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No monsters, no miracles: in nonlinear sciences hydrology is not an outlier!

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Abstract The end users of hydrological models may be justified for being tired of the excessive uncertainty of these models, not to mention their simplistic approximations and crude modelling. The ever-increasing sophistication of model parameter fitting is simply a smoke-screen that hides the models' lack of physical basis, their scale dependence, and their inability to fit widely diverse behaviours. More generally, we have to admit a lack of qualitative improvement in hydrological modelling in recent times. In fact, operational hydrology may have suffered for some time from ignoring the advances in theoretical hydrology, which have, in contrast, greatly stimulated the nonlinear sciences. For instance, more than a century ago fractals were considered as geometrical monsters, whereas decades ago river networks became classical fractal objects, and rainfall and discharges are now classical examples of multifractal fields. These hydrological characteristics are still often ignored by operational hydrology, whereas they explain not only its current limitations, but also how to overcome them. To illustrate these problems, this paper focuses on the fact that hydrological fields are most likely singular with respect to measures of time and volume. This would not only explain the ubiquitous scale dependence of hydrological observations, but would also give the possibility to transform them into scale-independent quantities. The upscaling of a rainfall time series from an hour to a year is therefore discussed in detail, and enables us to quickly introduce other examples.

Key words scales; measure; singularities; balance equation; fractals; multifractals

Ni monstres, ni miracles: l'hydrologie n'est pas un horsain des sciences non-linéaires!

Résumé Les utilisateurs de modèles hydrologiques peuvent légitimement déplorer l'incertitude excessive des modèles qui leur sont proposés, pour ne rien dire de leur caractère fruste et de leurs approximations simplistes. La sophistication croissante des procédures de calage des paramètres n'est qu'un rideau de fumée cachant l'absence de bases physiques des modèles, leur dépendance d'échelle, et leur incapacité à rendre compte de comportements contrastés. Plus généralement, nous devons reconnaître que la modélisation hydrologique n'a connu récemment aucune amélioration qualitative. En fait, l'hydrologie opérationnelle souffre depuis longtemps de son ignorance des avancées théoriques de l'hydrologie, qui ont par contre grandement stimulé les sciences non linéaires. Les fractals, par exemple, étaient considérés il y a un siècle comme des monstres géométriques mais les réseaux hydrographiques sont devenus des prototypes d'objets fractals, tandis que la pluie et les débits sont maintenant des exemples classiques de champs multifractals. Ces propriétés hydrologiques sont encore souvent ignorées par l'hydrologie opérationnelle alors qu'elles expliquent ses limitations actuelles et permettraient de les surmonter. Pour illustrer cette problématique, ce papier est centré sur le fait que les champs hydrologiques sont très probablement singuliers par rapport aux mesures du temps et des volumes. Ceci non seulement expliquerait l'omniprésence de la dépendance en échelle des champs hydrologiques, mais donnerait aussi le moyen de les transformer en quantités invariantes d'échelle. L'agrégation d'une heure à une année d'une chronique de pluie est ainsi discutée en détail et nous permet d'introduire rapidement d'autres exemples.

Mots clefs échelles; mesure; singularités; équation de bilan; fractals; multifractals

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HYDROLOGICAL MONSTERS OR AN INADEQUATE FRAMEWORK?

With floods and droughts of all magnitudes, hydrology seems to be as full of monsters as the *Court of Miracles* described by Victor Hugo (Hugo, 2002). However, beyond this layman's statement, the fundamental scientific question is: Do we have the appropriate theoretical framework to tame these monsters yet, or do they remain alien to our current framework? As a typical example, one may take the often invoked "outliers", which are usually defined as observations that are numerically distant from the rest of the data (Bernet & Lewis, 1994). Such a distance may be due either to measurement problems or to a probability distribution that admits large deviations, e.g. "fat-tailed" distributions (i.e. with power-law fall-offs).

Mathematicians immediately think about the preliminary characterization of the functional space (e.g. Lions, 1969), on which one tries to solve a given (hydrological) problem. For instance, should we look for smooth solutions over a given domain Ω, e.g. continuous and k times differentiable functions that define the classical functional space $C^k(\Omega)$, or, on the contrary, to irregular solutions, i.e. having discontinuities or singular behaviours, such as those belonging to the more recent Sobolev spaces, $W^{k,p}(\Omega)$, where the derivatives (up to the order k) may exist only in the sense of generalized functions (Schwartz, 1950–1951; Lighthill, 1959)? This characterization has, in fact, been undertaken by theoretical hydrology under various forms seeking to characterize the extreme variability of hydrological fields over a wide range of space–time scales. It was first attempted with the help of geometrical and phenomenological approaches. This brought a new understanding of hydrological fields and their geometrical complexity; in addition, it greatly stimulated what has been often called "fractal geometry" (Mandelbrot, 1983). Suddenly, the odd features of hydrological fields became categorized – as were many other geophysical fields – as being fractals, and became quantifiable with the help of the corresponding concepts, in particular that of fractal dimension. For instance, although everyone has been aware for some time that the borderline of a drainage basin could be as complex as coastlines, it is not yet widely known that both could be characterized by a fractal dimension, D, larger than the dimension of a smooth curve ($D = 1$). Furthermore, the larger this dimension is, the more crooked is the borderline (Perrin, 1913; Richardson, 1961; Mandelbrot, 1967; Bendjoudi & Hubert, 2002; Sapoval *et al*., 2004). More influential should have been the development of various techniques to assess and to simulate the variability of hydrological fields. To underline that hydrology was in many respects ripe for these developments, it is worth noting that this was often obtained by revisiting some pre-fractal techniques elaborated in hydrology. This is particularly the case for the "rescaled range analysis" (Mandelbrot & Wallis, 1969a), which is a refinement of the "range analysis" obtained by Hurst (1951) on the basis of an empirical analysis of the Nile floods (Klemeš, 1974; Sutcliffe, 1979). In a similar way, the fractal rainfall model "fractal sum of pulses" (Lovejoy & Mandelbrot, 1985; Lovejoy & Schertzer, 1985) is a scaling refinement and simplification of rainfall models, notably the Newman-Scott model, based on the idea of adding random structures of various sizes, corresponding to storms, rainfall super-cells and cells, respectively (Waymire & Gupta, 1981; Rodriguez-Iturbe *et al*., 1987; Arnaud & Lavabre, 1999; Onof *et al*., 2000). Among the significant consequences of these fractal developments were the clearer understanding of the following: a few relevant statistical exponents may define classes of stochastic processes, e.g. Lévy flights *vs* Gaussian walks (Painter, 1996); the importance of long-range interrelations, for both previous types of processes (Mandelbrot & Wallis, 1969b; Bras & Rodriguez-Iturbe, 1985; Koutsoyiannis, 2002); and the statistical classes of the extremes, e.g. fat-tailed *vs* thin-tailed pdfs, i.e. power-law *vs* exponential fall-offs (Turcotte & Greene, 1993; Turcotte, 1994; Chaouche *et al*., 2002; Koutsoyiannis, 2004).

Unfortunately, operational hydrology did not benefit as much from all these developments. An optimistic understanding would be that the limitations of these developments called for a paradigm shift: from geometry to physics; from the scaling sets of points to the scaling of fields; from phenomenology to symmetries of the generating equations; from additive to multiplicative stochastic processes; and from fractals to multifractals. Indeed, it might be significant that this shift did not occur in hydrology, but was imported to hydrology from turbulence (Schertzer & Lovejoy, 1987), whereas it was a rather direct answer to a long-lasting fundamental problem in hydrology: the scale sensitivity of the fundamental hydrological observables.

WHAT IS THIS PAPER ABOUT?

Although review papers and books with applications of (multi-) fractals to hydrology have been published fairly regularly (e.g. Mandelbrot, 1989; Turcotte, 1989; Foufoula-Georgiou *et al*., 1991; Schertzer &

Lovejoy, 1993; Lovejoy & Schertzer, 1995; Rodriguez-Iturbe & Rinaldo, 1997; Schertzer *et al*., 2002a), the long and lively discussion we had during the first day of the workshop on "The Court of Miracles of Hydrology" held in Paris in 2008 (Andréassian *et al*., 2010) indicated that there is a need to discuss in depth the corresponding paradigm shift that prevents hydrologists from being at ease with these concepts, even though they respond to what hydrologists have been seeking for a while. Therefore, contrary to our initial intention, this paper is not a review of the application of multifractals to hydrology, although a section of this paper reviews several application areas succinctly. On the contrary, this paper aims to uncover almost all of what lies "beyond the curtain", i.e. beyond the expressions "scale symmetry", "singularities" and "singular measure" commonly used in multifractals, which seem to have generated jargon and conceptual barriers. Still following the workshop discussions, we propose to "get to grips" with these concepts with the help of an almost everyday hydrological exercise: to upscale a time series, but with a slight variant that, among various consequences, enables one to *see* the singularities and scale invariance on a graph. This is so straightforward that any reader can repeat this exercise with her/his preferred time series. We therefore avoid, as much as possible, shorthand notations, as well as lengthy and cumbersome ones.

For this paper, we selected a rainfall time series, because rainfall records are the most common and the longest hydrological time series; however, this choice should not hide the generality of the demonstration. This generality may be understood first by the fact that rainfall is considered as the main driver of hydrological variability. We furthermore point out clearly how this extreme rainfall variability brings into question the usual smooth balance equation between various hydrological fields. This presumably corresponds to the core problem and main limitations of current hydrological models (e.g. Beven, 1989; Beven & Binley, 1992; Tchiguirinskaia *et al*., 2004; Beven, 2006; Vogel & Roth, 2003), as well to attempts to overcome them (e.g. Sposito, 1998; Sivapalan *et al*., 2003; Gupta, 2004; Viney & Sivapalan, 2004; Schertzer *et al*., 2007).

IS THE RAIN MEASURE REGULAR OR SINGULAR?

There is a common-sense intuition of precipitation intermittency: most of the time it does not rain, furthermore when it does rain, its intensity can be extremely variable. In spite of this intuition, an adequate mathematical framework had long been elusive, and has only been seriously elaborated during the last 25 years. Furthermore, this framework is ignored by operational hydrology.

The rain rate, which is the basic hydro-meteorological quantity, is indeed a surprising and outstanding example of a fundamental paradox: even though everyone is rather aware of the strong scale dependency of the rain rate, since it depends on the duration on which it is measured, almost everyone continues to consider an instantaneous point-wise rain-rate function, $r(\underline{\boldsymbol{x}}, t)$, defining the elementary rainfall, dR, across an infinitesimal surface, d$S(\underline{\boldsymbol{x}})$, and during an infinitesimal time duration, dt, as:

$$\mathrm{d}R = r(\underline{\boldsymbol{x}}, t)\ \mathrm{d}S\ \mathrm{d}t \tag{1}$$

When such a relationship holds between the (mathematical) measures dR and dS dt, the measure dR is said to be regular with respect to dS dt, and one can forget the notion of mathematical measure to only focus on the "density function", $r(\underline{\boldsymbol{x}}, t)$, which needs to be smooth enough to be measurable by dS dt. Nevertheless, and contrary to many classical mathematics textbooks, it is worth noting that the notion of (mathematical) measure is the most appropriate and natural notion ones needs for measuring something like rain: a mathematical measure, μ, is a systematic procedure to assign to each "suitable"[1] subset B a measure of its contents or its size:

$$\text{“suitable” subset } B \xrightarrow{\mu} \mu(B) = \int_B \mathrm{d}\mu \tag{2}$$

This concept corresponds to a broad generalization of the notion of length, surface and volume, which are assigned with the help of the Lebesgue measures $\mathrm{d}x_1$, $\mathrm{d}x_1\mathrm{d}x_2$, and $\mathrm{d}x_1\mathrm{d}x_2\mathrm{d}x_3$ for dimensions 1, 2 and 3, respectively.

We will show that the assumptions of scale dependency and regularity are in fact incompatible: scale dependency is a distinctive feature of singular measures with respect to Lebesgue measures (i.e. of the type $\mathrm{d}x_1 \ldots \mathrm{d}x_d$). The source of this distinction is that the action of dilation/contraction, T_λ, of the space on a regular measure is a trivial rescaling, whereas it is no longer the case on a singular measure. Let us give a very first example with the help of the simple (one-dimensional) time dilation/contraction, T_λ, of

[1] We use this vague term to avoid technical terms such as σ-algebra, borelian sets, etc.

(dimensionless) ratio λ (for $\lambda \leq 1$ and $\lambda \geq 1$, respectively):

$$T_\lambda(t) = t/\lambda \tag{3}$$

applied to the Lebesgue measure, dt:

$$\forall[\tau, \tau'[: \int_{[\tau,\tau'[} \mathrm{d}t = |\tau - \tau'| \tag{4}$$

and to the Dirac measure, δ, defined by:

$$\forall B : \int_B \delta_{t_0}(t) = 1_B(t_0) \tag{5}$$

with the indicator function, 1_B, of any set, B, defined as:

$$1_B(t) = 1 \text{ if } t \in B; = 0 \text{ otherwise} \tag{6}$$

The Dirac measure, often improperly called the Dirac function, is indeed singular because, if one attempts to define its density with respect to the Lebesgue measure, dt, one is compelled to consider a weird "function" which is zero everywhere, except at t_0, where it is infinite: the total mass (=1) is indeed concentrated in the "atom", t_0. The definitions of the measures (equations (5)–(6)) already show that the action of a time dilation/contraction, T_λ, will have quite different results: whereas the Lebesgue measure will be rescaled by the ratio, λ, as any time period $[\tau,\tau'[$, the Dirac measure like its atom will not be rescaled. This is in agreement with the dimension of their support, 1 and 0, respectively.

A FUNDAMENTAL CHOICE

The action of a transform, T_λ, such as a simple contraction of time (equation (3)) on a measure μ (e.g. dμ = dt, $\mu = \delta_{t_0}(t)$) corresponds to the general definition of a pushforward measure, $T_{\lambda,*}(\mu)$, or image measure (e.g. Bourbaki, 2004):

$$T_{\lambda,*}(\mu)(B) = \mu(T_\lambda^{-1}(B)) \Leftrightarrow \int_B T_{\lambda,*}(\mathrm{d}\mu) = \int_{T_\lambda^{-1}(B)} \mathrm{d}\mu \tag{7}$$

which indeed pushes forward μ to measure any suitable set B, with the help of the original measure of $B' = T_\lambda^{-1}(B)$: the transform $T_{\lambda,*}$ of the measure μ just compensates the inverse transform of the set T_λ^{-1} (see Fig. 1). One may note that the inverse T_λ^{-1} does not need to be defined pointwise: the pre-image $T_\lambda^{-1}(B)$ is the subset of points that are (pointwise) transformed into B. The mathematical interest of this definition (equation (7)) is that the continuity of T_λ assures that $B' = T_\lambda^{-1}(B)$ is a suitable set for any suitable set B. Furthermore, this definition applies to any generalized (space–time) contraction operator (Schertzer & Lovejoy, 1985) that is necessary to handle space–time processes and/or anisotropic media, e.g. atmospheric turbulence (Chigirinskaya *et al.*, 1994; Lazarev *et al.*, 1994) and subsurface media (Tchiguirinskaia, 2002). Although we will stick to the pedagogical example of a simple contraction of time in this paper (equation (3)), the results obtained *mutatis mutandis* apply to much more complex cases.

Noting that:

$$B = [\tau, \tau'[\Leftrightarrow T_\lambda^{-1}B = [\lambda\tau, \lambda\tau'[\tag{8}$$

the application of this definition (equation (7)), respectively, to the Lebesgue and Dirac measures (equations (5)–(6)) yields:

$$\int_{[\tau,\tau'[} T_{\lambda,*}(\mathrm{d}t) = \int_{[\lambda\tau,\lambda\tau'[} \mathrm{d}t = \lambda \int_{[\tau,\tau'[} \mathrm{d}t \\ \Rightarrow T_{\lambda,*}(\mathrm{d}t) = \lambda \mathrm{d}t \tag{9}$$

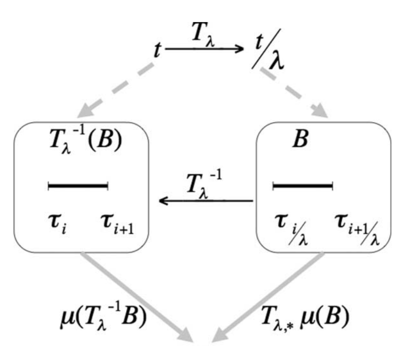

Fig. 1 Scheme of the "pushforward" transform $T_{\lambda,*}$ of a measure μ due to a time contraction T_λ with a scale ratio λ (see equation (7)).

$$\int_B T_{\lambda,*}(\delta_{t_0}) = \int_{T_\lambda^{-1}B} \delta_{t_0} = 1_{T_\lambda^{-1}B}(t_0)$$
$$= 1_B(t_0/\lambda) \Rightarrow T_{\lambda,*}(\delta_{t_0}) = \delta_{t_0/\lambda} \quad (10)$$

in agreement with our qualitative discussion at the end of the previous section. In this paper, we will be interested in singular measures, R, more general than the Dirac measure, i.e.:

$$T_{\lambda,*}(R) = \lambda^{1-\gamma}R \Leftrightarrow R(T_\lambda^{-1}B) = \lambda^{1-\gamma}R(B) \quad (11)$$

generalizing equations (9)–(10), with the help of the "singularity" γ ($\gamma = 0$ for the Lebesgue measure and $\gamma = 1$ for the Dirac measure).

Let us now emphasize the physical importance of dilation/contraction of the coordinates: it indeed belongs to the (extended) Galilean group formed by the space–time transforms which leave invariant the non-relativistic laws of nature. This corresponds to the Galilean invariance of the physics law of nature, which is also called Galilean relativity.

The attention in mechanics, especially in point mechanics, has been focused initially on the space shifts between two (Galilean) frameworks that differ only by a constant relative velocity or a given rotation, which define the pure Galilean group. However, with extended bodies and, therefore, continuous mechanics, this broadened to other transforms, such as scale dilations. In particular, Sedov (1972) pointed out in the wake of the Buckingham π theorem (Buckingham, 1914, 1915; Sonin, 2004) the key role of the latter in fluid mechanics, including for many applications (e.g. to estimate the blast wave of a nuclear explosion). More recently, Speziale (1985) demonstrated their relevance in selecting and defining relevant subgrid models in turbulence. It should also be mentioned that the fundamental role of scale change in fluid mechanics can be traced back to Aristotle (*Physica*, Chap. IV, Hardie & Gaye, 1930).

In fact, multifractals merely generalize this approach by considering not a unique dilation, but a hierarchy of dilations. However, presently it is sufficient to question whether hydrology can afford not to respect these symmetries, which are so fundamental in physics and mechanics. The previous discussion pointed out that we would violate this symmetry if we made the wrong hypothesis about the regularity/singularity of the rain measure. For example, the so-called "physically-based models" rely on balance equations written with the help of density functions, e.g. for the mass balance for surface runoff:

$$\partial s(\underline{x}, t)/\partial t = r(\underline{x}, t) - q(\underline{x}, t) - i(\underline{x}, t) \quad (12)$$

where s, q and i are the densities of storage, discharge and infiltration. Are these quantities well defined and, in particular, do they satisfy the (extended) Galilean invariance? We will first point out that most likely the rainfall rate does not exist, and therefore one has to recast the mass balance equation into another form.

WHAT CAN WE INFER FROM EMPIRICAL DATA?

Let us now illustrate the scale dependence of the rain rate with the help of Fig. 2(a), which displays (from top to bottom) the Nîmes time series of rain rates at increasingly coarser time resolutions (the constant surface of the raingauge does not intervene) from hourly to yearly resolution. It is striking that the intensity scale is decreasing with the resolution, e.g. the maximum decreases from 35 to 0.1 mm/h, which indicates that the variability decreases with decreasing resolution. Nevertheless, as emphasized by Ladoy *et al.* (1993), the variability is still there at coarser resolution.

To show that these observations are rather in opposition to the regularity of the rain measure with respect to the Lebesgue measure dt, we are compelled to introduce some notation. Let us first define the dimensionless resolution λ as the ratio of the external time scale, T, e.g. the total number N of hours of the time series, with the given duration Δt, e.g. a number n of hours:

$$\lambda = T/\Delta t = N/n \quad (13)$$

and the corresponding regular partition $\{\tau_i\}$ of the time period T into adjacent and disjoint windows of duration T/λ:

$$\sum_i [\tau_i, \tau_{i+1}[= [0, T[; \tau_{i+1} - \tau_i = T/\lambda \quad (14)$$

The data resolution, which is therefore the highest empirical resolution, will be denoted in general by Λ. Then the rain rate at a given resolution, λ, is the step function, $r_\lambda(t)$, defined by:

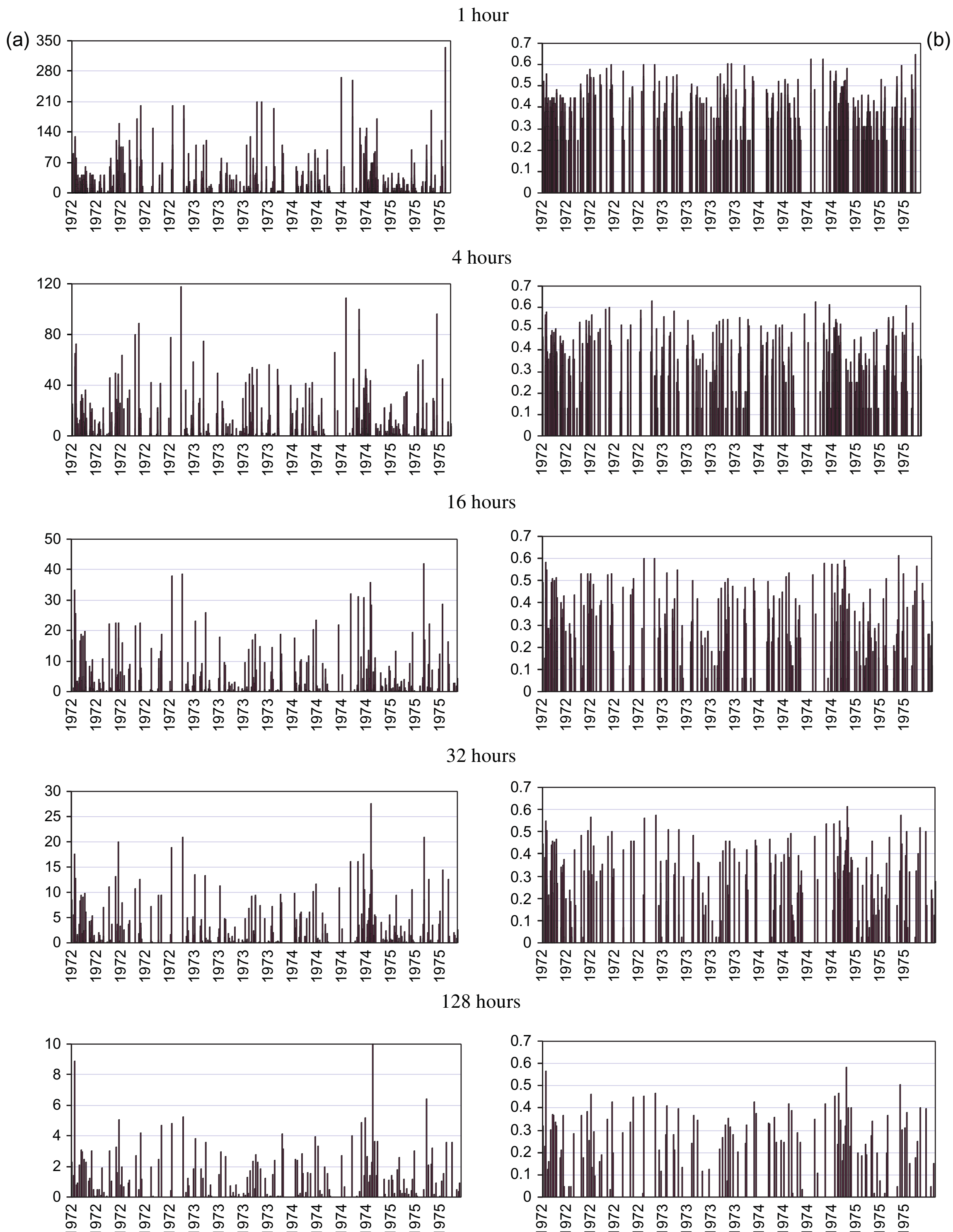

Fig. 2 Observed rainfall at Nîmes (1972–1975) from 1 hour to 1 year duration: (a) the rain rate $r_\lambda(t)$ (equation (15)) does exhibit a strong scale dependence, since its maximum value decreases from 35 to 0.1 mm/h from 1 hour to 1 year duration (the unit of the intensity scale corresponds to 0.1 mm/h). (b) The corresponding singularities $\gamma \approx \log_\lambda(r_\lambda)$, in contrast, show a remarkable scale independency over the same range of durations. Both show evidence of the singular nature of the rainfall measure (see text).

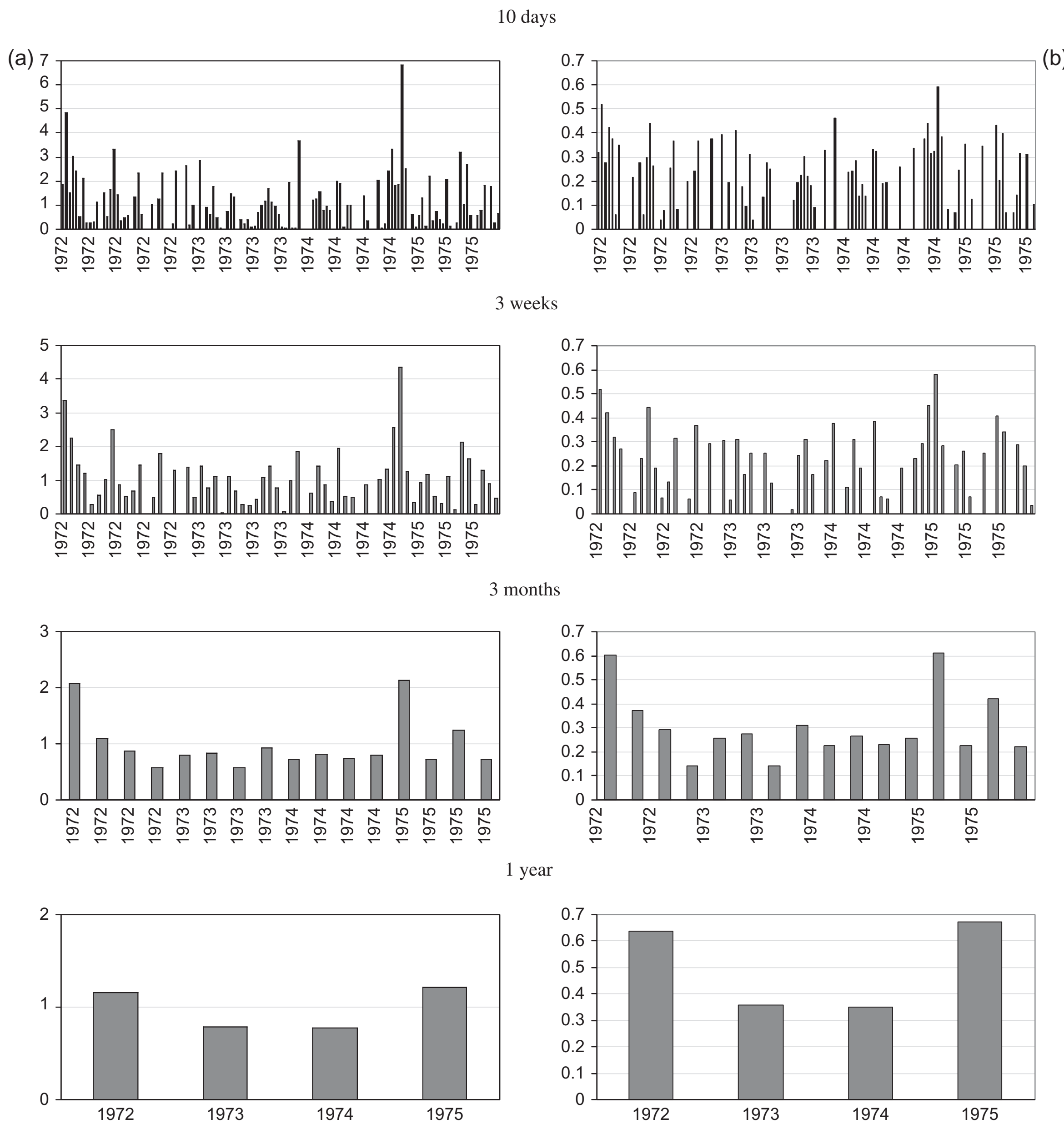

Fig. 2 (*Continued*).

$$r_\lambda(t) = \frac{\lambda}{T}\sum\nolimits_i 1_{[\tau_i,\tau_{i+1}[}(t)R([\tau_i,\tau_{i+1}[) \tag{15}$$

i.e. the series of the rain rates averaged over the windows $[\tau_i,\tau_{i+1}[$. For instance, Fig. 2(a) corresponds to the averages, $r_\lambda(t)$, of the hourly rain rate, $r_\Lambda(t)$, of the original time series over adjacent and non-overlapping windows of n hours ($n = 1, 2, \ldots, 24 \times 365$). In mathematical terms, the definition of $r_\lambda(t)$ given by equation (15) is too precise and one would really need a looser definition, but it is helpful to fix ideas and it is of practical interest. Indeed, such a transform is widely used to "upscale" time series.

Let us note now that the dilation/contraction T_λ (e.g. equation (3)) has a trivial but fundamental property of being a one-parameter (λ) multiplicative group, i.e.:

$$\forall \lambda_1, \lambda_2 : T_{\lambda_2} \circ T_{\lambda_1} = T_{\lambda_2\lambda_1} \Rightarrow T_{\lambda_1}^{-1} = T_{\lambda_1^{-1}} \tag{16}$$

and there is a corresponding group property to go from one curve to another: we can do this in a unique step (e.g. $\Lambda\rightarrow\lambda_2$) or in two or more steps (e.g. $\Lambda\rightarrow\lambda_1\rightarrow\lambda_2$). Indeed, the application of the pushforward transform, $T_{\lambda,*}$ to the measure $r_{\lambda 1}(t)\mathrm{d}t$ corresponds to:

$$r_{\lambda_2}(t)\mathrm{d}t = T_{\lambda_2/\lambda_1,*}(r_{\lambda_1}(t)\mathrm{d}t) \qquad (17)$$

and, due to equation (7), the group property of T_λ is transposed to $T_{\lambda,*}$.

How do these mathematical properties apply to data? At the moment, due to the finite resolution Λ of the data, i.e. there is no empirical r_λ with a resolution $\lambda > \Lambda$, we cannot always go upward on Fig. 2. Indeed, for the present time, we only know how to upscale the time series, i.e. degrade its resolution. The effective range of λ for $T_{\lambda,*}$ is therefore rather limited to $\lambda \geq 1$, i.e. the effective $T_{\lambda,*}$ forms only a semi-group.

Let us emphasize the fact that there is an unambiguous deterministic manner to degrade the resolution of the data (e.g. equation (15) for $\lambda_2 < \lambda_1$), because we are just losing some information from the time series, whereas in order to downscale the time series ($\lambda_2 > \lambda_1$), i.e. to upgrade its resolution, we need to add information. Therefore, there is generally not a unique manner to do it and the inverse of the transforms $T_{\lambda}*$ ($\lambda \leq 1$) can no longer be deterministic, but rather is stochastic in order to yield the set of all the possible realizations of the downscaled time series. For instance, when we go upwards on Fig. 2, we just see one possible upscaled realization. Otherwise, one must select a rather arbitrary deterministic scheme (Obregon *et al*., 2002; Puente & Sivakumar, 2007; Cortis *et al*., 2009). We refer to Sivakumar *et al*. (2001) and Schertzer *et al*. (2002b) for a discussion on the deterministic *vs* stochastic approaches in hydrology.

Before discussing further the question of the inversion of $T_{\lambda,*}$, let us come back to the question of the scale/resolution sensitivity of the $r_\lambda(t)$ as displayed in Fig. 2(a): the curves $r_\lambda(t)$ become spikier and spikier with increasing resolution. There are indeed many ways to show that this is in contradiction with the assumption that the rain measure dR is regular, i.e. admits a (smooth) density function $r(t)$ with respect to the Lebesgue measure (equation (1)). For instance, the application of the mean value theorem to equation (1) yields:

$$\forall\lambda, i, \exists s_i \in [\tau_i, \tau_{i+1}] : r(s_i) \equiv \frac{\lambda}{T}\int_{\tau_i}^{\tau_{i+1}} r(t)\mathrm{d}t \qquad (18)$$

$$r_\lambda(t) = \sum\nolimits_i 1_{[\tau_i,\tau_{i+1}[}(t)r(s_i) \qquad (19)$$

Equations (18–19) mean that the upscaling (with the help of $T_{\lambda,*}$, $\lambda < 1$) would merely correspond to a uniform sampling of the hypothetical function $r(t)$ (the uniformity being enforced by the integration with respect to dt involved in equation (18)), not to the generation of a rather different function. Obviously, Fig. 2(a) does not correspond to a series of uniform samplings of the same function $r(t)$.

In particular, this is the case of the original hourly time series $r_\Lambda(t)$ (more generally the time series at the largest possible resolution Λ), which can be understood as an upscaled version of the hypothetical (smooth) $r(t)$. The latter would correspond to a much larger resolution than Λ (remembering that for a given external time scale T, a larger and larger resolution corresponds to a smaller and smaller duration Δt – see equation (9) – for the time partition). The contradiction is that the function of $r(t)$, if it exists, cannot be smooth because its sampling at resolution Λ is not. Furthermore, a too "wild" function $r(t)$ would not be measurable by dt.

The existence of the function $r(t)$ is even more questionable when going in the opposite direction, i.e. downscaling the time series (with $T_{\lambda,*}$, $\lambda > 1$). Indeed, not only is $r_\Lambda(t) = T_{\Lambda/\lambda 1,*}r_{\lambda 1}(t)$ much "wilder" than the yearly rain rates $r_{\lambda 1}(t)$, but this would be amplified even more if we were to iterate $T_{\Lambda/\lambda_1,*}$, i.e. downscale to hour/24 × 365, hour/(24 × 365)^2, etc., up to a time where a strong mechanism introduces a break in these iterations (Desaulnier-Soucy *et al*., 2001; Lilley *et al*., 2006). Otherwise, iterations of $T_{\Lambda/\lambda 1,*}$ will just show how $r_\lambda(t)$ departs more and more from a smooth function and therefore from an increasingly questionable convergence to a hypothetical function $r(t)$. On the contrary, there is no difficulty for such a "wild" $r_\lambda(t)$ to (weakly) converge to a singular measure d$R(t)$ on any suitable subset B:

$$\int_B r_\lambda(t)\mathrm{d}t \overset{\lambda\rightarrow\infty}{\longrightarrow} \int_B \mathrm{d}R(t) \qquad (20)$$

which means that the measures of any given suitable set B obtained with the help of $r_\Lambda(t)\mathrm{d}t$ and d$R(t)$ are rather the same for large enough λ.

HOW POPULATED IS THE COURT OF MIRACLES?

We have shown how the apparent hydrological monsters could be mere manifestations of a singular measure that cannot be handled as a regular measure: monstrous fluctuations are the rule, they are not outliers, and occur more frequently than was usually thought. The most striking resulting feature is that the (theoretical) statistical moments can easily be infinite causing their empirical estimates to be erratic and to diverge with sample size. This can be understood in physical terms as resulting from the fact that the upscaling cannot smooth out intense small-scale events.

A more quantitative point of view can be obtained with the help of the group $T_{\lambda,*}$ of up-/down-scaling ($\lambda \leq 1$ and $\lambda > 1$, respectively). Whereas the upscaling $T_{\lambda,*}$ ($\lambda \leq 1$) is already defined in an unambiguous deterministic manner (equation (15)) with the help of the dilation T_λ, it is timely to further discuss the stochastic downscaling, $T_{\lambda,*}$ ($\lambda \geq 1$). In fact the answer has been known for a while as a cascade process, which generically breaks structures into smaller ones and can be traced back to a humorous footnote of Richardson (1922). This process can be seen theoretically as a stochastic solution of the multiplicative group property (equation (16), but for $T_{\lambda,*}$) obtained with the help of the generator of this one-parameter group. Let us first note that, if r_λ was only a number instead of being a step function with respect to time, the solution of equation (17) for all λ would be straightforward:

$$T_{\lambda,*} = \lambda^{\gamma}; r_\lambda = \lambda^{\gamma} r_1 \tag{21}$$

for any arbitrary "singularity" γ (when positive it does measure the divergence of γ with the increasing resolution λ). This already shows that, to avoid the scale dependency of $r_\lambda(t)$, it might be sufficient to consider the corresponding singularities $\gamma \approx \log_\lambda(r_\lambda)$ $\gamma = -8$, if $r_\lambda = 0$. This is confirmed strikingly by Fig. 2(b), which displays these singularities for the upscaled fields displayed by Fig. 2(a). However, not surprisingly, there is not a unique singularity. Nevertheless, the intensity of these singularities is rather independent of the scale. This is supported, furthermore, by Fig. 3 – on a longer period. To go further, one may look for singular behaviours of $R([\tau_i,\tau_{i+1}[)$ in equation (15), i.e. in agreement with equation (11):

$$R([\tau_i,\tau_{i+1}[) = \lambda^{\gamma_i -1} R([0,T[) \tag{22}$$

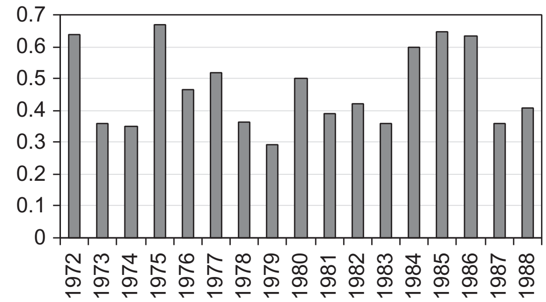

Fig. 3 Singularities of the annual rain rates $\gamma \approx \log_\lambda(r_\lambda)$ over the 17-year period (1972–1988) at Nîmes show that: the 4-year period 1972–1975 represented on Fig. 1(b) is representative of the full data record; while the "monsters" were met on a daily scale during this period, they remain rather hidden on the annual scale, e.g. 1988 is not exceptional on the yearly scale, whereas it includes a hydrological catastrophe on the daily scale.

Figure 2(b), which displays $\log_\lambda(r_\lambda(t))$, points out that the γ_i do not have a common and unique finite value γ, indeed:

$$\begin{aligned} r_\lambda(t) &= \frac{R([0,T[)}{T} \sum\nolimits_i 1_{[\tau_i,\tau_{i+1}[}(t)\lambda^{\gamma_i} \\ &\Rightarrow \log_\lambda(r_\lambda(t)) \\ &= \sum\nolimits_i 1_{[\tau_i,\tau_{i+1}[}(t)\left(\gamma_i + \log_\lambda\left(\frac{R([0,T[)}{T}\right)\right) \end{aligned} \tag{23}$$

where the last term is slowly decreasing with the resolution λ. It is obvious in Fig. 2(b) that there is no accumulation of γ_i around zero, which confirms the singular behaviour of the rainfall accumulation, d$R(t)$, with respect to the Lebesgue measure, dt. In fact, it shows more than that: d$R(t)$ is in fact multi-singular rather than uni-singular because of the variability of the singularities, γ_i. Thus, one is naturally led to measure the frequency of occurrence of a given singularity, γ, and, therefore, the probability of the periods of time when a singularity γ_i exceeds a given γ, i.e. to measure the support $\Sigma_\lambda(\gamma)$ of the singularity γ at resolution λ:

$$\sum_\lambda(\gamma) = \{r_\lambda > \lambda^\gamma\} = \sum_{\mathrm{i}} 1_{\gamma_! > \gamma}[\tau_{\mathrm{i}}, \tau_{\mathrm{i}+1}[\tag{24}$$

and to estimate the corresponding exceedence probability:

$$\Pr(r_\lambda \geq \lambda^\gamma) \approx \lambda^{-c(\gamma)} \tag{25}$$

The power law dependence on the resolution (r.h.s. of equation (25)) corresponds to the fact that this measure is expected to be singular ($c(\gamma)\neq 0$) and to have a group property like $T_{\lambda,*}$. The function $c(\gamma)$ is necessarily non-decreasing due to:

$$\forall \gamma_1 < \gamma_2 : \Sigma_\lambda(\gamma_1) \supseteq \Sigma_\lambda(\gamma_1) \tag{26}$$

The exceptional mono-/uni-fractal case corresponds to $c(\gamma)$ having a unique non-zero value, and the even more exceptional, regular case corresponds to $c(\gamma) \equiv 0$.

The scale sensitivity of the exceedence probability (equation (25)) is removed in the same manner as for $r_\lambda(t)$, i.e. with the help of the base-λ logarithm: Fig. 4 shows that indeed the estimates of $c(\gamma)$ obtained in this manner collapse together for durations from 1 to 32 h, which covers the duration range of general interest in hydrology. Furthermore, it can be shown that the exponent $c(\gamma)$ is the (statistical) co-dimension of the limit set $\Sigma(\gamma)$ of $\Sigma_\lambda(\gamma)$ for an infinite resolution λ (Schertzer *et al.*, 2002a): for $c(\gamma) < 1$, $\Sigma(\gamma)$ has a fractal dimension $D(\gamma) = 1 - c(\gamma)$. Therefore, this measure is generally multifractal in the sense that it corresponds to an infinite hierarchy of embedded fractals level sets, whereas the term multifractal was first coined (Parisi & Frisch, 1985) in a geometric and deterministic framework, which was therefore more restrictive than the present one. One may note that the multifractal genericity has been rigorously demonstrated in functional spaces such as the Sobolev spaces (Fraysse & Jaffard, 2006): multifractal elements of these spaces are prevalent, recalling that the concept of "prevalence" generalizes that of "almost every" on infinite dimensional spaces (Hunt *et al.*, 1992). The following simple example illustrates this mathematical result: among the infinite number of (continuous) curves joining two points, the curves, which are differentiable everywhere, are obviously extremely rare in comparison with more convoluted non-differentiable curves (e.g. obtained with the help of Brownian motion).

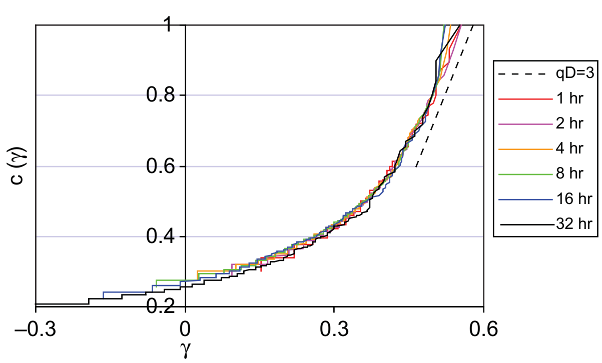

Fig. 4 Estimates of $c(\gamma)$ obtained with the help of $\log_\lambda(r_\lambda(t))$ (see equations (23)–(24)) and the Weibull plotting position for durations ranging from 1 to 32 h of the rain rates and their singularities displayed in Fig. 2. The good superposition of the corresponding co-dimension curves gives an empirical support to the scaling of the probabilities (equation (25)). The often cited asymptotic slope $q_D = 3$ (i.e. the exponent of the power-law fall-off of the probability tail) is displayed, and fits rather well the high singularity behaviour, in particular for short durations.

Overall, large fluctuations of the order of λ^γ (for large $\gamma > 0$) are neither monsters nor outliers, but have already a probability about $\lambda^{-c(\gamma)}$ of occurring. Below, we will show that it could be even larger.

HOW DO WE GENERATE APPARENT MONSTERS?

To downscale $r_\lambda(t)$ to higher resolutions than the data resolution Λ, i.e. to find solutions of equation (17) with $\lambda = \lambda_2 > \lambda_1 = \Lambda$ that also satisfy equation (23), one can consider a discrete cascade with an elementary scale ratio $\mu\lambda > 1$ (=2 usually). The latter downscales the rain rate $r_\lambda(t)$ according to the following elementary transform, $T_{\mu\lambda*}$ which corresponds to a pullback transform of a function, as discussed elsewhere in detail:

$$\begin{aligned} r_{\lambda(\mu\lambda)}(t) &= \tilde{T}_{\mu\lambda}(r_\lambda(t)) \\ &= \sum_i \sum_{j=1,\mu\lambda} 1_{[\tau_i^j,\tau_i^{j+1}[}(t) r_\lambda(\tau_i)(\mu\lambda)^{\gamma_i^j} \end{aligned} \tag{27}$$

where the refined partition $\{\tau_i^j\} = T_{\mu\lambda}^{-1}\{\tau_j\}$ ($\tau_i^j = \tau_{i+(j-1)\Delta t/\mu\lambda}$) and the singularities γ_i^j are independent identically distributed variables with the exceedence probability:

$$\Pr(\gamma_i^j \geq \gamma) \approx (\mu\lambda)^{-c(\gamma)} \tag{28}$$

The desired properties (e.g. equation (25)) have been enforced for the scale ratio $\mu\lambda$ and by iterating n times the transform $T_{\mu\lambda*}$, this also will be true for the ratio $(\mu\lambda)^n$. In spite of the finite data resolution Λ, this enables us to stochastically restore the full group "property" of $T_{\lambda,*}$, at least over a given set of discrete scales $T/(\Lambda(\mu\lambda)^n)$. It should be mentioned that the discrete scale constraint can be relaxed with the help of cascades continuous in scales, e.g. with coloured Lévy noises such as stochastic generators (Schertzer & Lovejoy, 1987, 1997), also referred to as infinitely divisible cascades. These continuous cascades have been used in various domains and are called "universal multifractals" due to their property of stability and attractivity, which correspond to a broad and multiplicative generalization of the central limit theorem.

The statistical moments are used for many applications and thanks to the Mellin transform (Schertzer *et al.*, 2002a) the scaling relationship of equation (25) transposes to:

$$\langle r_\lambda^q \rangle \approx \lambda^{K(q)} \tag{29}$$

where $\langle\cdot\rangle$ denotes the mathematical expectation and $K(q)$ is the scaling function of the moments of order q. The prior constraints on $K(q)$ are that it should be convex like $c(\gamma)$ and $K(0) = 0$ to assure the normalization of the probability. Both functions are conjugate with respect to the Legendre transform (Parisi & Frisch, 1985; Mandelbrot, 1999) with the consequence, among many others, that a non-constant $c(\gamma)$ corresponds to a nonlinear $K(q)$. The latter in turn implies that all the classical dimensionless ratios of statistical moments, such as the variation coefficient, skewness and flatness are scale dependent, with a scaling exponent of the type:

$$K(q, \eta) = K(q\eta) - \eta K(q) \tag{30}$$

with $\eta = 2$ and $q = 1/2, 3/2, 2$ for the variation coefficient, skewness and flatness, respectively. Obviously, $K(q,\eta)$ retains only the nonlinear part of $K(q)$ and therefore cancels only for mono-/uni-fractal or regular cases. In fact, the exponent $K(q,\eta)$ is the cornerstone of the double trace moment method to determine the exponents of the universal multifractals (Lavallée *et al.*, 1992; Veneziano & Furcolo, 1999).

There is a more subtle posterior constraint: $K(q)$ jumps to infinity when the order q reaches the critical value, q_D, which is a non-trivial solution ($q \neq 1$) of:

$$K(q) = D(q-1) \tag{31}$$

where D is the dimension of integration of the process, which is not necessarily integer, although we took $D = 1$ to upscale the data in this paper. The theoretical statistical moments for $q \geq q_D$ are infinite and their empirical estimates are erratic and diverge with the sample size (Schertzer & Lovejoy, 1992). This results from the fact that the usual statistical moment of $r_\lambda(t)$ bound above the "trace moments" that scale with the exponent $K(q) - D(q - 1)$ and therefore diverge when $q \geq q_D$ (Schertzer & Lovejoy, 1987). With the help of the Legendre transform, the corresponding codimension function and the probability distribution are shown to become linear with the slope q_D (above the corresponding critical singularity γ_D, see Fig. 4) and fat-tailed (for large enough thresholds, s), respectively:

$$\gamma \geq \gamma_D : c(\gamma) - D = q_D(\gamma - D) \tag{32}$$

$$s >> 1 : \Pr(r_\lambda > s) \approx s^{q_D} \tag{33}$$

The latter corresponds to an extreme case of intermittency, often called "self-organized criticality" (Bak *et al.*, 1988; Bak & Chen, 1991). Is it therefore appropriate to go on calling such extremes *monsters*, when they belong to the same statistical population as much more moderate events?

IS THIS HELPFUL?

The generality of up-/down-scaling is so wide and so ubiquitous in hydrology that the domain of application of what we have discussed in this paper covers a large part, if not all, of the hydrological sciences. Furthermore, the singular behaviour of the rain measure forces us to recast the balance equation (equation (12)) in terms of (singular) measures rather than their hypothetical densities with respect to the (space–time) Lebesgue measure ($d\underline{\boldsymbol{x}}\, dt$), i.e.:

$$\Delta S(B) = R(B) - Q(B) - I(B) \tag{34}$$

where ΔS, R, Q and I are the measures of the storage evolution, precipitation, discharge and infiltration, respectively, for any suitable space–time subset. This balance implies that the singular behaviour of one of these measures will contaminate at least one other. This explains why the multifractal rainfall analyses can and must be extended to the runoff, as emphasized by Tchiguirinskaia *et al.* (2007). This began to be carried out either for runoff series analyses explicitly in time (Tessier *et al.*, 1996; Pandey *et al.*, 1998; Labat *et al.*, 2002), or implicitly in space (Gupta & Waymire, 1990, 1998), by considering that the observation resolution corresponds to the size of the basin. Equation (34) may also explain the disturbing fact that conceptual and lumped models are often more successful than distributed physically-based ones (Hansen *et al.*, 2007), or that refining the latter is not always helpful (Wood, 1998). Obviously, equation (34) will be inextricable without theoretical knowledge on its pushforward by a change of space–time scales.

To illustrate that once you have the appropriate framework (almost) everything becomes rather straightforward, let us consider, as suggested by an anonymous referee, the puzzling so-called separation of skewness of the annual maximum flood series of more-or-less homogeneous hydrological regions (Matalas *et al.*, 1975). This separation was defined by the fact that fluctuations in empirical skewness estimates were found to be much larger than those drawn from any common regional distribution. As pointed out by Dawdy & Gupta (1997), the skewness scale dependence for a multifractal field, discussed above, presumably explains in a straightforward manner the separation of skewness: a purely homogeneous region but comprising basins of different sizes will indeed display strong skewness fluctuations due to the differences of scale/resolution. This also reinforces questions already raised by Bobee *et al.* (1989) about the interpretation and use of the separation of skewness as a criterion for discrimination between candidate probability distributions (Rossi *et al.*, 1984; Ahmad *et al.*, 1988).

In a general manner, the scaling moment function $K(q)$ can be estimated from any d-dimensional data set (*in situ*, radar, lidar, satellite, simulation outputs). This already provides a lot of information about the structure of the field (time series), in particular its extremes. Furthermore it enables us to downscale the field stochastically for simulations and predictions, for instance:

- Various remotely-sensed data have been used to show that the outer spatial scale of the hydrometeorological fluctuations is of the order of the planet (Lilly & Paterson, 1983; Schertzer & Lovejoy, 1984; Lovejoy *et al.*, 2001, 2004, 2008; Lilley *et al.*, 2004), i.e. these fields presumably fluctuate up to the largest possible spatial scale.
- We pointed out that singular measures easily provide such a strong intermittency that it corresponds to fat-tailed probability distributions for the extremes. This issue was often considered as controversial for applications, but more recently the discussion of this important topic has become less partisan (Schertzer *et al.*, 2006).
- The detection and quantification of the evolution of the hydrological extremes in a climate scenario (Royer *et al.*, 2008), which more than ever brings into question (Hubert *et al.*, 2007) the ubiquitous hypothesis of stationary statistics (a requisite for the classical frequency analysis):
  - (i) the multifractal parameters that rule the extremes were estimated on the resolved scales of the model (larger than 250 km), and their respective temporal evolution dates (from year 1860 to 2100) are opposite each other in the present century, whereas the hydrological extremes increase/decrease if both parameters do;
  - (ii) this result may already explain why it is so difficult to detect this evolution by classical methods on discharge time series (Kundzewicz *et al.*, 2005; Svensson *et al.*, 2006), and the maximum precipitation generated by the model; and
  - (iii) furthermore, the probable maximum singularity (Douglas & Barros, 2003; Hubert *et al.*, 2003), which can be analytically computed from these parameters, resolves this ambiguity not only on the resolved scales but all through the scaling range down to scales relevant to hydrology.
- The assessment of the intrinsic predictability limits of hydrological processes, which bounds above our effective predictive capacity, and therefore determines the required model quality to be achieved: multifractal predictability has been shown to be distinct from those of the "butterfly effect" (Schertzer & Lovejoy, 2004).

CONCLUSIONS

Having pointed out that the mathematical measure is indeed the appropriate mathematical tool to quantify rain and other hydrological fields, we stressed the important differences between singular and regular measures, the latter being (unfortunately) much more popular. We emphasized the fundamental role of changing scales by dilation/contraction and, thus, analysed the application of the up-/down-scaling on singular measures, which yields non-trivial results contrary to regular measures.

With the help of a rainfall time series and the mass balance equation, we argued that hydrological fields are most likely singular measures and that most (if not all) the so-called hydrological monsters result from this feature, more precisely from the fact that this singular measure is improperly handled as a regular measure. Furthermore, we pointed out what we can gain by handling this measure properly – in particular that the downscaling of a field merely corresponds to a cascade process – along with a few applications.

In summary, it is timely:

– to recognize that the hydrological court of miracles is rather depopulated,
– to fit concepts to experimental data rather than to fit parameters, and
– to consider the full extent of the statement of the US National Research Council (1991): "*The search for an invariance property across scales as a basic hidden order in hydrologic phenomena to guide development of specific models and new efforts in measurement is one of the main themes of hydrologic science.*"

When envisioning a new framework for hydrology, it may be appropriate to keep in mind two bold statements from the author of the court of miracles (Hugo, 1980, 2002a,b):

– "*Rien ne résiste à un acharnement de fourmi*" "Nothing is resistant to ant eagerness" (Hugo, 1980); and
– "*La raison, c'est l'intelligence en exercise; l'imagination c'est l'intelligence en erection*." "Reason is the application of intelligence, imagination is intelligence in erection" (Hugo, 2002b).

Both statements emphasize the eagerness and imagination that are required to develop an adequate framework for hydrology, that are out of reach for business-as-usual approaches.

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