Multifractal study of three storms with different dynamics over the Paris region

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Abstract Nowadays research is triggered by the permanent need to better relate the measured radar reflectivity to the surface rainfall. The knowledge of flow structure within cloud formation systems and the associated convective–stratiform separation may provide useful information in this respect. We will first discuss how stochastic multifractals can handle the differences of scales and measurement densities of the raingauge and radar data; and help to merge information from these data. Mosaics from the Météo-France ARAMIS radar network are used that correspond to horizontal projections of radar rainfall estimates for a 1 km × 1 km × 5 min grid over France. In particular, three storm events with different dynamics over the Paris region were selected to illustrate the efficiency of the multifractal framework. In spite of the difficulty that usually the same precipitation field comprises both stratiform and convective formations, their respective scaling properties allow to decipher and to classify the radar data.

Key words multifractals; convective-stratiform formations; rainfall extremes; power law

INTRODUCTION

Precipitation constitutes a natural phenomenon which has very strong socio-economical impacts, especially as heavy rainfall usually associated with storms. Climate change and sprawling urbanization are two drivers that would increase the extremes. Both already put into question the usual implicit approximation of statistical stationarity, and therefore classical operational notions. To take into account this aspect, the hydrological systems of the rainfall alert and forecast need to provide the communities with more detailed space–time information. Ideally, this should cover time scales from 1 second and spatial scales ranging from millimetres to about 50 km, i.e. from drop sizes to Paris region scale. This range of scales is crucial for urban hydrology and it also corresponds to the lesser known scale range for precipitation fields. There are many ongoing discussions on possible rainfall scaling breaks below 30 km (e.g. Gires *et al.*, 2011). A deeper understanding of the rainfall process across scales seems to be necessary for bridging the scale gap between radar and *in situ* measurements, which has been a long-lasting problem for radar calibration (Austin, 1987). In this paper we explore the scaling behaviour of horizontal projections of the radar rainfall estimates by properly dealing with the strong intermittency of rainfall that is more obvious at small scales. We first discuss interlinks between what could be termed convective/stratiform rainfall and a possible rainfall scaling break. Then we illustrate how the multifractal methods handle the differences of scales and measurement densities of the raingauge and radar data that are available f[?] 1 km × 1 km × 5 min grid. Finally we underline how merging the information from these data would increase our understanding of rainfall extremes. Empirical parameters allow an implementation of a procedure for simulation and nowcasting of rainfall fields and for cross-fertilization with operational applications.

DEFINITIONS ACROSS SCALES

Initially triggered by climatological studies, the discussion of convective *vs* stratiform rainfall was widely enlarged by the aspects of radar rainfall measurement. Most precipitation systems can be sorted as convective or stratiform, while there is no consensus on a precise definition of the terms convective/stratiform (see e.g. Steiner & Smith, 1998); notably their distinction is not always sharp. The most common cinematically-based classification into convective/stratiform formations

appeals to the characteristic scale and magnitude of the vertical velocity field. Indeed, Houze (1993) underlined that stratiform rainfall is more variable in the vertical than the horizontal, since hydrometeors (snow or rain) in stratiform clouds grow primarily by descent through a widespread updraft whose velocity magnitude is less than a metre per second. Thus, purely stratiform rainfall would result from mid-latitude frontal systems. Convective rainfall is associated with regions where rainfall remains equally variable in the horizontal and in the vertical directions, so the magnitudes of upward air motion would be sufficiently large in respect to the fall velocity of the hydrometeors. Steiner & Smith (1998) advocated that the definition of what might be termed convective/stratiform rainfall should hold across all scales. As a step in this direction, Seed (2004) investigated the extent of the vertical-horizontal anisotropy, depending on the physics dominating the rainfall production process. He proposed use of variograms of radar reflectivity in the vertical and horizontal dimensions to distinguish between convective and stratiform rainfall, although he admitted that this method requires very large samples of data. In particular, it requires vertical profiles of radar reflectivity that are not actually available among the ARAMIS products of Météo-France (Gilet *et al.*, 1983).

The same precipitation field often comprises formations of both types (Houze, 1997). In spite of this difficulty, meteorologists often select samples of predominantly stratiform or convective rainfall fields aiming to investigate their respective scaling properties. For example, Chumchean (2004) pre-classified over 800 fields of hourly rainfall accumulation over Sydney (Australia). He used the spectral analysis on 256×256 km^2 rainfall fields of 1 km resolution. The spatial spectrum $E(k)$ is the density of "energy" of the field (i.e. the squared field) per unit wave number k (km^{-1}). A random field is *scaling* when its spectrum follows a power law, $E(k) \propto k^{-\beta}$. Chumchean (2004) found differences in the estimates of the power law exponent, β, often called the "spectral slope", for stratiform and convective rainfall, with respective average values of 2.3 and 2.1. However, the significance of this 10% difference could be put into question due to uncertainties. In fact, rainfall is a complex space–time process. Hence, contrary to the usual approaches that take into account only spatial fluctuation estimators, we propose to analyse the time evolution of this estimator to classify convective and stratiform rainfalls. We therefore discuss the capacity of a more general space and time scaling analysis, often resulting in non-stationary estimates of scaling parameters. Then the time series of scaling exponents could be directly used, for example, to automatically sort out the convective rainfall episodes among numerous radar data archives.

We use the ARAMIS data of Météo-France (Parent du Châtelet, 2003) for three storms over the Paris region, each lasting 4 h. These storms were *a priori* classified by meteorologists as two mixed events (on 11 January 2008 and on 9 February 2009 from 00:12 h to 00:16 h) and a stratiform rainfall on 13 February 2009 (from 00:01 h to 00:05 h). However, this classification remains only an indicative one for the selected periods and location. We started from the (spatial)
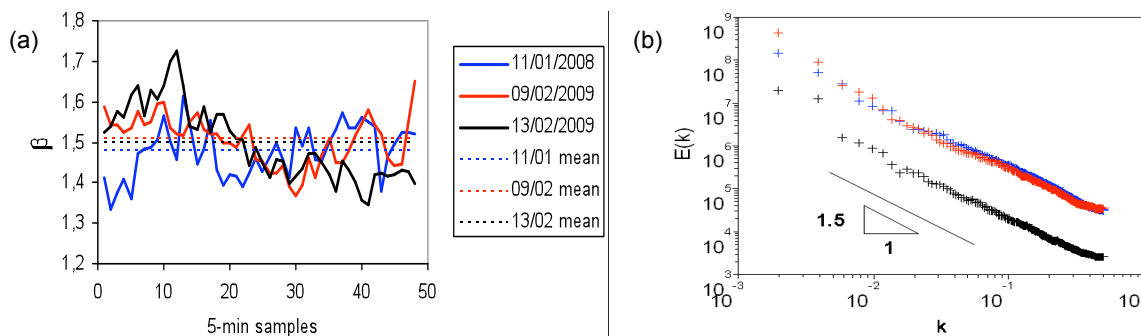

Fig. 1 (a) Temporal evolution of the spectral slopes obtained on power spectra in space (1 km resolution) of consecutive samples of 5 minute rain accumulation; (b) power spectra in space, averaged over the duration of an event: 11 January 2008 (blue), 9 February 2009 (red) and 13 February 2009 (black). All events yield the average spectral slope $\beta = 1.5$ (in space and time), independently of the physics that dominated the rainfall production process.

spectral analysis of 5 min time consecutive rainfall accumulation maps of $512 \times 512 \text{ km}^2$ with a 1 km resolution. For all three storm events, the time series of the spatial power law exponents fluctuate around the mean value of $\beta = 1.5$ (Fig. 1(a)), independently from the physics that dominated the rainfall production process. The same value of the β exponent characterises the spatial power spectra averaged over the duration of the event (Fig. 1(b)). Although the stratiform rainfall is notably less intense (i.e. the spectrum is lower than the two convective ones), there are no significant differences in scaling behaviour at larger scales of these spectra. The power law behaviour confirms the scaling nature of the rainfall field, although Fig. 1(a) points out the problem of fluctuations or non-stationarity of the power law exponent estimates (and more generally, of scaling parameters) during each event. These fluctuations put into question the possibility to distinguish in a reliable manner convective *vs* stratiform situations with only the help of the spectral slope, although it was quite appealing.

FRACTALS AND CONVECTIVE-STRATIFORM CLASSIFICATION

The simplest scaling feature of radar images, i.e. rainfall clustering over a fractal support, can be easily established with the help of the straightforward box-counting algorithm. The boxes are defined as the pixels of a regular 2D grid covering the radar image, where a mesh size l is consecutively increased by a given factor (usually 2). Then the fractal dimension Df of the rain support is the exponent of the scaling law of the number of pixels $n(l)$ containing at least one positive rainfall measurement: $n(l) \propto l^{-Df}$. Obviously a homogeneous 2D field will correspond to $Df = 2$, and lower and lower fractal dimensions correspond to more and more inhomogeneous fields. Figure 2(a)–(c) displays the temporal evolution of the two fractal dimensions estimated on national radar images ($512 \times 512 \text{ km}^2$) of 5 min rainfall accumulation, over two distinct sub-ranges of scales. Indeed, we detected a change in the scaling behaviour at about 20 km that corresponds to low values of the coefficient of determination estimated over the full range of scales, and was particularly visible on 11 January 2008. This implies that two distinct fractal dimensions are necessary over the small scale range (1–16 km) and over the large scale range (16–512 km), respectively. The small scale analysis (i.e. performed over the small scale range) confirms a general dependence of the support fractal dimension on the percentage of rainfall under the threshold of radar detection (i.e., the "zero rainfall"): the fractal dimension tends to decrease with an increase of zero rainfall, e.g. it decreases from 1.93 to 1.88 during the episode of 9 February 2009 when the zeros increase from 30% to 41%. Although the same $Df \approx 1.8$ is observed for two types of rainfall events: the convective type on 11 January 2008 with about 55% of zero rainfall during the episode and the stratiform type on 13 February 2009 with 70% of zeros. Therefore, a small scale fractal dimension would not be sufficient to sort out radar data. On the contrary, a comparison of small scale and large scale fractal dimensions gives a striking dissimilarity of the first two rainfall events (11 January 2008 and 9 February 2009) with respect to the third one (13 February 2009). While the large scale fractal dimensions remain close to 2 for the first two events and hence, remain distinct from the corresponding small scale fractal dimensions. In contrast, on 13 February 2009 the estimates of the large scale fractal dimension fluctuate around the small scale value of $Df \approx 1.8$. This result is in agreement with the fact that the horizontal extent of vertical air motions during a stratiform situation may grow up to hundreds of kilometres. Hence, the stratiform and decaying convective rainfall systems would naturally result in a more widespread precipitation, which in turn would result in similar estimates of fractal dimension over small and large scales. In contrast, a convective situation is characterized by significant vertical air motions over horizontal scales ranging from several metres to several kilometres. So, over a larger area, a convective rainfall system can be understood as a homogeneous distribution ($Df \approx 2$) of smaller scale precipitation cells, all having a lower fractal dimension. Therefore, the fractal concept provides a simple basis to automatically sort out convective rainfalls from radar data. It is

worth mentioning that for both rainfall types, the estimates of fractal dimension at small scales *Df*
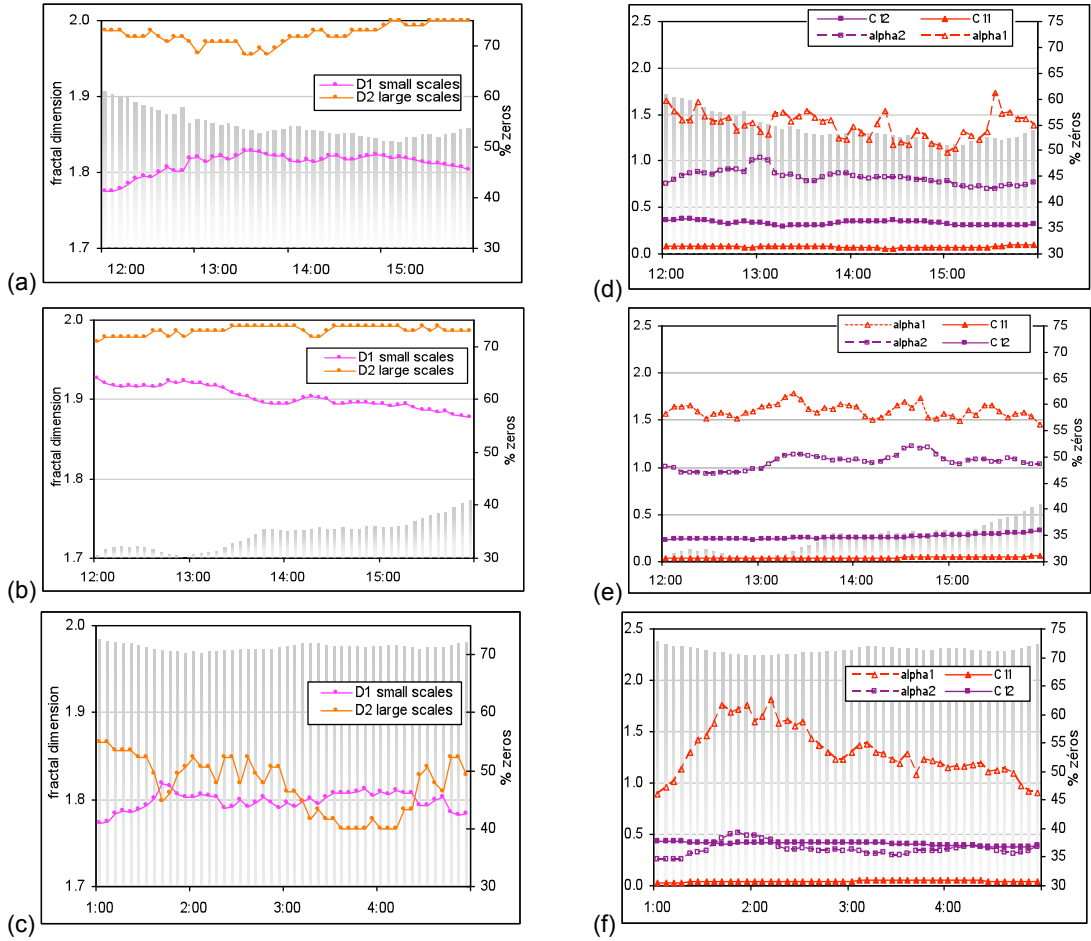

Fig. 2 Temporal evolution of the fractal dimensions (left column) and of the multifractal parameters (right column), all estimated on 48 images (512×512 km²) of 5 min rainfall accumulations: 11 January 2008 (a), (d), 9 February 2009 (b), (e) and 13 February 2009 (c), (f). Grey histogram indicates the percentage of pixels with "zero-rainfall", pink and red lines correspond to the estimates over the small scales (1–16 km), orange and purple lines correspond to the estimates over the large scales (16–512 km).

remain smaller than 2. This highlights that homogeneous distributions of rainfall would be particularly inappropriate for urban modelling. Indeed, at smaller scales, the proportion of pixels having non-zero rainfall should decrease as $l^{(2-Df)}$ to cope with reality.

PHYSICAL SIGNIFICANCE OF MULTIFRACTAL PARAMETERS

Different estimates of the fractal dimension exponent *Df* could be obtained over the same range of scales by simply changing the threshold that defines the non-zero rainfall: the rainfall intensities, therefore, correspond to a multifractal field (Schertzer & Lovejoy, 1987; Biaou *et al.*, 2003). Universal Multifractals (Schertzer & Lovejoy, 1997) can be defined with the help of a very limited number of parameters that have a strong physical significance and can be evaluated either theoretically or empirically. The parameter α measures the multifractality of the field. When $\alpha = 0$, the field is fractal and could be defined by a unique fractal dimension. C_1 is the co-dimension of the mean field and measures its mean fractality. A homogeneous field that fills the embedding space has $C_1 = 0$. When this parameter increases, the field becomes more and more intermittent, i.e. is concentrated on smaller and smaller fractions of the total area. The parameter estimation

methods are based on the upscaling of the field raised to various powers (Lavallée *et al.*, 1993). Figure 2(d)–(f) displays the temporal evolution of the multifractal parameters α and C_1 that are estimated on the same radar data and also separately over the same small and large ranges of scales, as for the fractal dimension estimates of Fig. 2(a)–(c). The multifractality of rainfall fields is stronger (α > 1) over small scales (i.e., over the range of 1–16 km). Over larger scales (i.e. over the range of 16–512 km), the high percentage of zero-rainfall reduces the multifractality while increasing the mean co-dimension C_1. The rainfall field appears to be more intermittent over large scales. This phenomenon is sometimes considered as the "unsolved issues regarding the treatment of the zero values" (Seed, 2004).
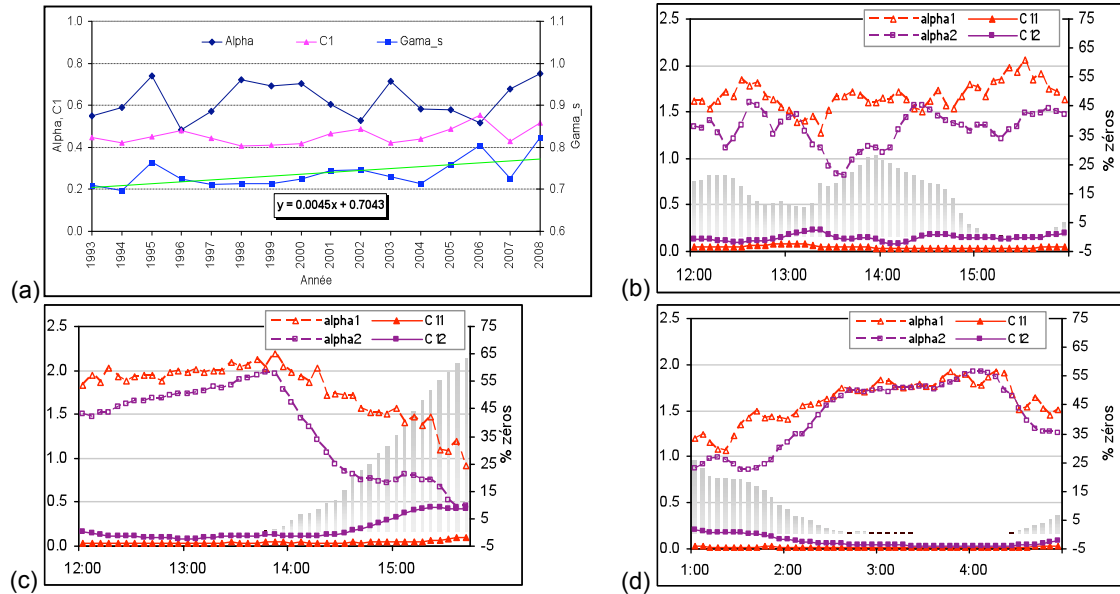

Fig. 3 Temporal evolution of multifractal parameters: (a) raingauge time series at Orly with *α* (dark blue), *C1* (pink), *γs* (blue) and the linear fit of the estimates of *γs* (green line); (b)–(d) centred radar images of 5 min rainfall accumulation, 11 January 2008, 12:00–16:00 h (b), 9 February 2009, 12:00–16:00 h (c) and 13 February 2009, 01:00–05:00 h (d). Grey histograms indicate the percentage of zero-rainfall pixels, dashed lines for *α*, dotted lines for C1 (red and purple for the small (1–16 km) and large (16–128 km) scale ranges, respectively).

TENDENCY IN THE EVOLUTION OF RAINFALL EXTREMES

Over large scales, α < 1 and C1 ≈ 0.25–0.5 (Fig. 2(d)–(f)) are qualitatively close to those estimated on continuous raingauge time series. This opens ways of combining the analysis of raingauge and radar data, in spite of their differences in scales and measurement densities. Figure 3(a) displays an example of the temporal evolution of multifractal parameters estimated on time series at Orly raingauge in the region of Paris during the period 1993–2008. The stochastic multifractal parameter estimates reflect the long-range dependencies and clustering of rainfall extremes, often having power law probability distributions. A simultaneous increase of both parameters directly implies an increase of extremes. Otherwise, the evolution of extremes can be investigated with the help of the maximum observable singularity: $\gamma_s = C_1 \alpha /(\alpha - 1)\left(C_1^{(1-\alpha)/\alpha} - 1/\alpha\right)$. Like for Orly's time series, a slight positive slope of the linear regression of its temporal evolution (Fig. 3*a*) indicates a possibly more important increase of the maximum intensity of rainfall, $R_{max}(\tau) \propto \tau^{-\gamma_s}$, which should not be neglected for small time scales τ. If such behaviour would be only observed on radar data, it could be attributed to a change in the QPE system during the year 2004. This illustrates a potential usefulness of completing the scaling analysis of radar data with the help of an analysis of available raingauge time series.

It is worthwhile noting that the difference in estimates between small and large scales

becomes less obvious when reducing the initial domain to 128×128 km^2, compatible with the surface area of the Paris region (12011 km^2). The storm events, selected for this paper, were indeed essentially located over this region. Reducing the initial domain, we therefore limited our analysis to the area of intense rainfall. Hence, the multifractal parameters obtained (Fig. 3(b)–(d)) are in good agreement with those estimated over the small scale range of the initial domain (Fig. 2(d)–(f)), and furthermore are compatible with those obtained for heavy rainfall episodes extracted from local raingauge time series (Hoang, 2008).

CONCLUSIONS

The aim of this study is to explore the scaling behaviour of horizontal projections of the radar rainfall estimates by properly dealing with the strong intermittency of rainfall. We start from the investigation of possible relations between radar rainfall scaling breaks and associated convective–stratiform separation. The differences of scales and measurement densities of the raingauge and radar data are easily handled in the framework of stochastic multifractals. This framework embodies all possible probability tail behaviours and clustering of the extremes, including possible long-range dependencies: two most important statistical features to fit empirical quantile estimates. Multifractal methods allow rainfall quantiles to be evaluated dynamically. For universal multifractals, the parameters a and C_1 fully define their statistics. This explains why the observed compatibility between the multifractal parameters for raingauge measurements and radar data is essential to merge the two types of data. Increased reliability of the parameters reduces the uncertainty in the predicted regional hydrological extremes. The results obtained illustrate a slight tendency for the hydrological extremes to become particularly local and intense in the Paris area. These preliminary results encourage more extensive use of radar archives for better detection of local climate trends.

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