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# Do stable atmospheric layers exist?

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[1] The notion of stable atmospheric layers is a classical idealization used for understanding atmospheric dynamics and thermodynamics. Using state of the art drop sonde data and using conditional, dynamical and convective stability criteria we show that apparently stable layers are typically composed of a hierarchy of unstable layers themselves with embedded stable sublayers, and unstable sub-sub layers etc. i.e. in a Russian Matryoshka doll-like fractal hierarchy. We therefore argue that the notion of stable atmospheric layers is untenable and must be replaced by modern scaling notions. **Citation:** Lovejoy, S., A. F. Tuck, S. J. Hovde, and D. Schertzer (2008), Do stable atmospheric layers exist?, *Geophys. Res. Lett.*, *35*, L01802, doi:10.1029/2007GL032122.

[2] There are two basic theoretical approaches for understanding atmospheric stratification: the statistical turbulent approach and the deterministic "dynamical meteorology" approach. In addition, there is the numerical modeling approach which is "agnostic": it aims at practical forecasting and simulation, it does not explicitly require the assumptions of either theory. Nevertheless in order to numerically integrate the equations it makes its own subgrid and often Boussinesq, hydrostatic, or anelastic approximations, the plausibility of which are largely informed by dynamical meteorology notions [*White et al.*, 2005]. At the same time, the mainstream turbulence approaches are all isotropic making them inappropriate for understanding stratification. Ultimately, we believe that the turbulence approach – when appropriately generalized from isotropic to anisotropic scaling (i.e. from 3D isotropic or 2D isotropic to 23/9D anisotropic so that the degree of stratification increases in a power law way with scale) is the best available - we develop this elsewhere [*Schertzer and Lovejoy*, 1985; *Tuck et al.*, 2004; *Lovejoy et al.*, 2007a; *Lilley et al.*, 2007; *Radkevitch et al.*, 2007; S. J. Hovde et al., manuscript in preparation, 2007]. In this paper we therefore concentrate on the central dynamical meteorology notion: the stable layer. This idealization plays a central role in the use of thermodynamic diagrams and in synoptic meteorology including in the interpretation of potential vorticity maps [*Hoskins et al.*, 1985]. In addition, the notion of stable, smoothly varying layers justifies ubiquitous linear theories. For example, *Nappo* [2002] states "Almost all of what we know about the nature of gravity waves is derived from the *linear theory*" (emphasis in the original). Using high resolution drop sonde data which allow the vertical structure to be measured to 5 m resolution, (i.e. 10–20 times better than operational radiosonde balloon data and 100 times better than the standard "significant levels"), we show that apparently stable layers are punctuated by a fractal hierarchy of unstable layers making it unlikely that linear theory is appropriate.

[3] Probably the best-known example of dynamical meteorology is its explanation for the near linear temperature fall-off with altitude $z$; the dry adiabatic lapse rate. Textbooks explain that when a parcel of air is vertically displaced, it will expand because of the vertical pressure gradient. The work required lowers the temperature of the parcel; if this process occurs adiabatically, then one obtains the dry adiabatic lapse rate ≈9.8 K/km. This explanation is only a first approximation, more interesting is the sign and magnitude of the deviations. Going back to *Väisälä* [1925] and *Brunt* [1927], consider an atmosphere with a uniform temperature gradient. When a parcel of air is vertically displaced by a small amount, it experiences a restoring force proportional to $g\partial\log\theta/\partial z = N^2$ where $\theta$ is the potential temperature and $N$ is the Brunt-Väisälä frequency. When $N^2 > 0$ the particle will oscillate about its initial position with frequency $N$, the atmosphere is stable. On the contrary, when $N^2 < 0$, the particle will accelerate away from its equilibrium position, the atmosphere is locally unstable. Since the result neglects the possible destabilizing effect of condensation of water vapor, $N^2 > 0$ implies only "conditional" stability. In a humid atmosphere, the same argument can be made taking into account the latent heat released by condensation of humidity, the "convective instability" criterion is the same with the "equivalent" potential temperature $\theta_E$ replacing the potential temperature $\theta$ and $N_E$ replacing $N$; both criteria are used below.

[4] The above analysis assumes that the air surrounding the parcel is motionless; $N^2 > 0$, $N_E^2 > 0$ are static stability criteria. However the atmosphere typically has large vertical shears, we must consider the dynamical stability. Surprisingly, this was actually considered somewhat earlier by *Richardson* [1920] who noted that buoyancy tended to stabilize shear flows and he quantified this effect by the eponymous dimensionless number $Ri = (N^2/s^2)$ where $s = \partial v/\partial z$ is the vertical shear. Layers with $Ri$ exceeding a critical value $Ri_c$ (usually taken ≈ 0.25) are considered "dynamically" stable, otherwise they are dynamically unstable (due to the scaling of $N^2$, $s^2$, changing $Ri_c$ will only change the fractal exponent characterizing the clustering of the layers). The atmosphere is thus sometimes classified: $Ri < 0$ "unstable stratification", $0 < Ri < Ri_c$ the "stable subcritical regime", $Ri > Ri_c$ the "supercritical regime".

[5] But do stable layers really exist? An early recognized symptom of problems caused by the strong atmospheric inhomogeneity is that – even at a fixed scale - $Ri$ is an

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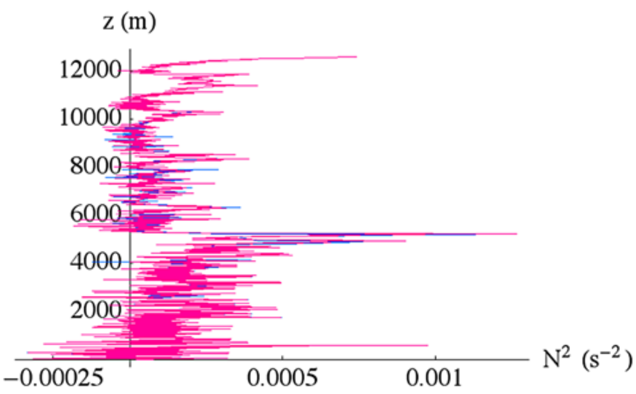


**Figure 1.** The Brunt Väisälä frequency squared as a function of altitude. The two sondes are indicated by red and blue traces. Most of the time they are indistinguishable indicating that the error in the measurement is less than the width of the lines.

incredibly variable quantity (its mean barely - if at all - converges [*Schertzer and Lovejoy*, 1985]). Indeed, less variable statistically based alternatives such as the "flux" Richardson number [e.g., *Garratt*, 1992] are frequently used instead. In addition $N^2$, $N_E^2$, $s^2$ and $Ri$ vary with scale [*Reiter and Lest*, 1968]. Radiosonde and drop sonde data show that the variation is in a scaling (power law) way with the layer thickness $\Delta z$ [*Schertzer and Lovejoy*, 1985; Hovde et al., manuscript in preparation, 2007] so that the derivatives defining both $N^2$ and $s^2$ tend to zero as the layers became thinner and thinner ($\Delta z \rightarrow 0$), implying that their true values will depend on the turbulent dissipation scale. More recently [*Dalaudier et al.*, 1994; *Muschinski and Wode*, 1998], thin (even sub metric) step-like structures called "sheets" were discovered in otherwise supposedly smoothly varying structures. Similar results have been reported in the ocean [*Gregg*, 1991; *Osborne*, 1998].

[6] In order to see if we could define smoothly varying stable layers, we used state-of-the-art drop sonde data from the NOAA Winter Storms 04 experiment over the Pacific Ocean, where 261 sondes were dropped from roughly 13 km altitudes by a Gulfstream 4 aircraft. These GPS sondes had vertical resolutions of ≈5 m, temporal resolutions of 0.5 s, horizontal velocity resolutions of ≈0.1 m/s [*Hock and Franklin*, 1999], and (due to technical improvements) temperature resolutions of ≈0.01 K. While the full analysis of the 2004 experiment is described by Hovde et al. (manuscript in preparation, 2007), we concentrate here on analysis of 8 pairs dropped within 0.3 s on the 2004/02/29. The two sondes within a pair are separated by roughly 30 m and therefore can be used to cross check each other's accuracy. Such intersonde comparisons put the following upper bounds on the resolutions: ±0.014 K, $\pm 1.4 \times 10^{-5}$ $s^{-2}$, $\pm 7 \times 10^{-5}$ $s^{-2}$ for temperature, $N^2$, $N_E^2$ respectively. These resolutions are sufficiently good that almost all of the layers discussed here are reproduced from one sonde to the other, even at the highest resolution. Figure 1 shows the comparison of $N^2$ calculated at 5 m resolutions for each sonde in the first pair. That the fluctuations cannot be

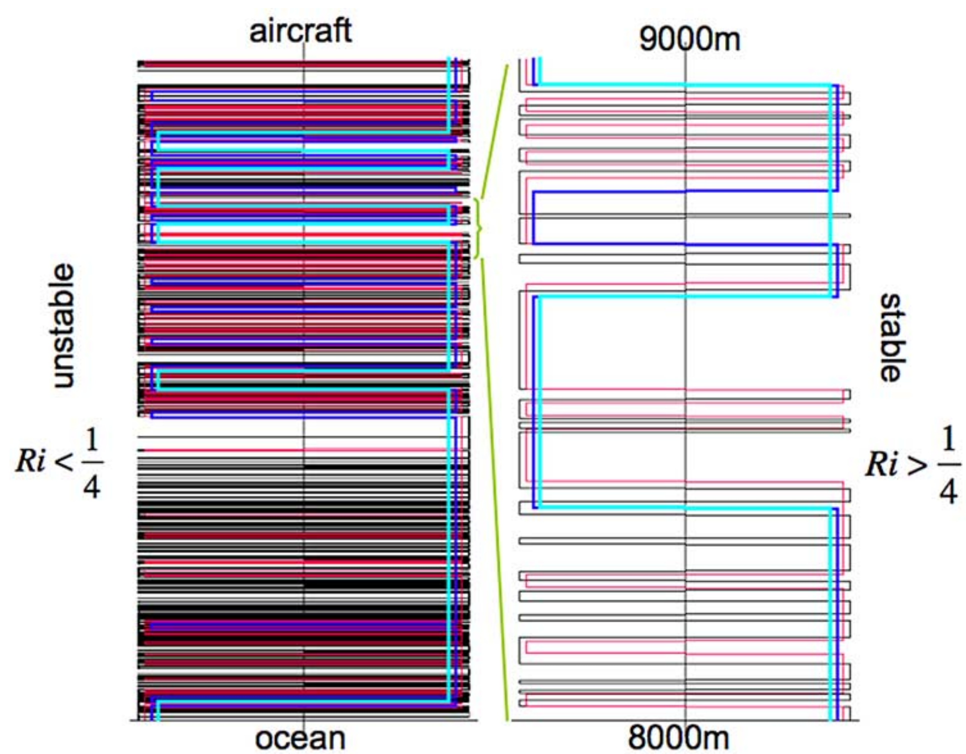


**Figure 2.** The stability of the atmosphere as determined by a drop sonde using the stability criterion $Ri > 1/4$ where the Richardson number is estimated using increasingly thick layers: 5, 20, 80, 320 m thick (black, red, blue, cyan respectively). The figure shows atmospheric columns, the left one from the ocean to 11520 m (just below the aircraft), the right is a blow up from 8000−9000 m. The left of each column indicates dynamically unstable conditions ($Ri < 1/4$) whereas the right side indicates dynamically stable conditions ($Ri > 1/4$). The figure reveals a Cantor set-like structure of unstable regions.

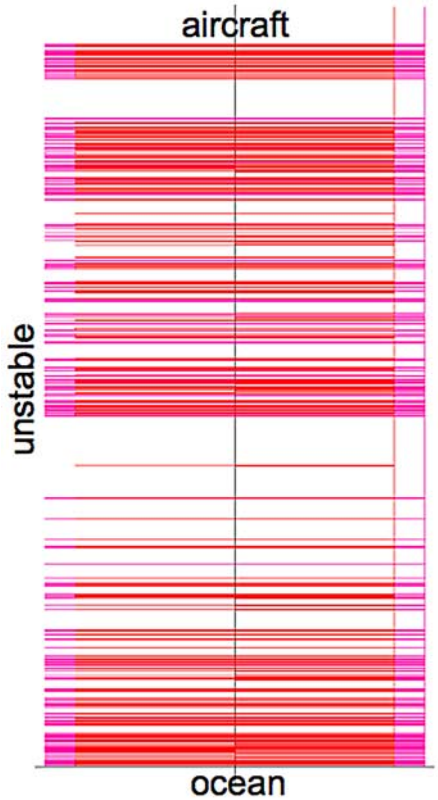


**Figure 3.** The stability of the atmosphere as determined by two drop sondes dropped about 30 m apart (indicated pink and red transitions), using the stability criterion $N^2 = g\left(\frac{\partial \log \theta}{\partial z}\right) > 0$ where $N^2$ is estimated using layers at 5 m thickness. The transitions from unstable (left) to stable (right) are shown as a function of altitude from the ocean (bottom) to 12 km altitude (top). Nearly the same fractal structure is found in both showing that the fractality is not an artifact of noise.

attributable to instrument noise is clear from their close agreement.

[7] Combining $N^2$ with velocity data, we can determine the dynamic stability ($Ri > 1/4$) at various resolutions. At low resolution (320 m, Figure 2), we obtain the usual first order approximation to the vertical structure familiar from operational radiosonde resolutions: unstable in the very lowest layer with only a few other fairly thin unstable layers higher up. At low resolution, there appear to exist reasonably wide layers which are stable, perhaps allowing the application of quasi-linear gravity wave theories. However this hope is dashed when we turn to the finer resolutions (80, 20, 5 m superposed). Many of the apparently stable sublayers are found to consist of a hierarchy of unstable subsublayers, themselves embedded with stable subsubsub layers etc. with the same "Russian doll" hierarchical structure holding in reverse for the initially unstable layers; the blow-up on the right hand side of Figure 2 shows this particularly clearly. Figure 3 shows the same profile using the static stability criterion $N^2 > 0$; we note (1) dynamical and static criteria are qualitatively similar, and (2) both sondes infer almost all the same layers. To show that the unstable layers are indeed fractal subsets of the vertical, we calculated (Figure 4) the (conditional) probability $P(\Delta z)$ of finding a (5 m thick) unstable layer at a distance $\Delta z$ from a given (5 m thick) unstable layer. $P(\Delta z)$ is roughly a power law; its (absolute) exponent is the correlation codimension $C_c = 1 - D_c$ ($D_c$ is the correlation dimension) which characterizes the sparseness of the unstable layers. Figure 4 shows the results for 24 sondes using the "conditional stability" $N^2 > 0$ criterion, the dynamical stability criterion ($Ri > 1/4$) as well as the "convective stability" criterion $N_E^2 > 0$. The bars show the amplitude of the sonde to sonde variations. If $C_c$ is estimated on each sonde individually, we obtain: $C_{cN} = 0.36 \pm 0.056$, $C_{cRi} = 0.22 \pm 0.037$, $C_{cNE} = 0.15 \pm 0.016$ based on $N$, $Ri$, $N_E$ respectively. This implies an ordering of decreasing sparseness from conditional instability, dynamic instability to convective instability. The deviation of the mean behavior from perfect power laws is less than 10% over the layers with separations in the range 5 m to 1.5 km. The result $C_{cN} > C_{cRi}$ is a (mathematical) consequence of the fact that the conditionally unstable layers are subsets of the dynamically unstable layers. The total 261 sondes from 10 separate flight days form a consistent data set, with the flight data from 2004/02/29 being typical. We should note that vertical scaling laws for $N^2$, $Ri$ were also found over land [*Schertzer and Lovejoy*, 1985] so that it is likely that our results are also valid over land.

[8] Our views of the atmosphere originated in a world of low resolution data when it appeared possible to phenomenologically divide the atmosphere into homogeneous stable structures and regimes and then to model each separately. In today's golden age of meteorological data, wherever we look we see on the contrary strong wide scale range heterogeneity. However, it is becoming increasingly clear that the statistics can nevertheless follow relatively simply scaling laws, although these must be different in the

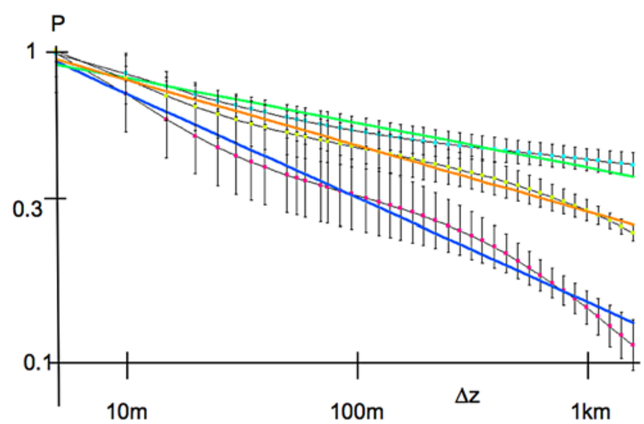


**Figure 4.** The conditional probability P of finding an unstable layer at a distance Δz from another unstable layer (at 5 m resolution, the average over 24 sondes); $P \propto \Delta z^{-C_c}$ where $C_c$ is the correlation codimension ($= 1 - D_c$ where $D_c$ is the correlation dimension of the unstable layers). The top is for convectively unstable layers, the middle is for dynamically unstable layers and the bottom is for conditionally unstable layers. The best fit absolute slopes are $C_{cN} = 0.36$, $C_{cRi} = 0.22$, $C_{cNE} = 0.15$ implying that the fractal correlation dimensions of the unstable layers are 0.64, 0.78, 0.85, respectively.

horizontal and vertical [see e.g., *Van Zandt*, 1982; *Schertzer and Lovejoy*, 1985; *Dewan and Good*, 1986; *Gardner*, 1994; *Dewan*, 1997; *Lilley et al.*, 2004; *Tuck and Hovde*, 1999a, 1999b; *Lovejoy and Schertzer*, 2005; *Lovejoy et al.*, 2004; *Lovejoy et al.*, 2007b]. In this context, the finding that unstable layers are distributed over sparse fractal sets is not surprising: presumably it is a consequence of the wide range vertical scaling of $N^2$, $N_E^2$, $s^2$ and indeed of all the meteorologically significant variables (see Hovde et al., manuscript in preparation, 2007, for a systematic overview).

[9] It has become commonplace to observe that most of the atmospheric energy fluxes – and hence dynamical processes – are concentrated in sparse fractal sets. We now see that the same is likely to be true of thermodynamic processes. This is because, as pointed out by *Dutton* [1976], "The atmosphere, then, finds the thermodynamic profits quite handsome indeed in unstable regions, and so it carries on much of its business where *Ri* is small." Our results quantify this by showing that convectively unstable thin layers are the most uniformly distributed (least sparse, smallest $C_c$) with the dynamically unstable layers somewhat sparser, and the conditionally unstable layers being the sparsest. This underlines the role of latent heating and has implications for our view of the general circulation of the atmosphere. Since many of the turbulent, buoyant mechanisms also operate in the ocean, it is possible that our results also hold there.

[10] There are also important consequences of our findings for the mainstream theories used to interpret vertical sounding data. These are the quasi-linear gravity wave theories notably the Saturated Cascade Theory [*Dewan and Good*, 1986; *Dewan*, 1997] and the Diffusive Filtering Theory [*Gardner*, 1994] which require layers with well defined and real, smoothly varying, Brunt-Väisälä frequencies (*N*). If the stable propagating gravity waves are broken up by a sparse fractal distribution of unstable layers, it is not obvious that those theories can be saved. However, there is a strongly nonlinear alternative; *Lovejoy and Schertzer* [2006] and *Lovejoy et al.* [2007a] show that one can readily make strongly nonlinear models based on localized turbulence fluxes which have wavelike unlocalized velocity fields, and this respecting the observed horizontal and vertical scaling. This turbulent anisotropic scaling can give rise to (nonlinear) dispersion relations not so different than those predicted by linear theory so it may be sufficient to reinterpret the empirical studies of waves in this anisotropic scaling framework.

[11] Finally, the concept of a stable layer plays a central role in synoptic meteorology not only through thermodynamic diagrams, but more importantly through the product of $N^2$ with the absolute vorticity, i.e. the potential vorticity, PV. PV maps are interpreted with the help of balance conditions which are only strictly valid in stable layers [*Hoskins et al.*, 1985]. At the moment PV analyses are mostly used in modeling the large scales with vertical resolutions such that layers are stable. However, as the models improve in resolution we may anticipate that the 'Russian Matryoshka doll' picture of the fractal embedding of stable and unstable layers will become visible and will have to be taken into account.

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