GEOPHYSICAL RESEARCH LETTERS, VOL. 34, L15802, doi:10.1029/2007GL029359, 2007

Is isotropic turbulence relevant in the atmosphere?

S. Lovejoy,[1,2] A. F. Tuck,[3] S. J. Hovde[3] and D. Schertzer[4,5]

Received 15 January 2007; revised 18 May 2007; accepted 4 June 2007; published 3 August 2007.

[1] The problem of turbulence is ubiquitous in the Earth sciences, astrophysics and elsewhere. Virtually the only theoretical paradigm that has been seriously considered is strongly isotropic in the sense that scaling exponents are the same in all directions so that any remaining anisotropy is "trivial." Using 235 state-of-the-art drop sonde data sets of the horizontal wind at ≈5 m resolution in the vertical, we show that the atmosphere is apparently outside the scope of these isotropic frameworks. It suggests that anisotropy may frequently be strong requiring different scaling exponents in the horizontal and vertical directions. **Citation:** Lovejoy, S., A. F. Tuck, S. J. Hovde, and D. Schertzer (2007), Is isotropic turbulence relevant in the atmosphere?, *Geophys. Res. Lett.*, *34*, L15802, doi:10.1029/2007GL029359.

1. Introduction

[2] If we include intermittency, Kolmogorov's [*Kolmogorov*, 1941] landmark proposal that fully developed turbulence has an "inertial subrange" with isotropic energy spectrum $E(k) \approx k^{-\beta}$ with $\beta \approx 5/3$ has apparently been spectacularly confirmed in both the horizontal direction and in the time domain (k is a wavenumber). For gradients over a horizontal distance Δx this implies $\langle|\Delta v(\Delta x)|\rangle \approx \Delta x^{H_h}$ ($H_h = 1/3$ corresponds to $\beta = 5/3$; "$\langle . \rangle$" indicates ensemble averaging). Remarkably, H_v for gradients over vertical distances Δz ($\langle|\Delta v(\Delta z)|\rangle \approx \Delta z^{H_v}$) has not been seriously investigated (note that ignoring intermittency, for either 1-D or isotropic spectral exponents we have $\beta = 1 + 2H$). Using state-of-the-art drop sonde data of horizontal wind, we find that from scales of 5 m to >10 km from the surface layer through to the top of the troposphere, H_v is close to (or larger) than the Bolgiano [*Bolgiano*, 1959]-Obukhov [*Obukhov*, 1959] value 3/5. $H_v > H_h$ implies that (1) the atmosphere becomes progressively less stratified at smaller scales although in a scaling way [*Schertzer and Lovejoy*, 1985a]; and (2) that at most a single (roughly) isotropic "sphero-scale" exists (often in the range 1–100 cm [*Lilley et al.*, 2004; *Lovejoy et al.*, 2004]).

[3] Kolmogorov's theory is based on two key assumptions: (1) that there exists an "inertial" range where the turbulence is isotropic depending only on the energy flux ε and the viscosity, and (2) that within the inertial range, an inertial subrange exists where only the scale-by-scale transport of energy is important; this is the $k^{-5/3}$ regime. The main reason for supposing the existence of an isotropic range in the atmosphere is that in turbulence, structures at a given scale are mostly coupled with structures at neighbouring scales so that the effects of large scale boundary conditions are progressively "forgotten" at small scales. Classically this tendency to "return to isotropy" [*Rotta*, 1951] has been modeled using second order closure techniques; however even within this framework, when buoyancy forces are included, they are found to be relatively large [*Moeng and Wyngaard*, 1986], just as in laboratory flows it is found that even small buoyancy forces readily destroy isotropy [*Van Atta*, 1991]. Even recent theoretical advances [*Arad et al.*, 1998; *Arad et al.*, 1999] assume *a priori* that fluctuation statistics follow the form $\langle|\Delta v(\underline{\Delta r})|\rangle = \Theta_{(\widehat{\Delta r})}|\underline{\Delta r}|^H$ where $\widehat{\Delta r}$ is the direction vector, $|\underline{\Delta r}|$ is the length of the separation vector $\underline{\Delta r}$. They thus assume that $H_h = H_v$ and introduce the "trivial anisotropy" function $\Theta_{(\widehat{\Delta r})}$; indeed they introduce a hierarchy of such terms each with different H's and Θ's. Since the theory ignores buoyancy, when it was checked in the atmosphere, the data were restricted to the horizontal [*Kurien et al.*, 2000]. Indeed, virtually all empirical surface layer atmospheric tests of isotropy (i.e. those with the best quality data) simply assume that $H_h = H_v = 1/3$ and test the anisotropy at unique scales. It is even common to study the spatial anisotropy of scalars by using single point time series of gradients, converting time to space with "Taylor's hypothesis" of frozen turbulence, and then using the skewness to determine the forward/backward trivial anisotropy [*Sreenivasan*, 1991]. Even in the analysis of laboratory (Rayleigh-Bénard) convection where there is a debate about whether $H = 1/3$ or 3/5, isotropy is still assumed (i.e. $H_h = H_v$) and one still uses data from time series at single points [*Ashkenazi and Steinberg*, 1999; *Shang and Xia*, 2001].

[4] Perhaps the most serious attempt to develop a specific theoretical framework for atmospheric turbulence is [*Charney*, 1971] whose 3D quasi-geostrophic turbulence was constructed as a broad 3D generalization of the 2D cascades pioneered by [*Kraichnan*, 1967]. Charney demonstrated that the pseudo-potential vorticity plays basically the same role as the vorticity, i.e. it is an invariant of the motion and it prevents the development of an energy cascade towards high wave-numbers yielding instead a pseudo-potential vorticity cascade. Without taking into account intermittency effects, for wave-numbers suitably renormalized along the vertical, the latter yields a 3D isotropic k^{-3} spectrum. This implies $H_h = H_v = 1$ although due to the renormalization, the vertical structures will be somewhat stratified (i.e. there is a trivial anisotropy $\Theta_{(\widehat{\Delta r})}$). Note that the pure 2D (Kraichnan) regime corresponds to $H_v = \infty$.

[5] Charney admitted that rigorous demonstration of the invariance of the pseudo-potential vorticity requires rather particular boundary conditions: the surface must be either isentropic or isothermal. He was only able to weaken this

[1]Department of Physics, McGill University, Montreal, Quebec, Canada.
[2]Centre GEOTOP UQAM-McGill, Université du Québec à Montréal, Montréal, Québec, Canada.
[3]Chemical Sciences Division, NOAA Earth System Research Laboratory, Boulder, Colorado, USA.
[4]Centre d'Enseignement et de Research Eau Ville Environnement, Ecole Nationale des Ponts et Chaussées, Marne-La-Vallee, France.
[5]Centre National de Recherches Météorologiques, Météo-France, Paris, France.

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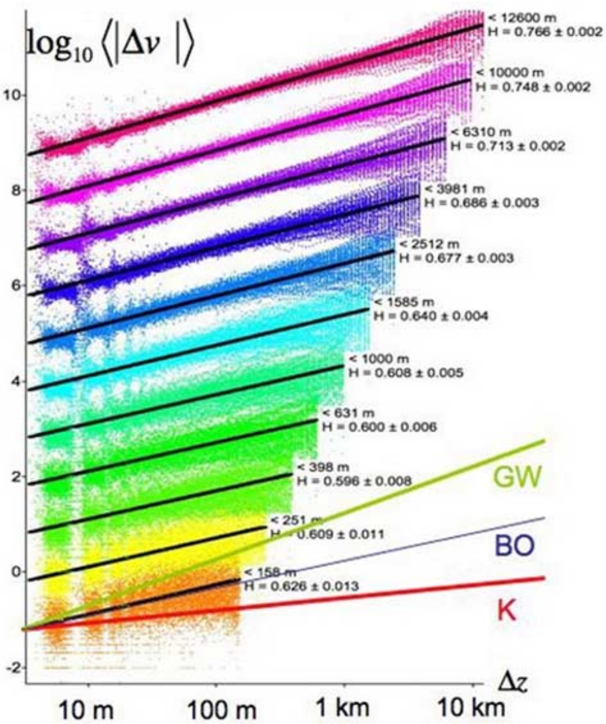

Figure 1. Rms fits to the sonde mean absolute vertical shears of horizontal wind for layers of thickness increasing logarithmically. The reference lines have slopes H_v = 1/3 (Kolmogorov), H_v = 3/5 (Bolgiano-Obukhov), H_v = 1 (gravity waves, pseudo potential vorticity). The rms H_v estimates are given next to the lines. The data for each level are offset by one order of magnitude for clarity, units m/s.

condition by minimizing the role of fronts. This contrasts with the regular observation of sharp temperature gradients, in particular in the vicinity of fronts. In any case - as in our previous studies - we neither observed the two different cascades predicted for large and small scales, nor a regime with $H_h = H_v$.

[6] Gravity is the source of anisotropy which leads to stratified turbulence and it acts at all scales through buoyancy effects. It is precisely buoyancy effects which lead to the hypothesis of a central role for the buoyancy variance flux [*Bolgiano*, 1959; *Obukhov*, 1959]. While the original isotropic Bolgiano-Obukhov regime was never observed, the buoyancy flux survives in the "hybrid" 23/9D anisotropic scaling model [*Schertzer and Lovejoy*, 1985a, 1985b] which postulates that the energy flux dominates in the horizontal while the buoyancy variance flux dominates in the vertical so that H_h = 1/3 but H_v = 3/5. Whereas classical approaches to turbulence deduce two distinct isotropic cascades from the existence of two invariants, the 23/9D model deduces a single anisotropic cascade from two invariants.

2. Empirical Analysis

[7] Most of our knowledge of the vertical structure of the atmosphere comes from radiosonde balloons designed for synoptic forecasting rather than research; they typically have vertical resolutions of the order 150 m. In addition to their low resolutions, balloons suffer from swaying payloads and disturbances on ascent caused by the balloon's wake (see however *Harrison and Hogan* [2006]). In spite of these difficulties, experimentalists largely interpret the vertical spectrum in terms of quasi-linear gravity waves with exponent H_v = 1 (but with $H_h \approx 1/3$) [see e.g., *Allen and Vincent*, 1995; *Dewan*, 1997; *Fritts et al.*, 1988; *Gardner*, 1994]. This follows from dimensional analysis if the layers are stable and homogeneous with well-defined Brunt-Väisäla frequencies, see S. Lovejoy et al., Scaling turbulent atmospheric stratification, part I: Turbulence and waves, submitted to *Quarterly Journal of the Royal Meteorological Society*, 2007, for a critique. In comparison, the older Lumley-Shur [*Lumley*, 1964; *Shur*, 1962] model predicts an *isotropic* $H_h = H_v = 1$ regime (as for pseudo potential vorticity). In order to test the Kolmogorov law in the vertical, we used state-of-the-art drop sonde data from the NOAA Winter Storms 04 experiment over the Pacific Ocean (ranging over latitudes of about 20–60° north), where 261 sondes were dropped by a NOAA Gulfstream 4 aircraft from roughly 13 km altitudes. These GPS sondes had vertical resolutions of ≈5 m, temporal resolutions of 0.5 s, horizontal velocity resolutions of ≈0.1 m/s and temperature resolutions of ≈0.1 K [*Hock and Franklin*, 1999]. While the full analysis of the 2004

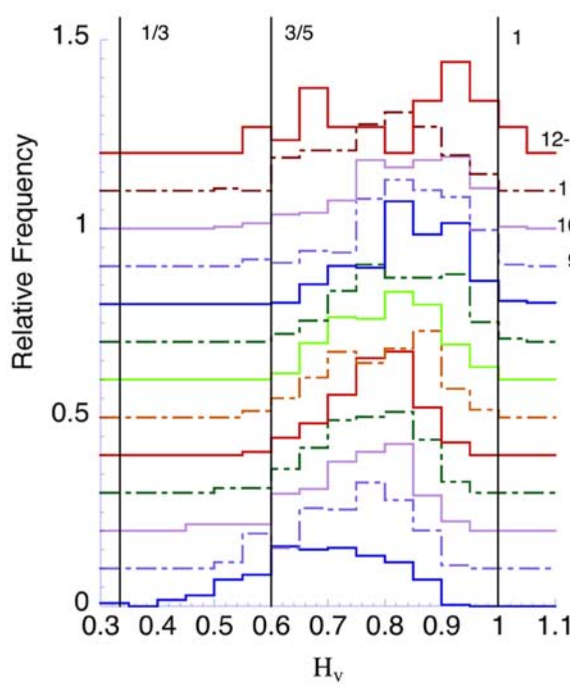

Figure 2. Histograms of H_v. Sonde by sonde, layer by layer, the figure displays the frequency distribution of H_v values calculated from the modulus of the vector velocity differences (offset in the vertical for clarity by 0.1). Successive histograms are for increasingly high 1 km thick layers, the histogram baselines are evident from the level of the far left horizontal lines. There were no values <0.30, nor >1.07. For each histogram, H is estimated over all points whose mean altitude is between the indicated altitude and 1 km below it, the fits are over the scales 5–1000 m. The vertical reference lines indicate the critical values 1/3 (Kolmogorov), 3/5 (Bolgiano-Obhukov), and 1 (pseudo-potential vorticity, gravity wave models).

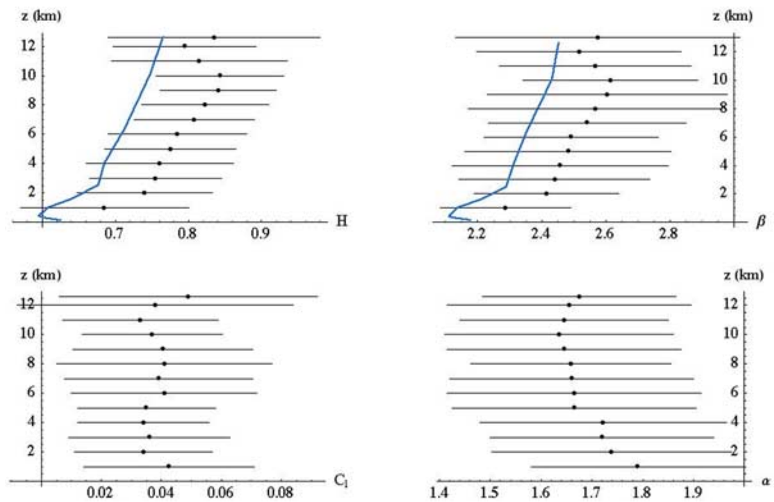

Figure 3. Scaling exponents as functions of altitude. (top left) The means and standard deviations of the H values calculated from the moduli of the vector differences in horizontal winds. The blue curve is from the H values in Figure 1, i.e., over all pairs of points below the altitude indicated, estimated over the entire range of scales available (i.e., up to 12.6 km at the highest altitudes). The points are fits from individual sondes, as indicated in Figure 2. The error bars indicate the sonde to sonde variability (235 sondes were used). (top right) Same as for Figure 3 (top left) but for the corresponding spectral exponents β, (nonintermittent) Kolmogorov theory yields $\beta = 5/3$, Bolgiano Obukhov, $\beta = 11/5$. The blue lines are a bit to the left since they are weighted to be near the indicated altitude whereas the black points are from data within a kilometer of the indicated altitude. (bottom left) The C_1 values corresponding to the north-south components. (bottom right) The corresponding α values.

experiment is described by S. J. Hovde et al. (Vertical scaling of the atmosphere: I. Dropsondes from 13 km to the surface, submitted to *Quarterly Journal of the Royal Meteorological Society*, 2007, hereinafter referred to as Hovde et al., submitted manuscript, 2007), we concentrate here on analysis of the horizontal velocities; see also *Tuck et al.* [2004] for the relation between H_v and jet streams. Our experiment is in many ways an update of one of the largest vertical scaling study to date: [*Lazarev et al.*, 1994] which used 287 radiosondes (at 50 m resolution) over the tropical Pacific; and came to conclusions similar to those below but without being able to analyze the fairly thin layers considered here (c.f. also the Landes (France) experiment using 80 sondes at about 40° north [*Schertzer and Lovejoy*, 1985a]).

[8] Figure 1 shows the composite analysis of the most complete 235 sondes; of these near complete data sets, outages were most frequent at the higher altitudes. For each sonde, the mean absolute shears $\Delta v(\Delta z) = |\underline{v}(z_1) - \underline{v}(z_2)|$ ($\underline{v}$ is the horizontal velocity vector) were calculated using all pairs of points with $\Delta z = |z_1 - z_2|$ in logarithmically spaced intervals, and for all $z_1, z_2, < z_t$ where z_t is the indicated altitude threshold. This method is particularly effective since while there are $\approx$1400 data points per sonde (at 2 Hz), there are many more pairs of points (roughly 10^6); the method also overcomes the irregular vertical spacing of the data without requiring potentially problematic interpolations. Note that for each layer the logarithmic spacing of layers gives predominant weight to the upper part of the range; for layers with constant thicknesses, see Figures 2 and 3.

[9] Four features of Figure 1 are particularly striking: (1) the overall scaling – even for the thickest layers spanning the entire troposphere – is excellent; the standard errors in the slope (H) estimates are <±1%; (2) the slopes at the lower levels (which are not too affected by the ever present strong jet streams) are very close to the BO value 3/5, but increase at higher altitudes; (3) there is no evidence for $H_v = 1/3$ (Kolmogorov) behaviour, even at the smallest scales (5 m) and in the lowest layer (<158 m) which for technical reasons are inaccessible to radiosondes; this is especially significant; and (4) there is no evidence for $H_v = 1$ (gravity wave, pseudo potential vorticity) even at the largest vertical scales. However, since Figure 1 pools the data from all the sondes, the result might be an artifact of mixing data from profiles some of which might have $H_v = 1/3$ or $H_v = 1$ scalings. Figure 2 shows histograms, altitude by altitude giving the distribution of H_v values. In this case, the layers are spaced linearly, and regressions are made over layers 1 km thick with $z_t -$ 1 km < $(z_1 + z_2)/2 < z_t$ km, for 5 m < Δz < 1 km. With the exception of the top (12–12.6 km) histogram (which due to missing data is based on only 29 sondes whereas all the others are based on >200 sondes), the distributions are generally unimodal. Of the total 2727 H_v values estimated, only a single one at the lowest 1 km level has $H_v \approx 1/3$, only 9 have $H_v > 1$, and only 1 has $H_v > 1.05$. In order to quantitatively characterize the mean and spread of these values, we refer to Figure 3 (top left) which gives the one standard deviation spread of values around the mean. One can see that the Kolmogorov $H_v = 1/3$ value is systematically 2–4 standard deviations below the mean, while $H_v \approx 1$ is about 2 standard deviations above it. Analysis of the near simultaneous aircraft statistics in the (roughly) horizontal direction confirm that $H_h \approx 1/3$ (and hence $H_h < H_v$) although the large distance behaviour requires careful analysis due to the effect of the small vertical aircraft displacements even on "horizontal" legs (see the discussion in *Lovejoy et al.* [2004]).

[10] We now examine several factors that may affect the result. First, the structure function - although simple to apply - is limited to series with $0 < H_v < 1$; we therefore also applied a version of the Detrended Fluctuation Analysis [*Kantlelhart et al.*, 2002] (adapted to irregular data spacing) which systematically removes linear trends, effectively redefining the fluctuation Δv as the difference between v and a linear estimate; this change only made a small change in the H_v estimates (it increased them slightly). Second, we may wonder how much the estimates are affected by intermittency; although a priori this effect will be small for the first order moments (indeed, this is the advantage of choosing moments of order $q = 1$ compared to the more standard $q = 2$ value). To quantitatively characterize it, we calculated the qth order structure function exponent $\xi(q)$: $\langle|\Delta v|^q\rangle \approx \Delta z^{\xi(q)}$. Increasing q yields statistics more and more sensitive to (rare) large fluctuations; Figure 3 (top) shows the H values cited above (= $\xi(1)$) and the spectral exponent $\beta = 1 + \xi(2)$. To characterize $\xi(q)$, we fit it to the following "universal multifractal" [*Schertzer and Lovejoy*, 1987] parametric form: $\xi(q) = Hq - K(q), K(q) = \frac{C_1}{\alpha - 1}(q^\alpha - q)$ where C_1 is the codimension of the mean fluctuation and α is the Levy index characterizing the degree of multifractality. C_1 quantifies the effect of intermittency on the mean; if this was large enough it could perhaps explain the large H_v values. Figure 3 (bottom left) shows that C_1 is quite small (much less than H), so that we cannot explain the deviations from Kolmogorov's law due to intermittency effects. The α values (Figure 3, bottom right) show that the effects of intermittency increase rapidly with q ($1.6 < \alpha < 2.0$ i.e. α is large).

3. Conclusions

[11] Using 235 state-of-the-art drop sondes over the northern Pacific Ocean, we have shown that there is no evidence that the Kolmogorov law ($H_v = 1/3$) – or its intermittent generalizations – holds; this includes tropospheric scales from 5 m and up, for over all layers, even those within 158 m of the surface. Similarly, the gravity wave and Charney exponent $H_v = 1$ (corresponding to k^{-3} vertical spectra) occurred in only 0.3% of the layers. Note that since $H_v > H_h$, structures will be at least approximately isotropic at a unique "sphero-scale". However this has nothing to do with a "return to isotropy" or independence of the turbulence with the large scale forcing; rather it is a "cross-over" phenomenon, i.e. a consequence of two power laws "crossing" at a unique scale. We emphasized that this is quite different from the classical return to isotropy for fully developed turbulence, which - in the absence of gravity - is expected at high turbulent energy rates, but which may be prevented by the action of gravity at all scales. This is clearly the case in the 23/9 D model since it is built on a balance between the (horizontal) turbulent energy flux and the (vertical) buoyancy force flux, which maintains on the contrary a scaling anisotropy.

[12] While it is true that absence of empirical evidence for isotropic turbulence is not the same as empirical evidence for its absence, the number of concordant studies is growing and includes notably those in the tropics [*Lazarev et al.*, 1994] and temperate latitudes [*Schertzer and Lovejoy*, 1985a] over land and over the oceans (for a review see M. Lilley et al., Scaling turbulent atmospheric stratification, part II: Spatial stratification and intermittency from lidar data, submitted to *Quarterly Journal of the Royal Meteorological Society*, 2006, and for new results Hovde et al., submitted manuscript, 2007). Let us emphasize that all these data analyses also showed that atmospheric turbulence is strongly intermittent, i.e. regions with very high turbulent energy inside of regions of with not so high turbulent energy: they did not display an effective return to isotropy. Although we cannot empirically rule out the possibility that in specific regions with very intense turbulence, the exponents converge - becoming isotropic - it is not easy to see how this could happen.

[13] Our results are precise enough to be more conclusive than before. Indeed, their high precision has brought to the fore a systematic tendency for the H_v values to increase from the near surface Bolgiano-Obukhov value 3/5 to values closer to 0.77 in higher layers subject to large (jet) shears. While the exact explanation for this increase is unclear at present, it should be recalled that like the usual turbulence laws, the 23/9D model presupposes spatial statistical homogeneity, which is violated by the strongly altitude dependent jets. In this respect it is significant that an analysis of data over land and in the lower 4 km (without strong shears) using aerosols as passive tracers [*Lilley et al.*, 2004] (detected by high resolution lidars), found $H_v \approx 0.60 \pm 0.03$ (compared to $H_h \approx 0.33 \pm 0.02$, close to the standard Corrsin-Obukhov value 1/3).

[14] **Acknowledgments.** We are grateful to the staff, aircrew, and ground crew from the NOAA Air Operations Center, who operated the Gulfstream 4. We also thank Manu Nunes-Carrier for help with the data analysis.

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S. J. Hovde and A. F. Tuck, Chemical Sciences Division, NOAA Earth System Research Laboratory, 325 Broadway, Boulder, CO 80305, USA.
S. Lovejoy, Department of Physics, McGill University, 3600 University Street, Montreal, Que., Canada H3A 2T8. (lovejoy@physics.mcgill.ca)
D. Schertzer, Centre d'Enseignement et de Research Eau Ville Environnement, Ecole Nationale des Ponts et Chaussées, 6–8, avenue Blaise Pascal, Cité Descartes, F-77455 Marne-La-Vallee Cedex, France.