required by scaling), with* $H = 0.4$.

defined scaling regimes (the entire range in figure 5*a*, and the 16 and 64 year curves in figure 5*b*), the shift is reasonably close to 0.4 log 4 as expected for $H = 0.4$, $\lambda = 4$.

Concentrating on the extremes (large ΔT), figure 5 displays another important feature — the probability « tail » is nearly straight. For reference straight lines of slope $-\alpha$ separated by H log 4 are shown with $H = 0.4$, $\alpha = 5$.

Empirically almost exactly this type of hyperbolical probability tail (i.e. $\Pr(\Delta T' > \Delta T)\,\Delta T^{-\alpha}$ for large ΔT), has been found in local and regional daily temperatures (Lovejoy and Schertzer, 1986*a*) as well as in a variety of other meteorological fields such as the rain, wind and potential temperatures (see e.g. Lovejoy, 1981; Lovejoy and Mandelbrot, 1985; Schertzer and Lovejoy, 1985*a*; Lovejoy and Schertzer, 1985). Theoretically, hyperbolic probability tails may be expected to occur as a result of cascade processes (Mandelbrot, 1974; Schertzer and Lovejoy, 1983, 1985*b*).

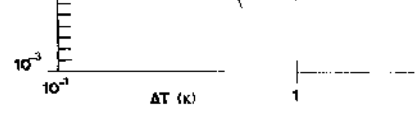

Figure 5*b*

The same as figure 5a except for the northern hemisphere annual average (Jones et al., *1982) data. The curves (from left to right) are for* $\Delta t = 1$, 4, 16, 64 *years. The straight lines are for the same values of* H, α, *as in figure 5a. This figure confirms that the scaling regime is obtained only for* $\Delta t > 4$ *years.*

6. INTERPRETATION

6.1. Global scaling

If climatological temperature fluctuations are to be eventually explained by the action of invariant physical laws, then it is important to consider the question of the stationarity of the temperature. The preceding analysis shows that we must make a basic distinction between the true stationarity of the process, and its apparent non-stationarity for periods of less than 40000 years. True stationarity means that the process is invariant with respect to translations along the time axis — hence that the ensemble mean temperature is independent of time. In the present case, temperature averages over periods of less than 40000 years will vary due to the scaling behaviour. This apparent non-stationarity is responsible for the difficulty in defining climatological temperatures for shorter periods. However, over longer periods, the break in the scaling does permit average temperatures to be well defined. If, we examine temperature increments (i.e. the series defined by the first differences of T), the situation is much simpler, because, Lovejoy and Schertzer (1983) have verified that the latter are apparently stationary over the entire range of 5 to 9×10^5 years.

With the above discussion in mind, it is possible to imagine many different mechanisms to account for the empirical $S(\Delta t)$ function. For example, dynamical systems theory might lead us to interpret the change in slope at $\Delta t \sim 40000$ years as marking the boundary (characterized by the magnitude $\pm$ 2.7 K) of a regime dominated the cascade of temperature variance flux to shorter and shorter time scales *via* non-linear interactions. Another interpretation, not necessarily in contradiction to the first, is that in the scaling regime, the temperature changes by the accumulation of random fluctuations. If these fluctuations were independent, then the temperature would vary in the same way as the co-ordinate of a Brownian particle ; we would have obtained (as in Hasselman, 1976) $H = 1/2$. The fact that $H < 1/2$ indicates that fluctuations have a tendency to cancel, even over very long time periods. Physically, this could result from the existence of negative feedback between the various parts of the climate system operating over a wide range of time scales. Note that the background spectrum while being typical of cascade processes, does not in itself permit us to draw conclusions about the forcing mechanism — whether astronomical or otherwise (for the former, see the review Berger, 1980).

6.2. The local spectral plateau

For local temperatures, we have argued that there is spectral plateau extending down to $f \sim (450 \text{ years})^{-1}$. However, we have seen that for hemispherically averaged temperatures, the plateau, if it is real, ends at f $(3 \text{ years})^{-1}$. We therefore, expect the plateau to diminish in size as the area over which the temperature is averaged is increased. A detailed study of this phenomenon would be illuminating.

7. CONCLUSIONS

We may summarize our results as follows :

a) Local temperatures have a scaling regime extending from at least several minutes to month with $\beta \sim 1.8$ ($H \sim 0.4$).

b) For scales of 1 month to 450 years, the local temperatures have a flat spectrum which we call the local spectral plateau, of amplitude $\pm$ 0.44 K. After 450 years, they are dominated by hemispheric variations, and apparently follow the scaling law $S(\Delta t) \sim 0.077\,\Delta t^H$ with $H \sim 0.4$ up to $\Delta t \sim 4 \times 10^4$ years.

c) For $\Delta t \gtrsim 5$ years, the hemispheric fluctuations follow the scaling law $0.077\,\Delta t^{0.4}$ until $\Delta t \sim 4 \times 10^4$ years. For $\Delta t \lesssim 5$ years, these fluctuations are apparently characterized by a spectral plateau, although this could be an artifact, if the climate network has an accuracy lower than $\pm$ 0.075 K. In this case, the scaling may continue to significantly higher frequencies.

d) For $\Delta t \gtrsim 4 \times 10^4$ years, the temperature variations are roughly stationary, yielding oscillatory behaviour with period $2 \times 4 \times 10^4 = 8 \times 10^4$ years with an amplitude $\pm$ 2.7 K.

Although we believe that the various data sources are fairly representative of the climatological phenomena discussed and that the above conclusions are valid, it is obvious that considerable additional work will be necessary, including the examination of other series.

If the above analysis is correct, then it may be possible to calibrate paleotemperatures by fitting the empirical $S(\Delta t)$ curve to that shown in figure 2. This method would work if the required calibration in either temperature or age is approximately linear.

Finally, it is interesting to note that the fundamental difference between meteorological and climatological temperatures is determined by the manner in which the high and low frequency parts of the spectrum join and how they vary when averaged over various time and space scales. In this regard, the very similar form ($f^{-1.8}$) of the climatological and turbulent spectra is very suggestive.

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