Stochastic and scaling climate sensitivities: Solar, volcanic and orbital forcings

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[1] Climate sensitivity (λ) is usually defined as a deterministic quantity relating climate forcings and responses. While this may be appropriate for evaluating the outputs of (deterministic) GCM's it is problematic for estimating sensitivities from empirical data. We introduce a stochastic definition where it is only a statistical link between the forcing and response, an upper bound on the deterministic sensitivities. Over the range ≈30 yrs to 100 kyrs we estimate this λ using temperature data from instruments, reanalyses, multiproxies and paleo spources; the forcings include several solar, volcanic and orbital series. With the exception of the latter - we find that λ is roughly a scaling function of resolution Δt: $\lambda \approx \Delta t^{H_\lambda}$, with exponent $0 \approx < H_\lambda \approx < 0.7$. Since most have $H_\lambda > 0$, the implied feedbacks must generally increase with scale and this may be difficult to achieve with existing GCM's. **Citation:** Lovejoy, S., and D. Schertzer (2012), Stochastic and scaling climate sensitivities: Solar, volcanic and orbital forcings, *Geophys. Res. Lett.*, *39*, LXXXXX, doi:10.1029/2012GL051871.

1. Introduction

[2] Even if one accepts that orbital forcing is the "pacemaker of the ice ages" [*Hays et al.*, 1976], over the range ≈30 yrs to ≈30 kyrs, there is no doubt that most of the variance in paleotemperature records is associated with the continuous spectral "background" [*Lovejoy and Schertzer*, 1986; *Wunsch*, 2003] (for a recent spectrum see Figure S1 in Text S1 in the auxiliary material).[1] This strongly suggests that other internal and/or external mechanisms are needed to explain the multidecadal, multicentennial and multimillenial variability. The discussion of these issues has been strongly tinted by the development of GCM's and their response to various external climate forcings. However, if the amplification factors are large – as they must be – then it will be hard to distinguish nominally external forcing paradigms from purely internal ones.

[3] The usual approach to evaluating climate forcings is via the climate sensitivity (λ) defined as the equilibrium change in a quantity, (here the temperature) per unit of radiative forcing. Sensitivities (λ) are commonly estimated with the help of (deterministic) numerical models; the usual example being the doubling of CO_2. The change in conditions (compositional in this example) simultaneously leads to changes in the typical mean global temperature (ΔT) and to the earth's radiative equilibrium from which the radiative forcing (ΔR_F) is determined by:

$$\Delta T = \lambda \Delta R_F \qquad (1)$$

This definition of climate sensitivity is convenient for numerical experiments with strong anthropogenic forcings. In this case, the response is relatively regular (smooth) so that the estimate $\lambda = \Delta R_F(\Delta t)/\Delta T(\Delta t)$ is well defined, insensitive to Δt. However, for natural forcings, it has several shortcomings. First, GCM outputs fluctuate over a wide range of Δt so that – except for very small time scales comparable to the model integration time steps - fluctuations $\Delta T(\Delta t)$ (and presumably) ΔR_F (Δt) typically have nontrivial scaling behaviours $\Delta T(\Delta t) \approx \Delta t^{H_T}$ and $\Delta R_F(\Delta t) \approx \Delta t^{H_R}$ implying $\lambda(\Delta t) \approx \Delta t^{H_\lambda}$ with $H_\lambda = H_T - H_R$ generally noninteger. Second, the usual definition of climate sensitivity is only valid if there is a causal link: the fluctuations ΔT and ΔR_F must have the same underlying cause such as a change in solar output. Strictly speaking, it therefore cannot be used empirically since in the real world there is only a single realization of climate. From the climate record, we can only measure correlations, not causality. In addition to the causality assumption, empirical estimates of λ must rely on model outputs in order to estimate ΔR_F [e.g., *Harvey*, 1988; *Claquin et al.*, 2003; *Chylek and Lohmann*, 2008; *Ganopolski and Schneider von Deimling*, 2008].

[4] As a consequence of these difficulties, λ has not been systematically explored as a function scale and it mostly known from models - not empirically. We therefore give a new stochastic definition of climate sensitivity which allows us to empirically estimate it for any physical forcing process whose consequent radiative forcing can be determined.

2. The Scaling of Temperatures, CO_2 Concentrations and Solar, Volcanic and Orbital Forcings

[5] Before considering potential climate drivers, let us first recall the variation with time scale Δt of temperature fluctuations ΔT. For this purpose, it turns out that it is *not* sufficient to define the fluctuation as the absolute difference ΔT (ΔT = $|T(t + \Delta t) - T(t)|$. Instead, we should use twice the absolute difference of the mean of the temperature between t and t+ $\Delta t/2$ and between t+ $\Delta t/2$ and t+ Δt. Technically, this corresponds to defining fluctuations using Haar wavelets rather than "poor man's" wavelets. While the latter is adequate for fluctuations increasing with scale (i.e., $\Delta T \approx \Delta t^{H_T}$ with $H_T > 0$), on average, absolute differences cannot

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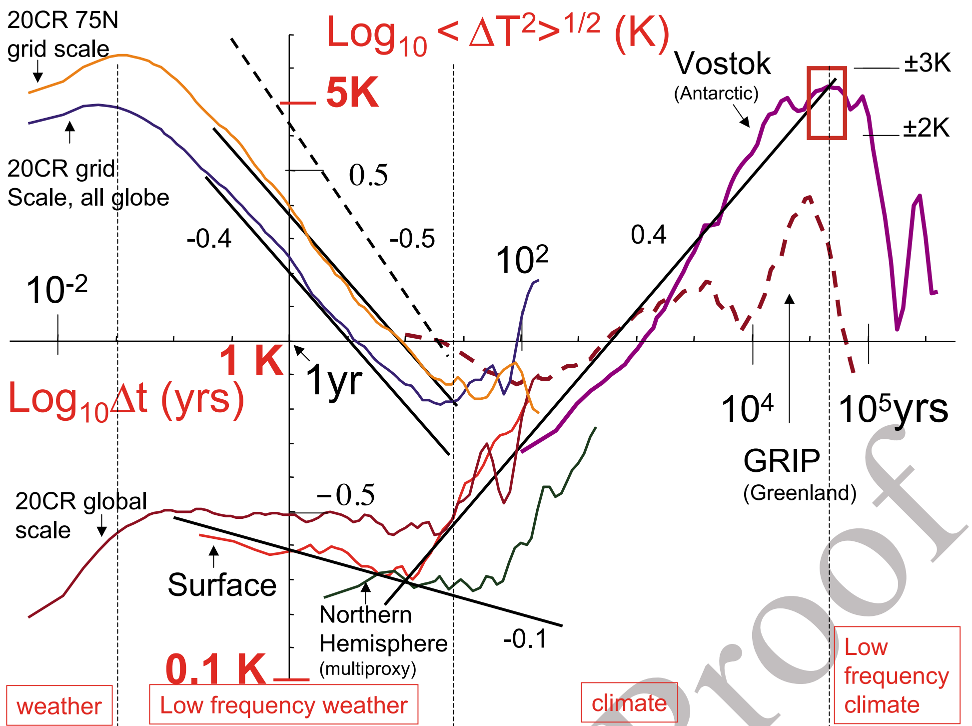

Figure 1. The RMS Haar structure function for temperatures including daily resolution 20th Century Reanalysis (20CR) data. On the left top we show grid point scale ($2° \times 2°$) daily scale fluctuations for both 75°N and globally averaged along with reference slope $\xi(2)/2 = -0.4 \approx H$ (20CR, 700 mb). On the lower left, we see at daily resolution, the corresponding globally averaged structure function. Also shown are the average of three in situ surface series as well as a multiproxy structure function (northern hemisphere). At the right we show both the GRIP (55 cm resolution, with calibration constant 0.5 K/mil) and the Vostok paleotemperature series. Also shown is the interglacial "window". See *Lovejoy and Schertzer* [2012b] for the figure and a full description of the data.

decrease and so when $H_T < 0$, do not correctly estimate fluctuations. The Haar fluctuation (which is useful for $-1 < H_T < 1$) is particularly easy to understand since (with proper "calibration") in regions where $H_T > 0$, it can be made very close to the difference fluctuation and in regions where $H_T < 0$, it can be made close to another simple to interpret "tendency fluctuation" (for discussion, see *Lovejoy and Schertzer* [2012b]).

[6] The variation of the fluctuations with scale can be defined using their statistics; the "generalized" qth order structure function $S_q(\Delta t)$ is particularly convenient:

$$S_q(\Delta t) = \langle \Delta T(\Delta t)^q \rangle \quad (2)$$

where "<.>" indicates ensemble averaging. In a scaling regime, $S_q(\Delta t)$ is a power law; $S_q(\Delta t) \approx \Delta t^{\xi(q)}$, where the exponent $\xi(q) = qH - K(q)$ and $K(q)$ characterizes the scaling intermittency (satisfying $K(1) = 0$). Below, with the exception of the volcanic series (where $K(2) \approx 0.2$), $K(2)$ is small ($\approx 0.01 - 0.03$), so that the RMS variation $S_2(\Delta t)^{1/2}$ has the exponent $\xi(2)/2 \approx \xi(1) = H$. Note that when $q = 2$ (the classical structure function), we have the useful relation $\xi(2) = \beta - 1$ where β is the spectral exponent defined by the spectral density $E(\omega) \approx \omega^{-\beta}$ where ω is the frequency.

[7] When $S_2(\Delta t)^{1/2}$ is estimated for various in situ, reanalysis, multiproxy and paleo temperatures, then one obtains Figure 1 (see Table S1 in Text S1). The key points to note are a) the three qualitatively different regimes: weather, low frequency weather and climate with RMS fluctuations respectively increasing, decreasing and increasing again with scale ($H_w > 0$, $H_{lw} < 0$, $H_c > 0$) and with transitions at $\tau_w \approx$ 5–10 days and $\tau_c \approx$ 10–30 yrs, b) the difference between the local and global fluctuations, with the former decreasing from ≈ 5 *K* (10 days) to ≈ 0.6 *K* at ≈ 25 yrs, increasing to ≈ 5 *K* at 50 kyrs c) the "glacial/interglacial window" corresponding to overall ± 3 to ± 5K variations over scales with half periods of 30 – 50 kyrs. This basic multiscaling regime picture is similar to that of *Lovejoy and Schertzer* [1984, 1986], *Pelletier* [1998], and *Huybers and Curry* [2006]. For comparison, we could note that unforced GCM's (control runs) at grid scale resolution have $H_{lw} \approx -0.4$ and do not yield any climate regime; i.e., $\tau_c \to \infty$ [see *Lovejoy and Schertzer*, 2012a].

[8] The problem of climate forcing is thus to determine what forcings might end the (decreasing, $H < 0$) low frequency weather regime and cause the fluctuations to start to increasing again when $\Delta t > \tau_c$ (i.e., $H > 0$)? To answer this, let us consider various possible external drivers as functions of scale; these may be conveniently classified according to whether they are scaling or nonscaling. This is useful because nonscaling climate forcings - i.e., at well defined frequencies – would leave strong signatures in the form of breaks in the temperature (and other) scalings which are generally not observed over the range of time scales between $\tau_c \approx 10 - 30$ yrs and $\tau_{lc} \approx$ 50–100 kyrs.

[9] An important nonscaling driver is the narrow-band orbital forcings at scales somewhat shorter but close enough to the upper time scale τ_{lc}. Although this break may well be

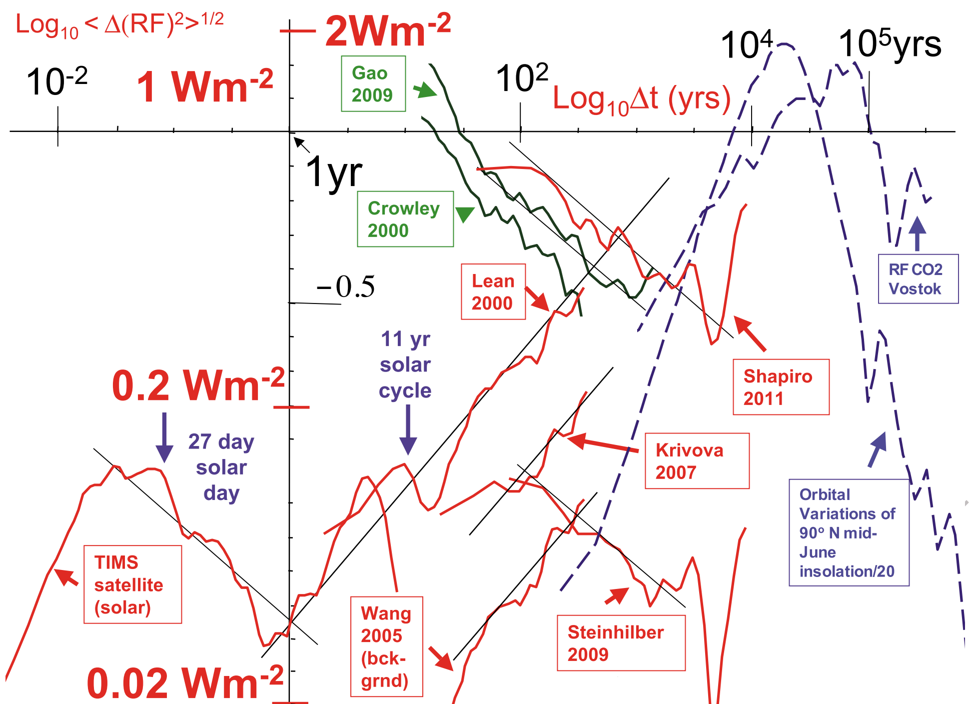

Figure 2. An intercomparison of RMS Haar fluctuations for various solar, volcanic, orbital and CO_2 data in units of radiative forcing (R_F) the solar radiances, the values of estimated Total Solar Insolation were converted into R_F using an albedo = ~~0.7~~ and geometric factor 1/4. The TIMS satellite data is for 8.7 yrs from 2003 to the present at a 6 hr resolution. Note that the Lean, 2000 reconstruction includes the 11 solar cycle whereas the Wang 2005 curve is only for the background. The Krivova 2007 curve has a 10 yr resolution. The Shapiro curve (the last 8963 yrs) was degraded to 20 yr resolution to average out the solar cycle, the Steinhilber curve was at a 40 yr ~~and~~ resolution over the last 9300 yrs. The volcanic series were from reconstructions of stratospheric sulphates using ice core proxies; due to the use of a 31 yr smoother, only results for longer scales are indicated in the figure (for reference the raw ~~D~~R_F(~~D~~$t = 2$ yrs) ≈ 5 Wm^{-2}). The Vostok paleo CO_2 series were converted to R_F using 3.7 W/m^2 per CO_2 doubling, the solar insolation at the north pole on June 15th was divided by 20, it is not a true R_F. The orbital variation curve was interpolated to 100 yr resolution and the low and high frequency fall-offs have logarithmic slopes −1, 1, i.e., they are the minimum and maximum possible for these Haar fluctuations. All the structure functions have been increased by a factor of 2 so that they are roughly "calibrated" with the difference ($H > 0$) and tendency ($H < 0$) fluctuations.

compatible with the observations, this is not trivial since the main signal in the temperature is nearer 100 kyr corresponding to orbital eccentricity variations. At least at high latitudes, these are much weaker not only than the higher frequency precessional and obliquity variations, but also than the lower frequency 400 kyrs eccentricity variations whose signal is virtually absent in the paleoclimate record; the "100 kyr" and "400 kyr" problems [*Ganopolski and Calov*, 2011; see also *Berger et al.*, 2005]. To quantify the orbital forcing, Figure 2 shows $S_2(\Delta t)^{1/2}$ of the solar irradiance variations at the north pole (every June 15th) determined from astronomical calculations [*Berger and Loutre*, 1991]. While this is not a true radiative forcing, it indicates its dominant time scales. One sees that the variability is confined to a fairly narrow range of scales and in Figure 3 we see that this range is about 3–4 times smaller than that of the peak in the paleotemperature variability; this is the 100 kyrs problem.

[10] Turning to the higher frequency continuous background, an (apparently) attractive possibility is to invoke greenhouse gas forcings. For example, using the recommended value 3.7 W/m^2 for a CO_2 doubling [*Intergovernmental Panel on Climate Change*, 2007], Vostok paleo CO_2 concentrations can be converted into radiative forcings (Figure 2). While to within a constant factor (Figure 3) this is very nearly the same as the corresponding temperature structure function, cross spectral temperature - CO_2 analysis (Figure S2 in Text S1) shows that over the whole range up to $\omega \approx (6\ \text{kyr})^{-1}$, that the phase of the CO_2 fluctuations lags those of the temperature by $\approx 74 \pm 22°$ so that (contrary to contemporary anthropogenic CO_2) – the paleo CO_2 is a "*follower*" not a "driver" (although it may play a role in solving the 100, 400 kyr "problems" [*Ganopolski and Calov*, 2011]), it is shown in Figures 2 and 3 for reference only.

[11] Quantifying solar variability is extremely difficult. Since 1980, a series of satellites have estimated the Total Solar Irradiance, yet the relative calibrations are not known with sufficient accuracy to establish the decadal and longer scale variability. Figure 2 shows $S_2(\Delta t)^{1/2}$ from the 8 year long series from the TIMS satellite; we see clearly the 27 (earth) day long solar "day" followed by a low frequency rise. To go further requires proxy based "reconstructions", Figure 2 shows $S_2(\Delta t)^{1/2}$ from several of these using sunspots and ^{10}Be records. The earliest [*Lean*, 2000] used a two component model, one of which had an 11 year cycle based on the recorded sunspots back to 1610, the other was a

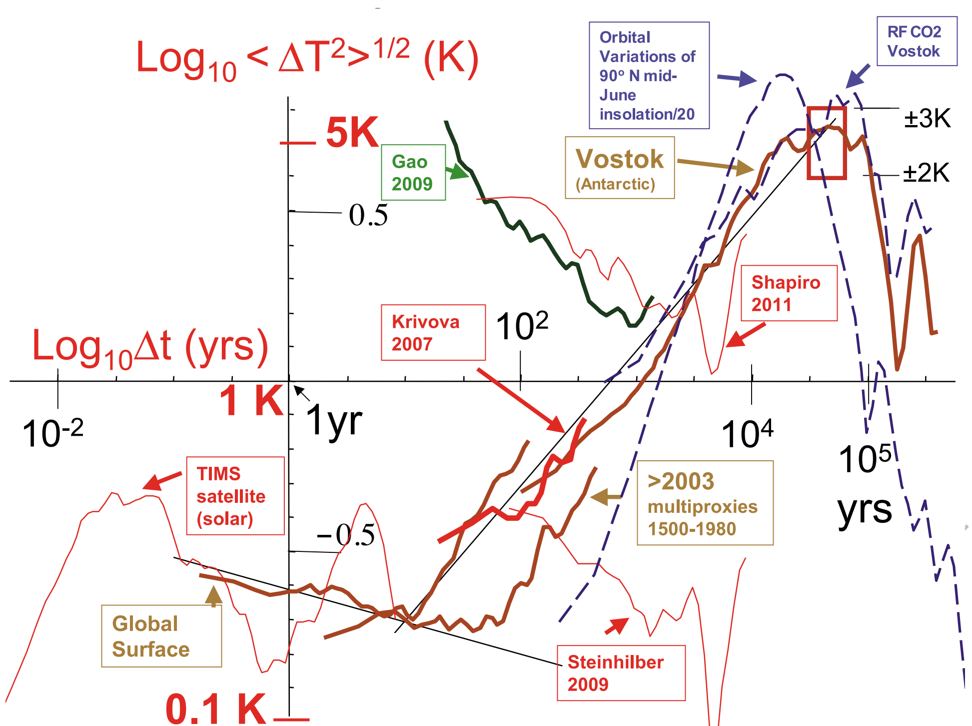

Figure 3. The RMS structure functions of the selected forcings from Figure 2 were converted into RMS temperature structure functions using a unique (and scale independent) climate sensitivity $\lambda = 4.5$ K/(Wm^{-2}). The reference lines have slopes of -0.1 and $+0.4$. It can be seen that the main orbital insolation fluctuations occur at time scales roughly 3–4 times smaller than the main temperature fluctuations.

"background". Combining the two results leads to an annual series featuring an overall 0.21% variation in the background since the 17th century "Maunder Minimum". Figure 2 shows that this reconstruction actually meshes quite nicely with the TIMS data with exponent $\xi(2)/2 \approx H_{RF} \approx 0.4$, i.e., close to H_T (Figure 3). *Wang et al.* [2005] updated this series and found typical fluctuations ≈4–5 times lower (Figure 2). A little later an intermediate (but still sunspot based) estimate yielded a variation of 0.1% since the Maunder minimum, again with $\xi(2)/2 \approx 0.4$ [*Krivova et al.*, 2007].

[12] The situation changed dramatically with the ≈9 kyr long reconstructions of *Steinhilber et al.* [2009] and *Shapiro et al.* [2011]. Both used ice core ^{10}Be concentrations to estimate the flux of cosmic rays, itself a proxy for the solar magnetic field and hence of solar activity. Although both were calibrated using the satellite observations, their assumptions were quite different, notably about a hypothetical "quiescent" solar state. The $S_2(\Delta t)^{1/2}$ for these reconstructions are remarkable for two reasons. First, they differ from each other by a large factor (≈8–9, see Figure 2); second, their slopes are the opposite to the sunspot based estimates: rather than $\xi(2)/2 \approx H \approx 0.4$, they have $\xi(2)/2 \approx H \approx -0.3$! While the large factor between them attracted attention, the change in the sign of H was not noticed even though it is probably more important as it would imply amplification mechanisms that increase quite strongly with scale.

[13] Another important driver is explosive volcanism. Volcanoes mainly influence the climate through the emission of sulphates that reflect incoming solar radiation; stratospheric sulphates can persist for months or years after an eruption. The two main volcanic reconstructions [*Crowley*, 2000; *Gao et al.*, 2008] are based on ice core particulate concentrations. First, sulphate concentrations are estimated and then with the help of models the corresponding global radiative forcings are determined; for $S_2(\Delta t)^{1/2}$, see Figure 2. It is remarkably similar to that of the ^{10}Be solar variabilities with $\xi(2)/2 \approx -0.3$, it nearly coincides with $S_2(\Delta t)^{1/2}$ from the *Shapiro et al.* [2011] solar reconstruction. The slightly longer (1500 yrs) *Gao et al.* [2008] series was converted into equivalent radiative forcings by scaling the mean to the *Crowley* [2000] series, the $S_2(\Delta t)^{1/2}$ results for the two series are very similar (Figure 2). Although very strong at small Δt, the volcanic forcings decrease rapidly at longer intervals so that any mechanism responsible for temperature fluctuations must on the contrary involve an amplification that strongly increases with scale.

3. Stochastic and Scaling Climate Sensitivities

[14] We would like to be able to compare the T and R_F fluctuations (Figures 1 and 2) but strictly speaking, the deterministic definition (equation (1)) doesn't allow it. To interpret our forcing and temperature statistics it is therefore convenient to introduce a stochastic definition of climate sensitivity:

$$\Delta T \stackrel{d}{=} \lambda \Delta R_F \quad (3)$$

where, "$\stackrel{d}{=}$" means equality in the sense of random variables [illegible] the random variables a, b satisfy $a \stackrel{d}{=} b$ if and only if Pr [illegible] $s) = \Pr(b > s)$ for all s, "Pr" means "probability"). Notice that while both deterministic and stochastic definitions (equations (1) and (3)) predict that the statistical moments are related by the equation $\langle \Delta T^q \rangle = \lambda^q \langle (\Delta R_F)^q \rangle$, the stochastic definition doesn't even require that R_F and T be correlated. A convenient interpretation is to regard the stochastic λ (equation (3)) as an upper bound on the deterministic λ with equality in case of full (and causal) correlation. The advantage of adopting equation (3) is that by fixing λ, we may convert Figure 2 into equivalent temperature fluctuations; Figure 3 shows the resulting superpositions using $\lambda = 4.5$ K/(Wm^{-2}) throughout. To put this value in perspective, we can compare it to $\lambda_0 \approx 0.3$ K/(Wm^{-2}), the sensitivity of the simplest energy balance model involving a homogenous atmosphere and radiative equilibria. We see that a (large) "feedback" factor $f = \lambda/\lambda_0 = 4.5/0.3 \approx 15$ is necessary to justify the overlaps shown in the figure.

[15] From equation (3) - and for simplicity only considering the mean ($q = 1$) behaviour - we see that if $\langle \Delta T(\Delta t)\rangle \propto \Delta t^{H_T}$ and $\langle \Delta \cancel{RF}(\Delta t)\rangle \propto \Delta t^{H_{RF}}$, then $H_\lambda = H_T - H_{RF}$. If we take $H_{RF} \approx -0.3$ (volcanic and ^{10}Be solar estimates), $H_{RF} \approx 0.4$ (sunspot based solar) and $H_T \approx 0.4$, then we find $H_\lambda \approx 0.7$ and ≈ 0 respectively. From Figure 2 we see that the volcanic and *Shapiro et al.* [2011] solar forcings require a feedback factor $f \approx 0.3$ at 30 year scales, rising to roughly ≈ 20 at 10 kyrs. If we consider instead the scale independent amplification factors ($H_\lambda \approx 0$), i.e., the Krivova and Wang reconstructions, we find the (scale independent) factors $f \approx 15$, 30 respectively. However, for this to apply at multimillenial scales, solar variability must continue to grow reaching ≈ 1 Wm^{-2} at 10 kyr scales.

4. Conclusions

[16] After decreasing over several decades of scale, to a minimum of $\approx \pm 0.1$ K at around 10–100 yrs, temperature fluctuations begin to increase, ultimately reaching ± 3 to ± 5 K at glacial-interglacial scales. In order to understand the origin of this multidecadal, multicentennial and multimillenial variability, we empirically estimated the climate sensitivities of solar and volcanic forcings using several reconstructions. To make this practical, we introduced a stochastic definition of the sensitivity which could be regarded as an upper bound on the usual (deterministic) sensitivity with the two being equal in the case of full (and causal) correlation between the temperature and driver. Although the RMS temperature fluctuations increased with scale, the RMS volcanic and ^{10}Be based solar reconstructions all decreased with scale, in roughly a power law manner. If any of these reconstructions represented dominant forcings, the corresponding feedbacks would have to increase strongly with scale (with exponent $H_\lambda \approx 0.7$), and this is not trivially compatible with existing GCM's. Only the sunspot based solar reconstructions were consistent with scale independent sensitivities ($H_\lambda \approx 0$), these are of the order 4.5 *K*/(Wm^{-2}) (i.e., implying large feedbacks) and would require quite strong solar forcings of ≈ 1 Wm^{-2} at scales of 10 kyrs.

[17] A recent analysis of $S_2(\Delta t)^{1/2}$ for forced GCM outputs over the past millennium S. Lovejoy et al. (Do GCM's predict the climate…. Or low frequency weather?, submitted to *Nature Climate Change*, 2012) showed that they strongly underestimate the low frequency variability – even when for example strong solar forcings were used. Our findings here of the occasionally surprising scale-by-scale forcing variabilities helps explain why they were too weak. It seems likely that GCM's are a missing an important mechanism of internal variability. A possible candidate is land-ice whose fluctuations are plausibly scaling over the appropriate ranges of space-time scale but which is not yet integrated into existing GCM's.

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