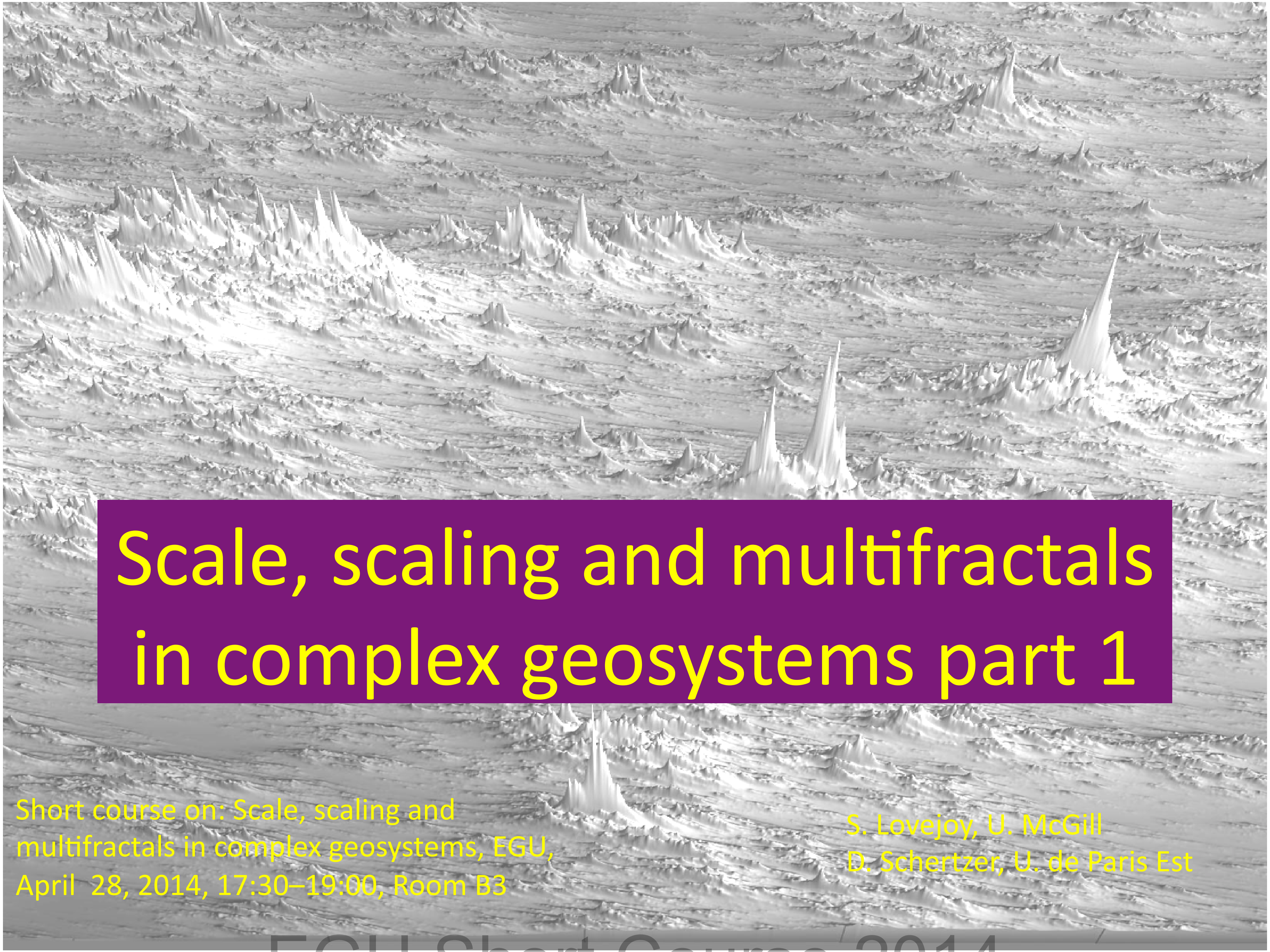
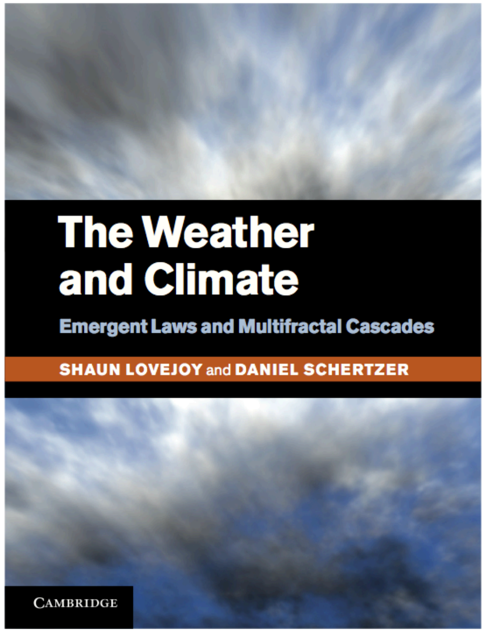


Scale, scaling and multifractals in complex geosystems part 1

Short course on: Scale, scaling and
multifractals in complex geosystems, EGU,
April 28, 2014, 17:30–19:00, Room B3

S. Lovejoy, U. McGill
D. Schertzer, U. de Paris Est

Graduate course at McGill: Multifractals and turbulence



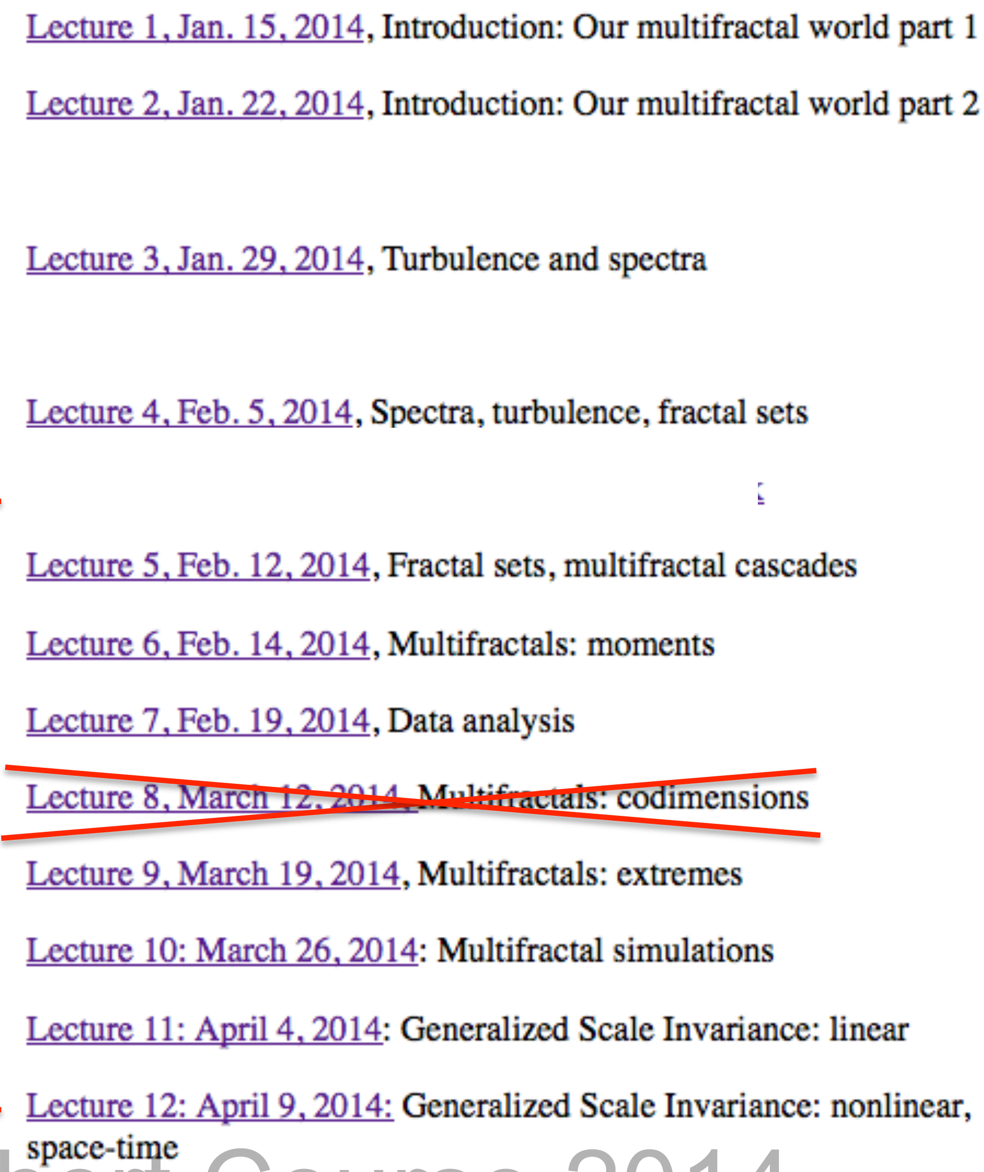
The Weather and Climate

Emergent Laws and Multifractal Cascades

SHAUN LOVEJOY and DANIEL SCHERTZER

CAMBRIDGE

12x2 hours, slides available:



Lecture 1, Jan. 15, 2014, Introduction: Our multifractal world part 1

Lecture 2, Jan. 22, 2014, Introduction: Our multifractal world part 2

Lecture 3, Jan. 29, 2014, Turbulence and spectra

Lecture 4, Feb. 5, 2014, Spectra, turbulence, fractal sets

Lecture 5, Feb. 12, 2014, Fractal sets, multifractal cascades

Lecture 6, Feb. 14, 2014, Multifractals: moments

Lecture 7, Feb. 19, 2014, Data analysis

~~Lecture 8, March 12, 2014, Multifractals: codimensions~~

Lecture 9, March 19, 2014, Multifractals: extremes

Lecture 10: March 26, 2014: Multifractal simulations

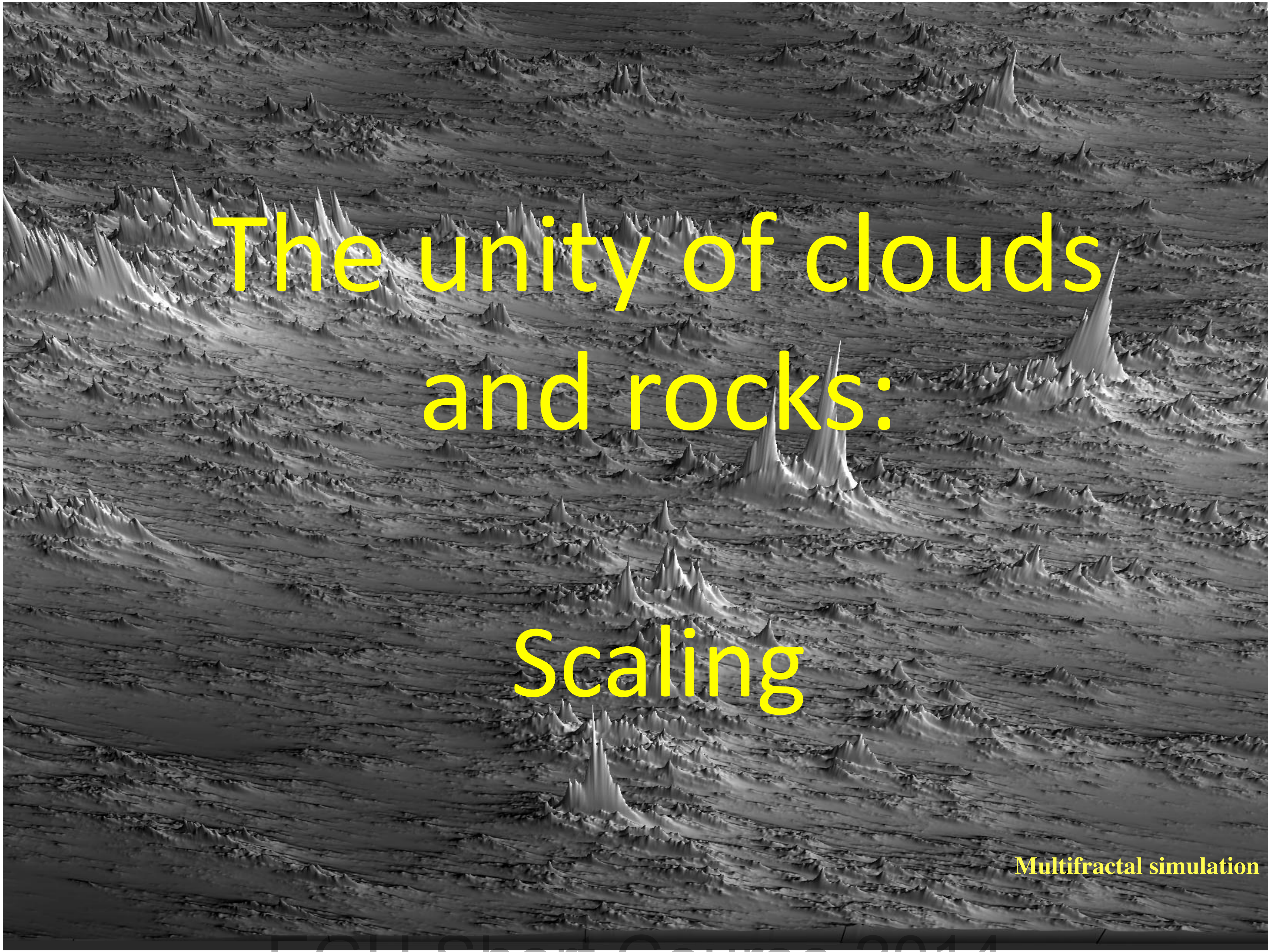
Lecture 11: April 4, 2014: Generalized Scale Invariance: linear

Lecture 12: April 9, 2014: Generalized Scale Invariance: nonlinear, space-time

Course synopsis

Today

http://www.physics.mcgill.ca/~gang/PHYS616/PHYS616.home.htm



The unity of clouds and rocks:

Scaling

Multifractal simulation

Emergent scaling laws

Fluctuations ≈ (turbulent flux) x (scale)^{H}

- Differences, tendencies, wavelet coefficients → Fluctuations
- Cascading driving flux → (turbulent flux)
- Anisotropic Space-time Scale function → (scale)
- Fluctuation /conservation exponent → H

Fourier domain:

$$\left(\frac{Variance_{observables}}{wavenumber}\right)=\left(\frac{Variance_{flux}}{wavenumber}\right)(wavenumber)^{-2H}=(wavenumber)^{-\beta}$$

Space: $E(k) \approx k^{-\beta}$

Time: $E(\omega) \approx \omega^{-\beta}$

Energy Spectra

Scaling geometric sets of points = fractals
Scaling fields=multifractals

"β = scale invariant"

$$E(k) \propto k^{-\beta}$$

E(k)= "scaling"

k=2π/L= wavenumber, β=spectral exponent

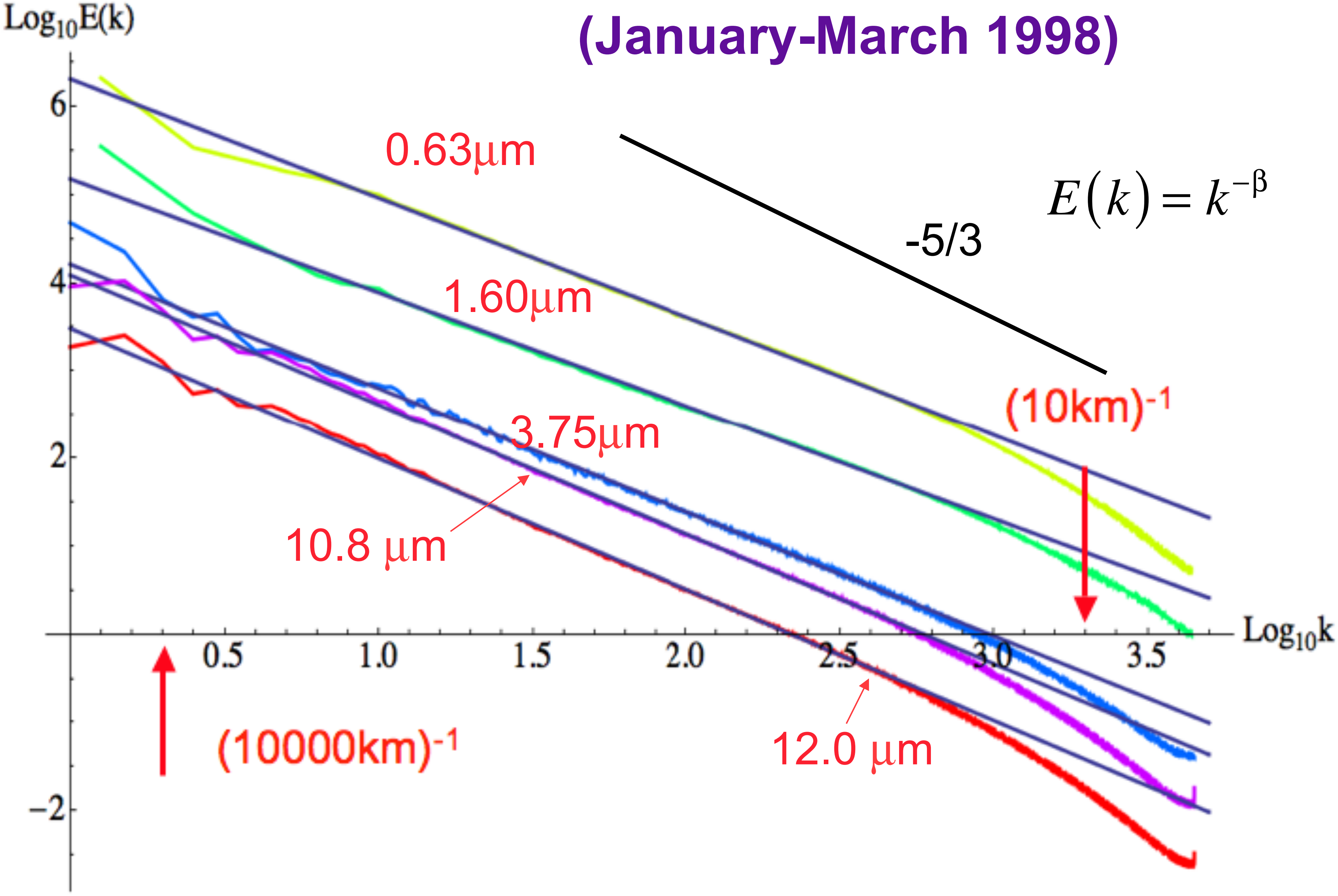
Scale invariance

$$E(\lambda^{-1}k) = \lambda^{\beta} E(k)$$

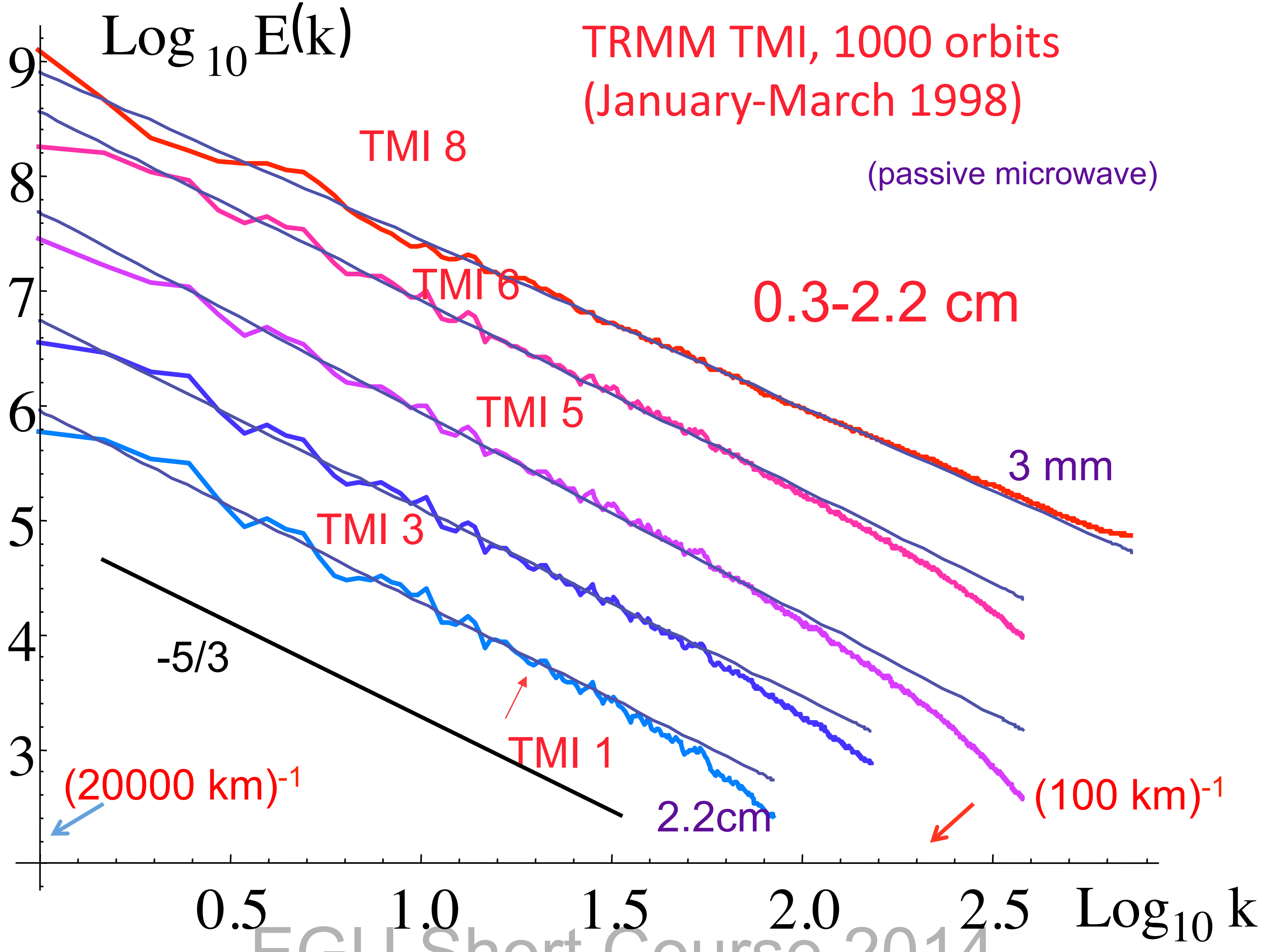
β Invariant under zoom by factor λ in real space.

Scaling in space, a guided tour

TRMM VIRS, 1000 orbits (January-March 1998)



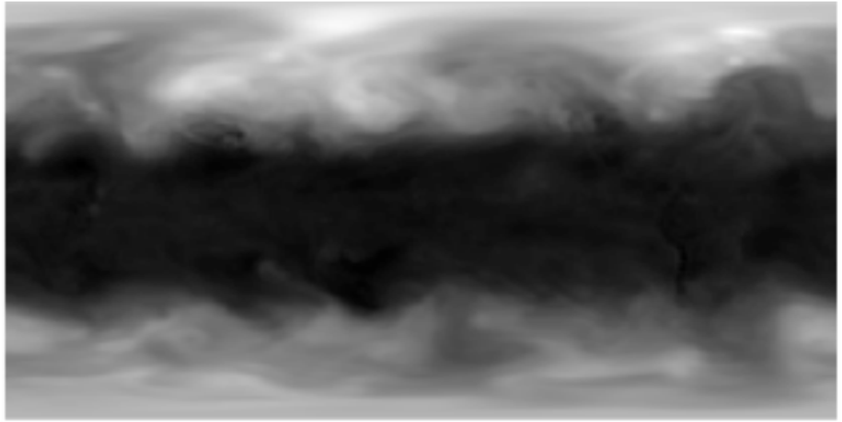
Visible, near infra red, thermal infra red



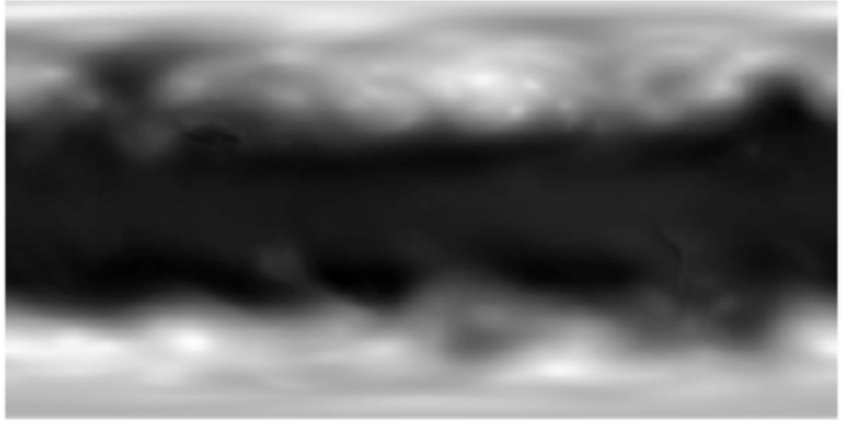
TRMM TMI, 1000 orbits
(January-March 1998)

(passive microwave)

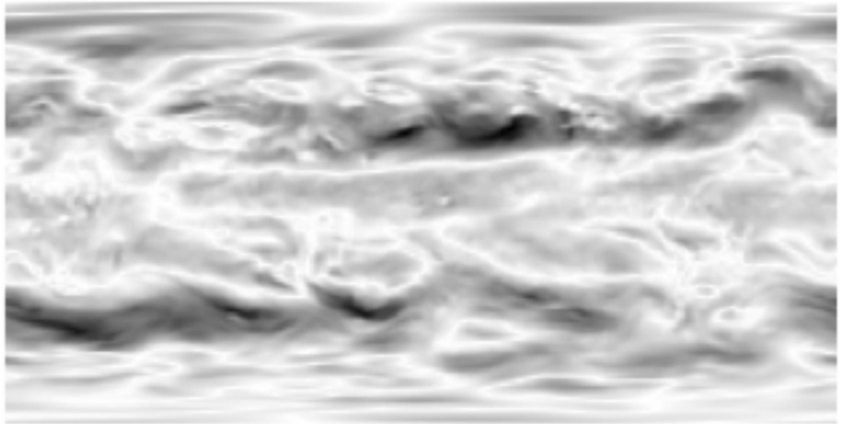
1.5a:



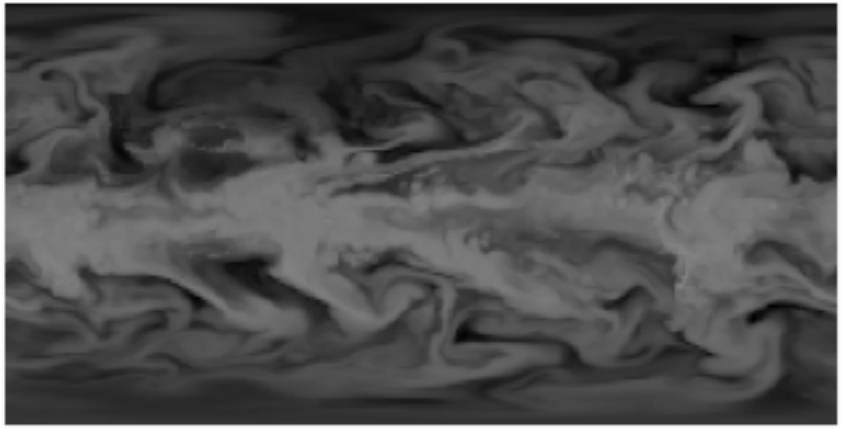
h



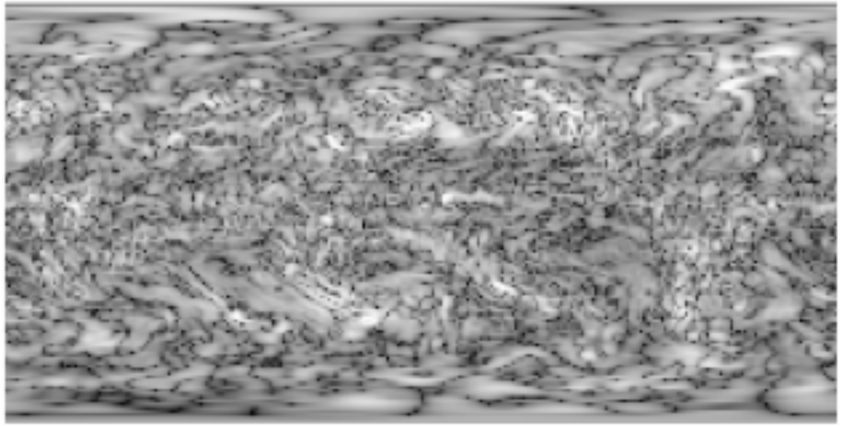
T



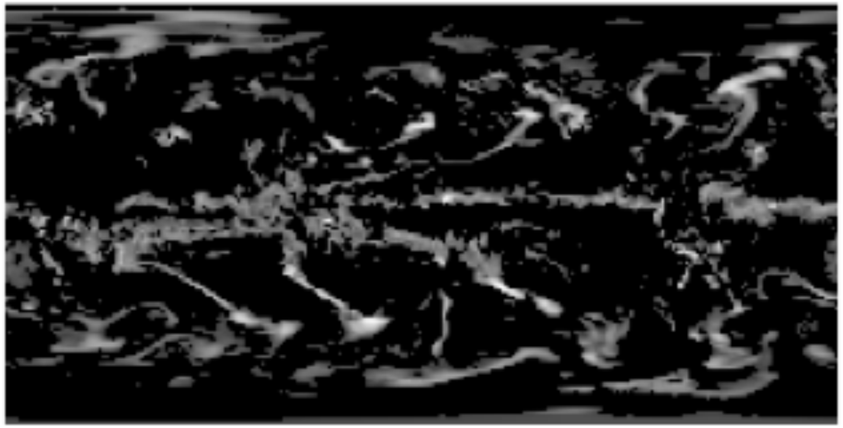
u



v

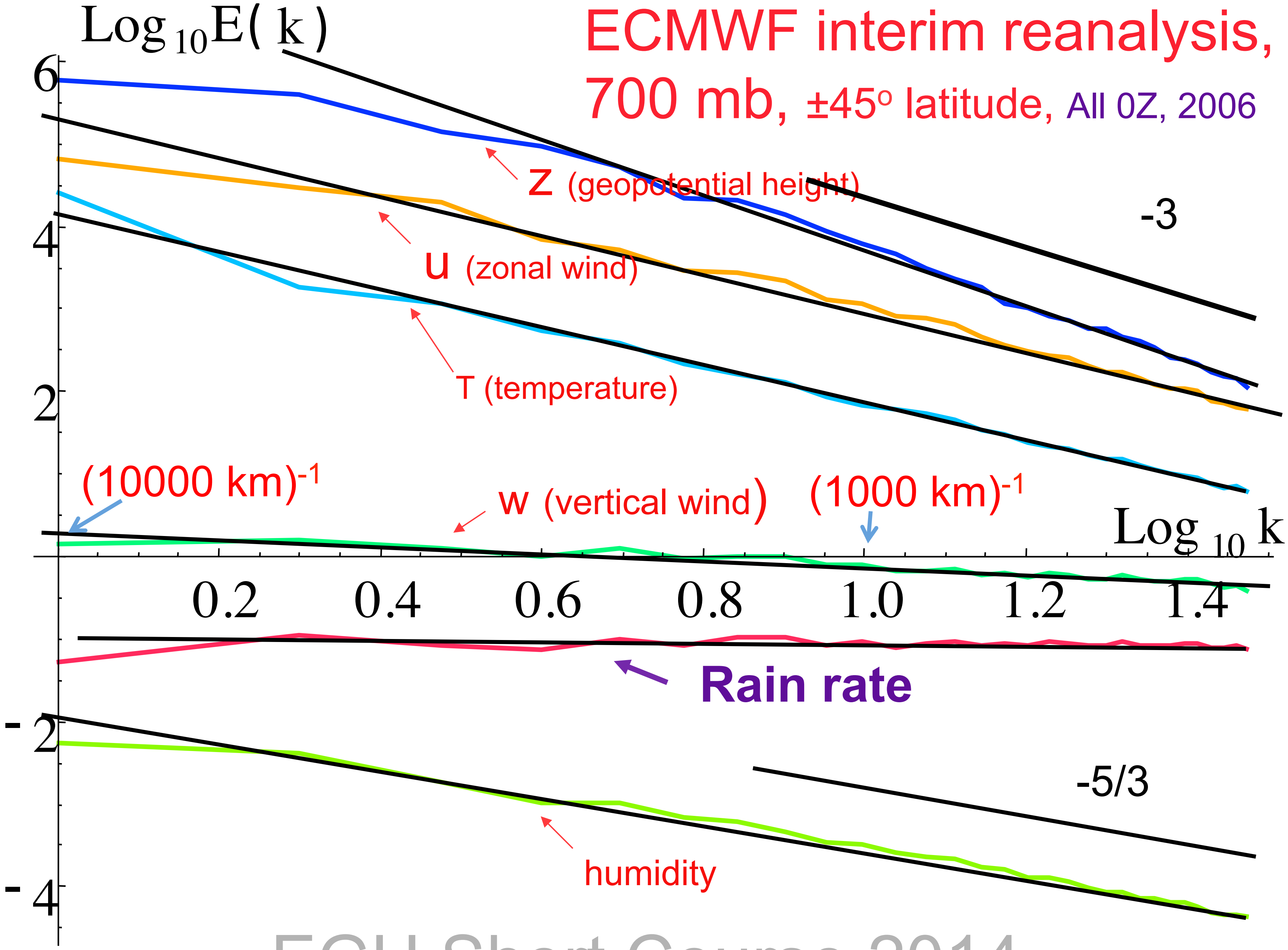


w

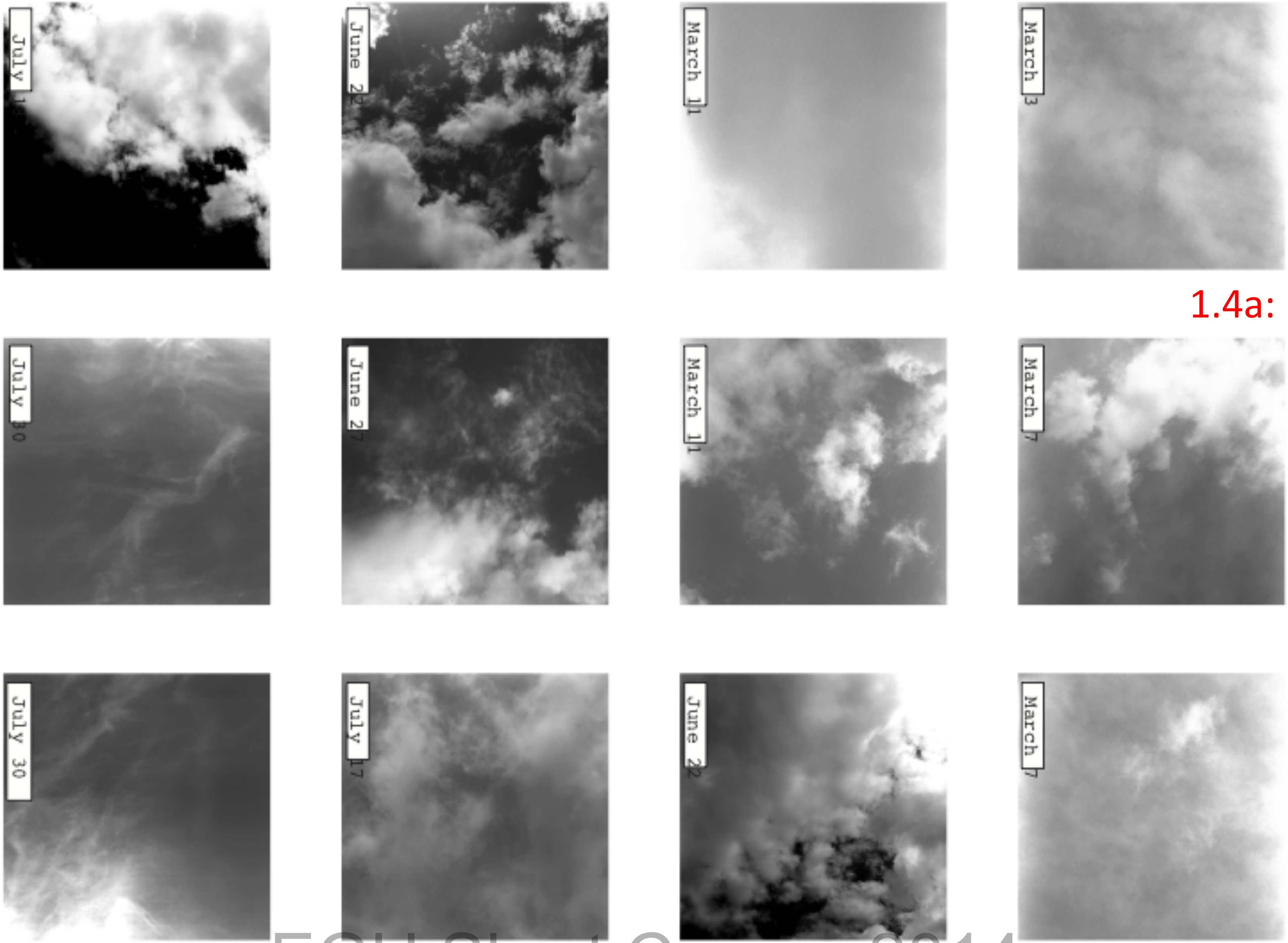


z

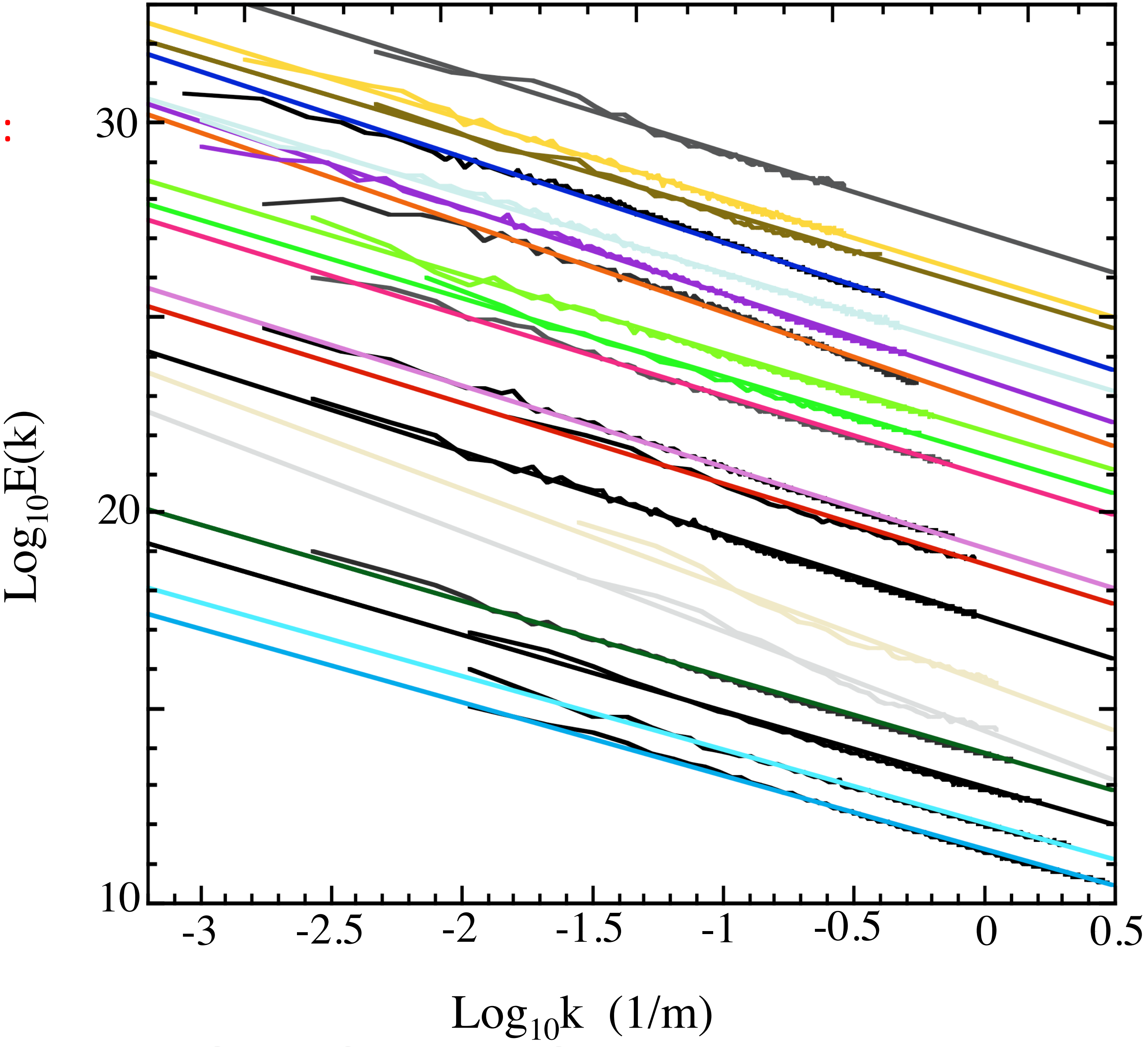
ECMWF interim reanalysis, 700 mb, ±45° latitude, All 0Z, 2006



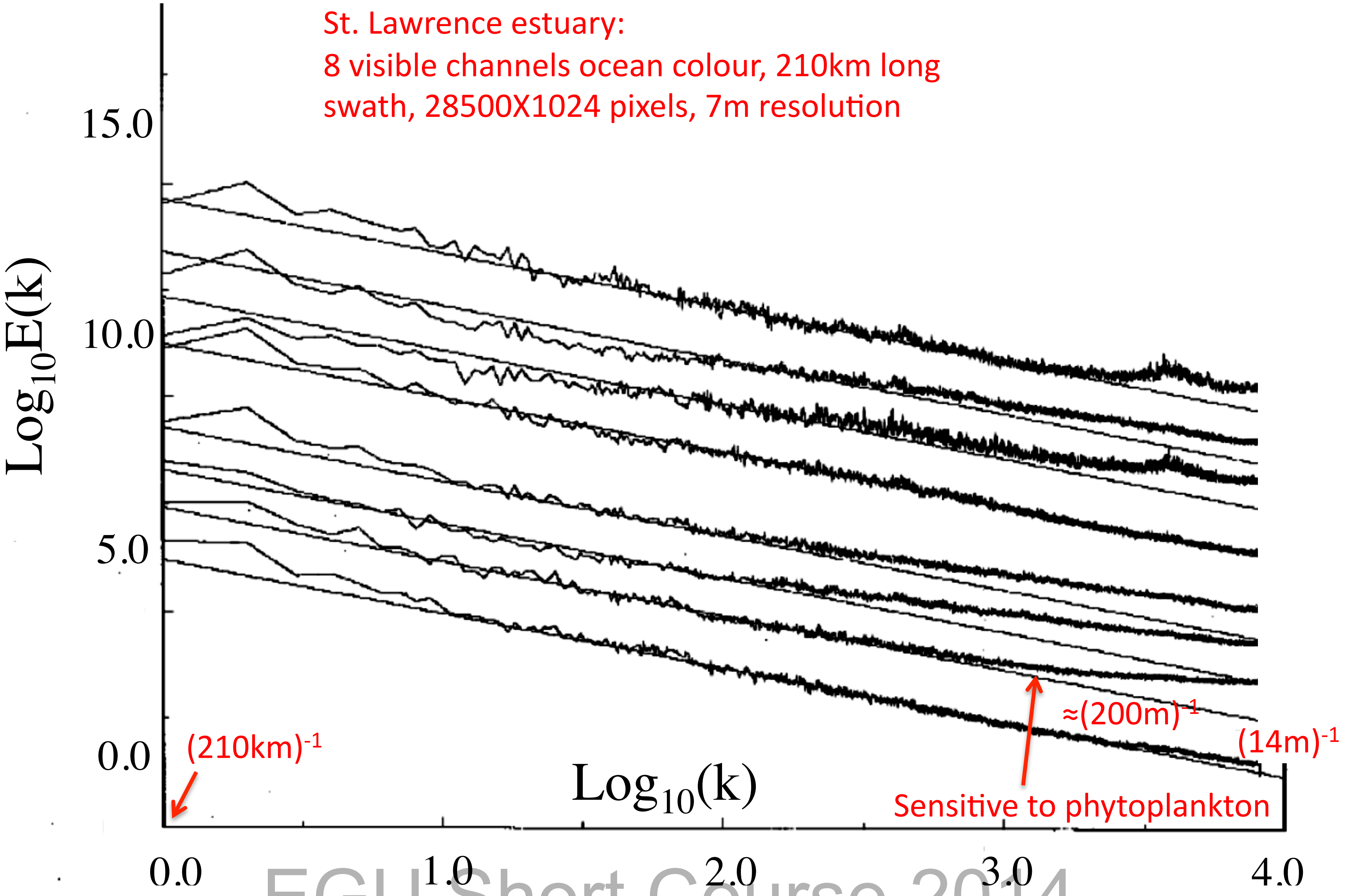
1.4a:


July 1
June 22
March 11
March 3
July 30
June 27
March 11
March 7
July 30
July 17
June 22
March 7

1.4b:



Ocean surface



St. Lawrence estuary:
8 visible channels ocean colour, 210km long swath, 28500X1024 pixels, 7m resolution

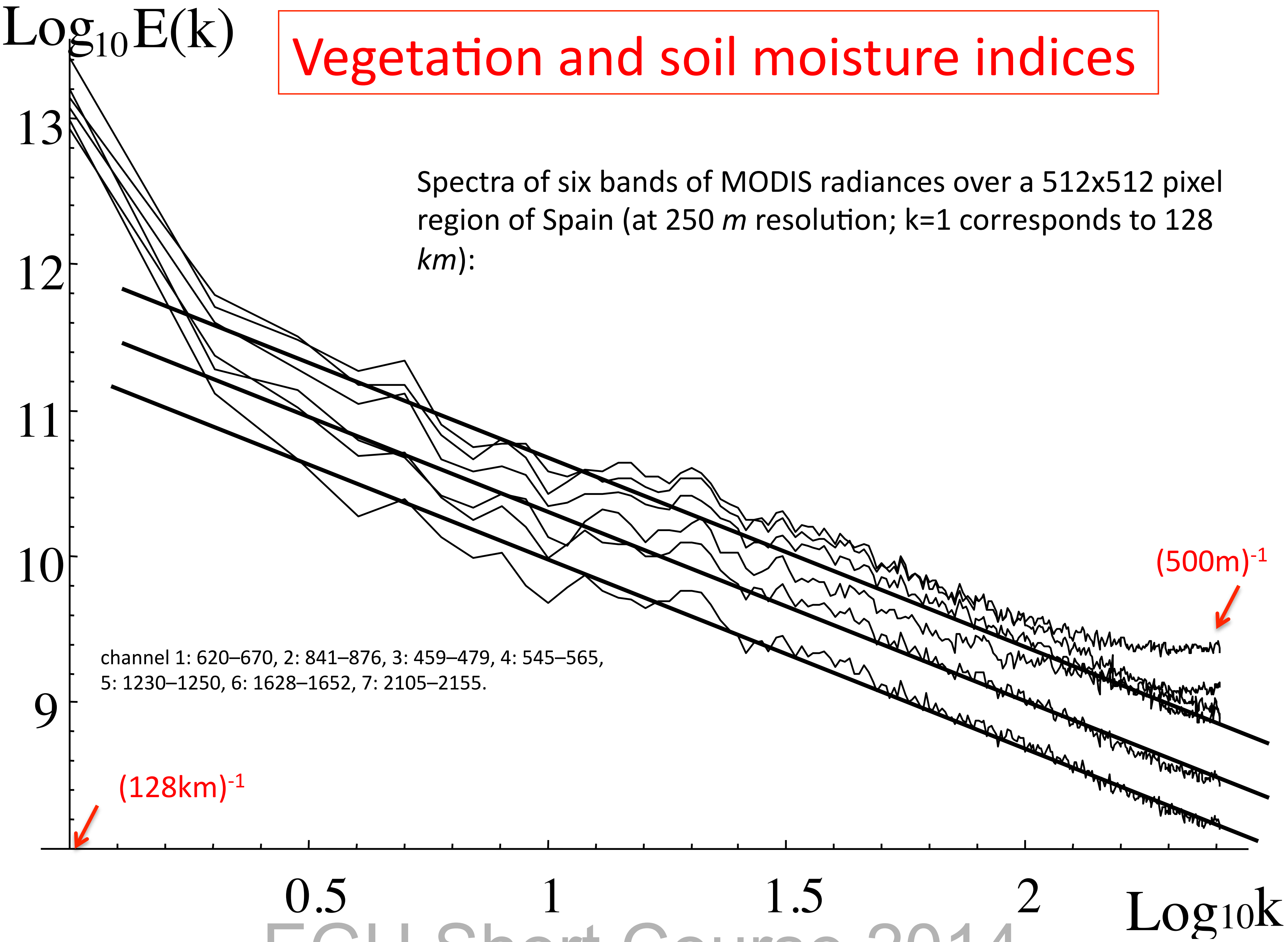
$(210\text{km})^{-1}$

$\approx(200\text{m})^{-1}$

$(14\text{m})^{-1}$

Sensitive to phytoplankton

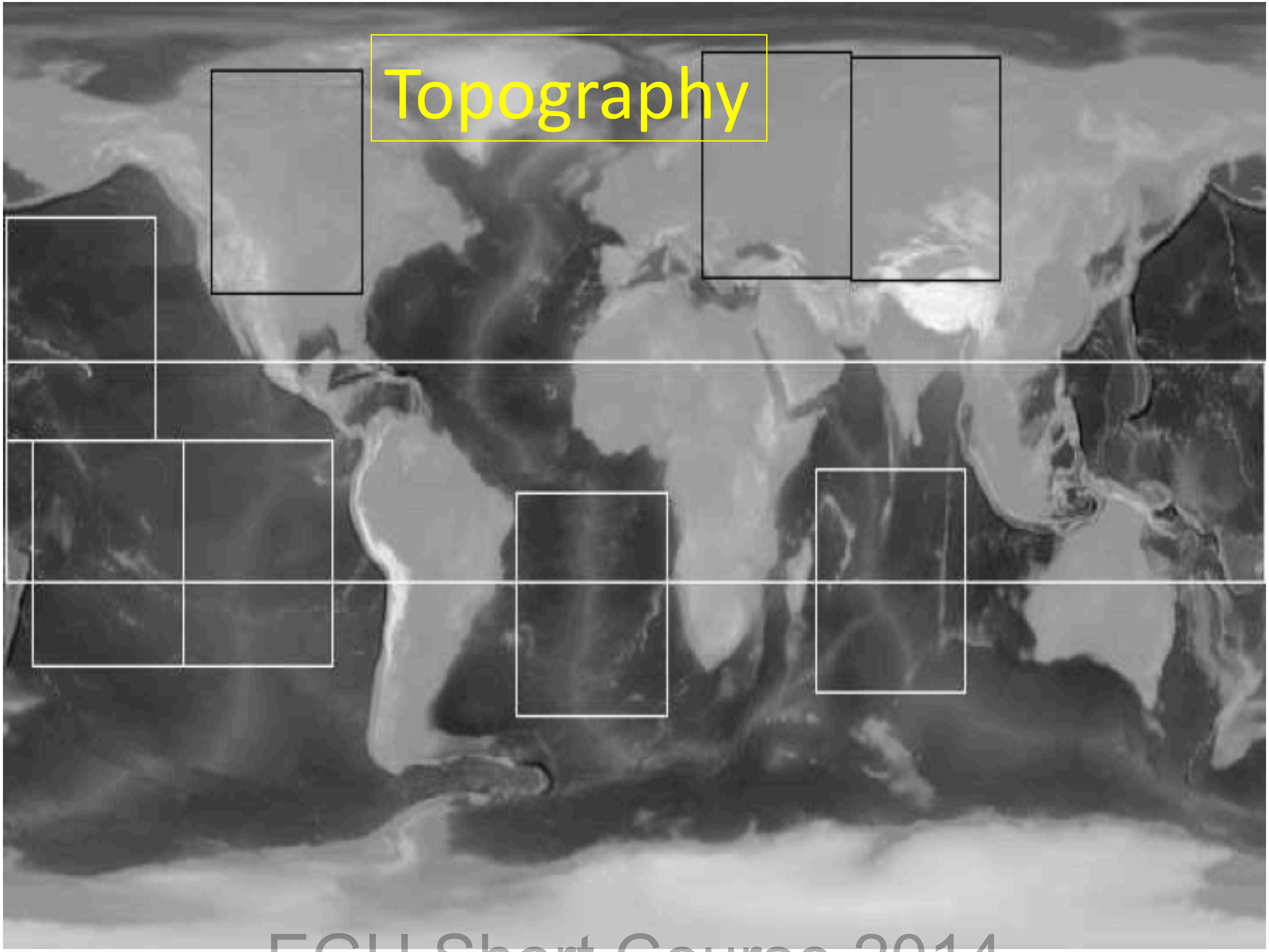
Vegetation and soil moisture indices

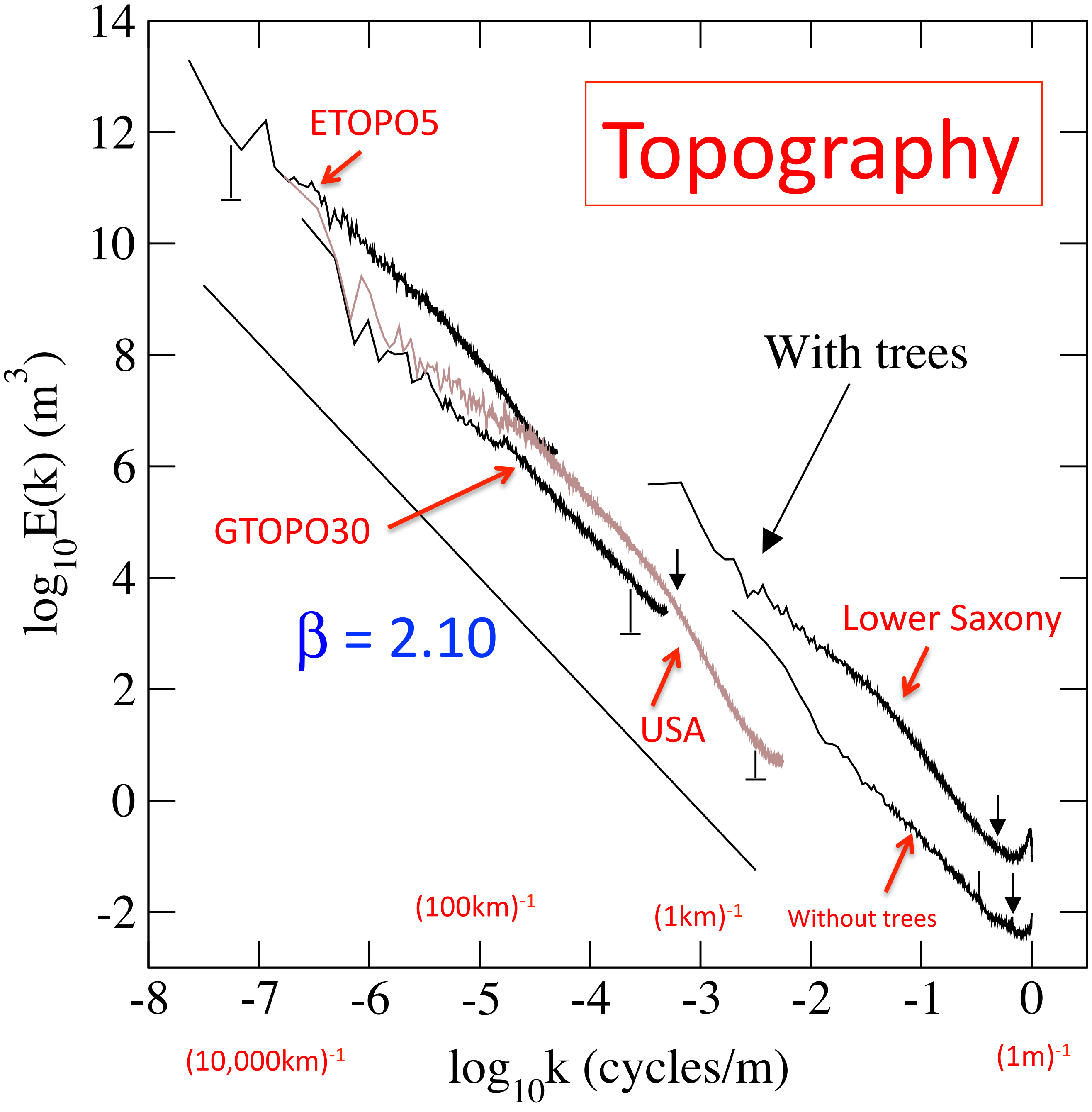


Spectra of six bands of MODIS radiances over a 512x512 pixel region of Spain (at 250 *m* resolution; k=1 corresponds to 128 *km*):

channel 1: 620–670, 2: 841–876, 3: 459–479, 4: 545–565,
5: 1230–1250, 6: 1628–1652, 7: 2105–2155.

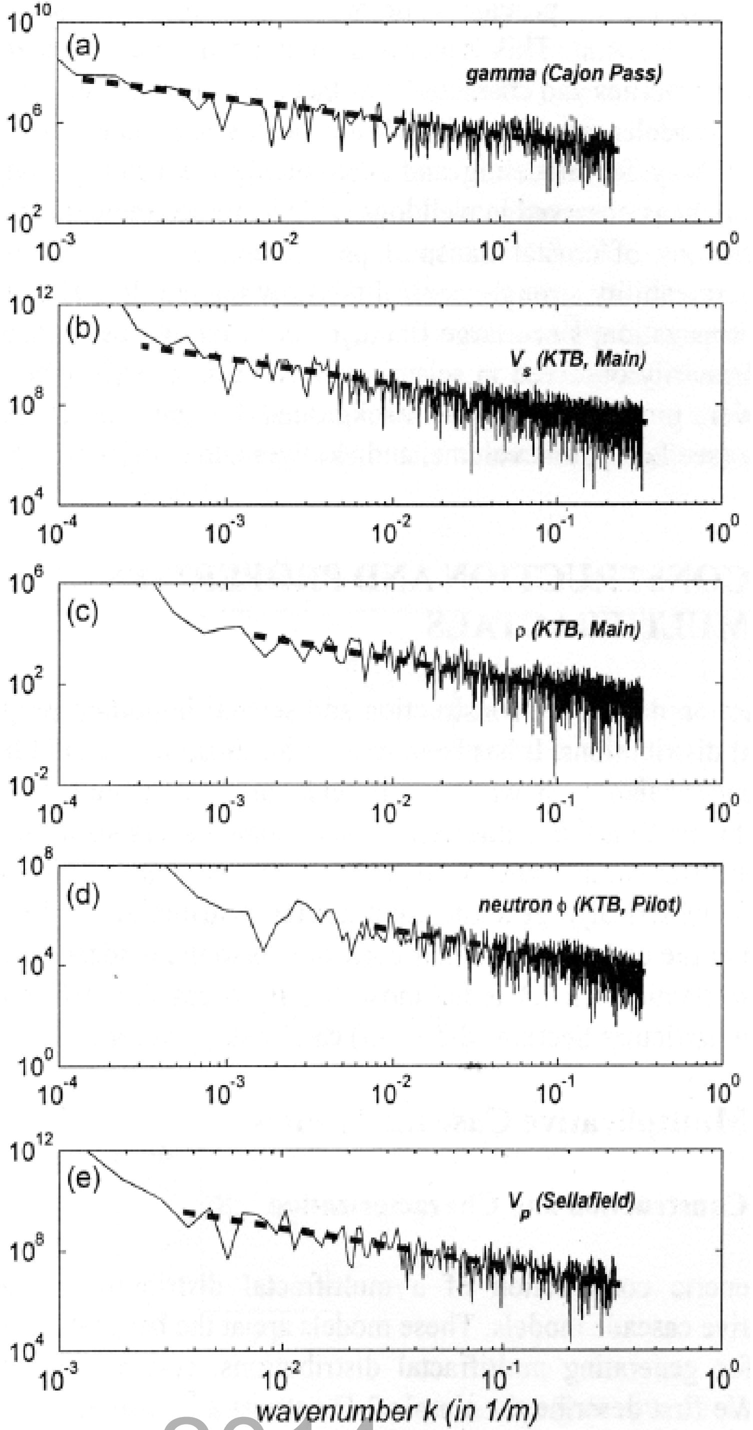
Topography





Gagnon, Lovejoy and Schertzer, 2006

The scaling of the KTB borehole



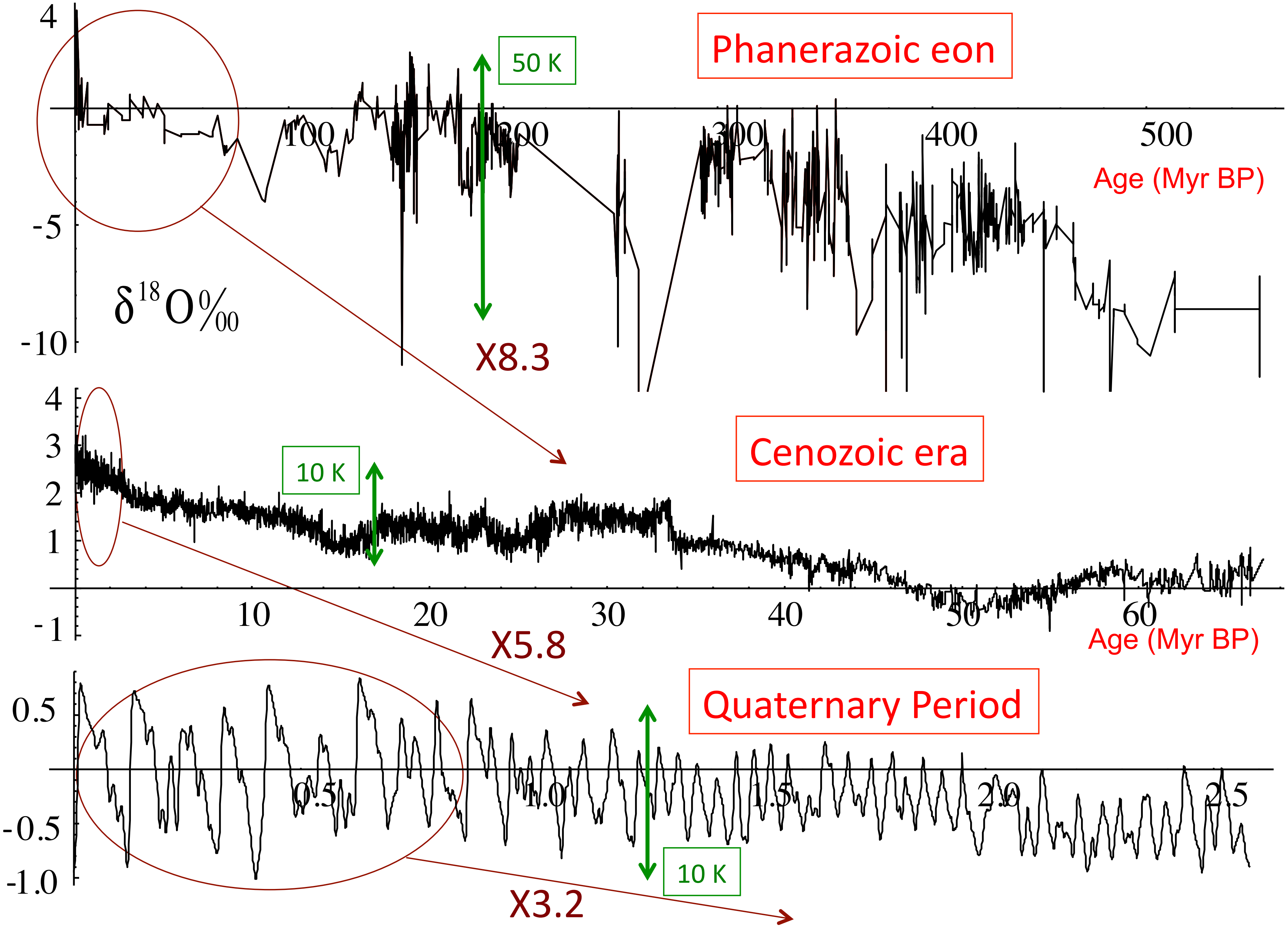
Marsan and Bean (2003)

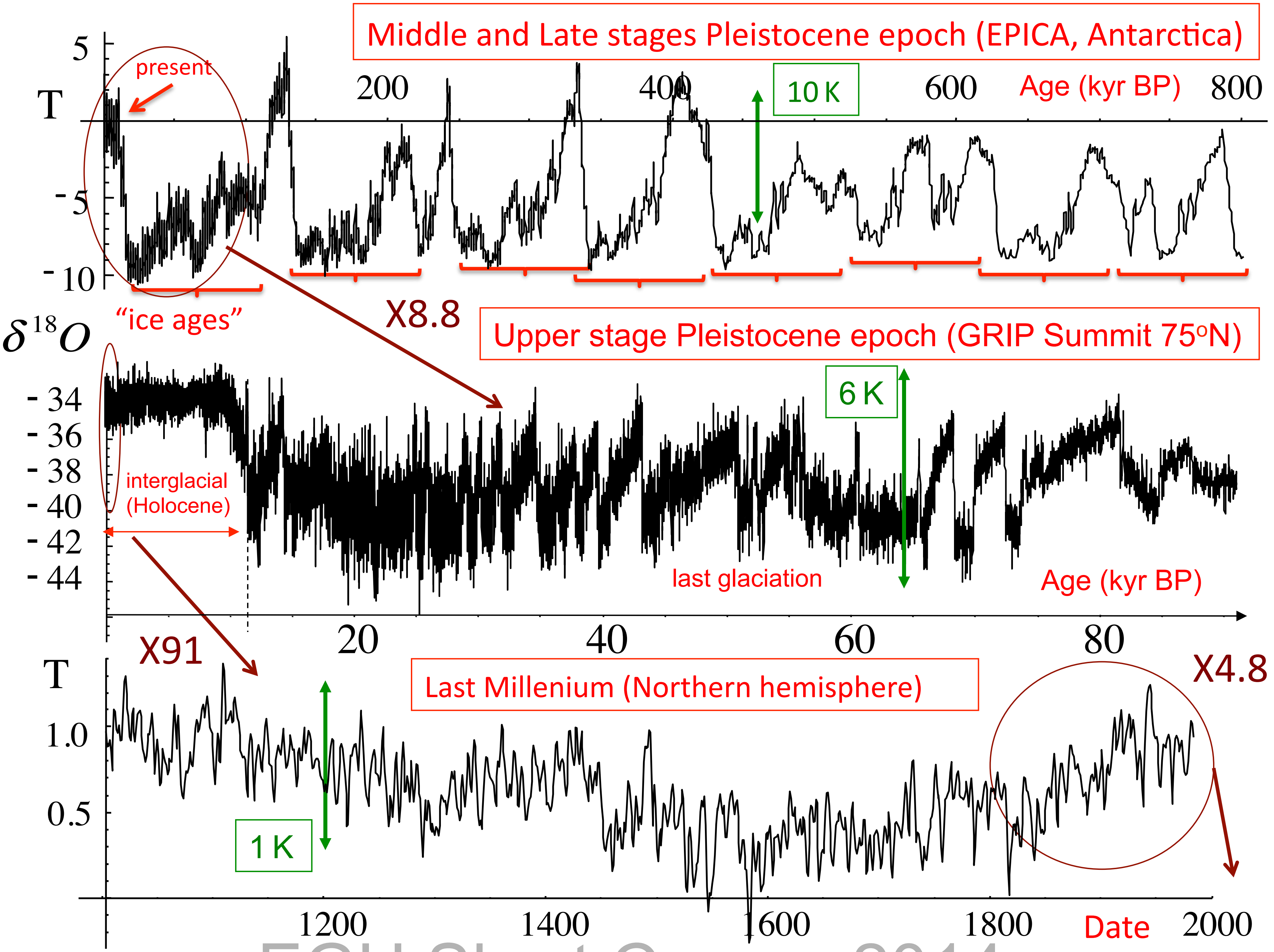
Scaling in time:

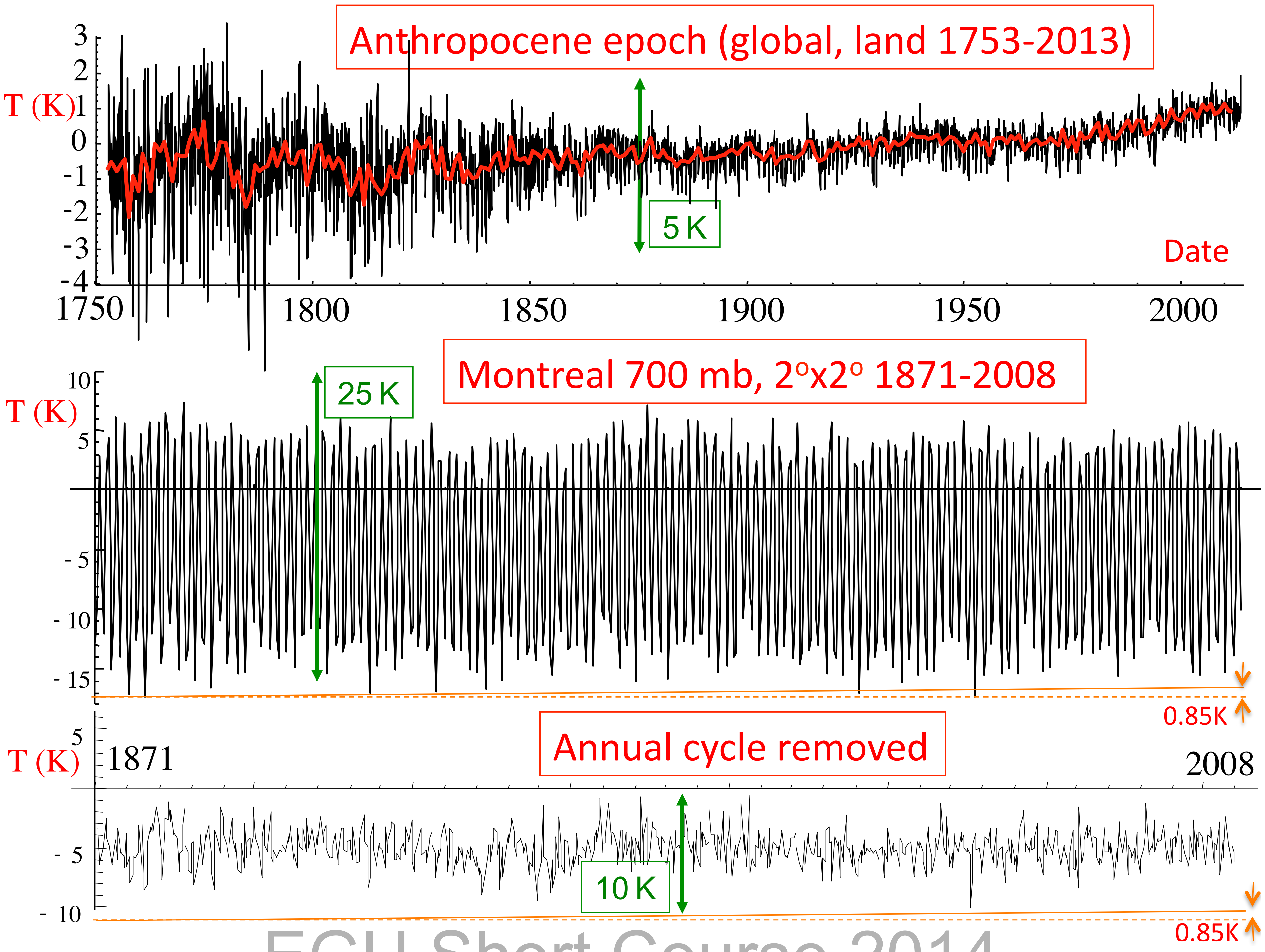
From the age of the earth to the viscous dissipation scale: $4.5x10^9$ years - 1 ms:

20 orders of magnitude in time

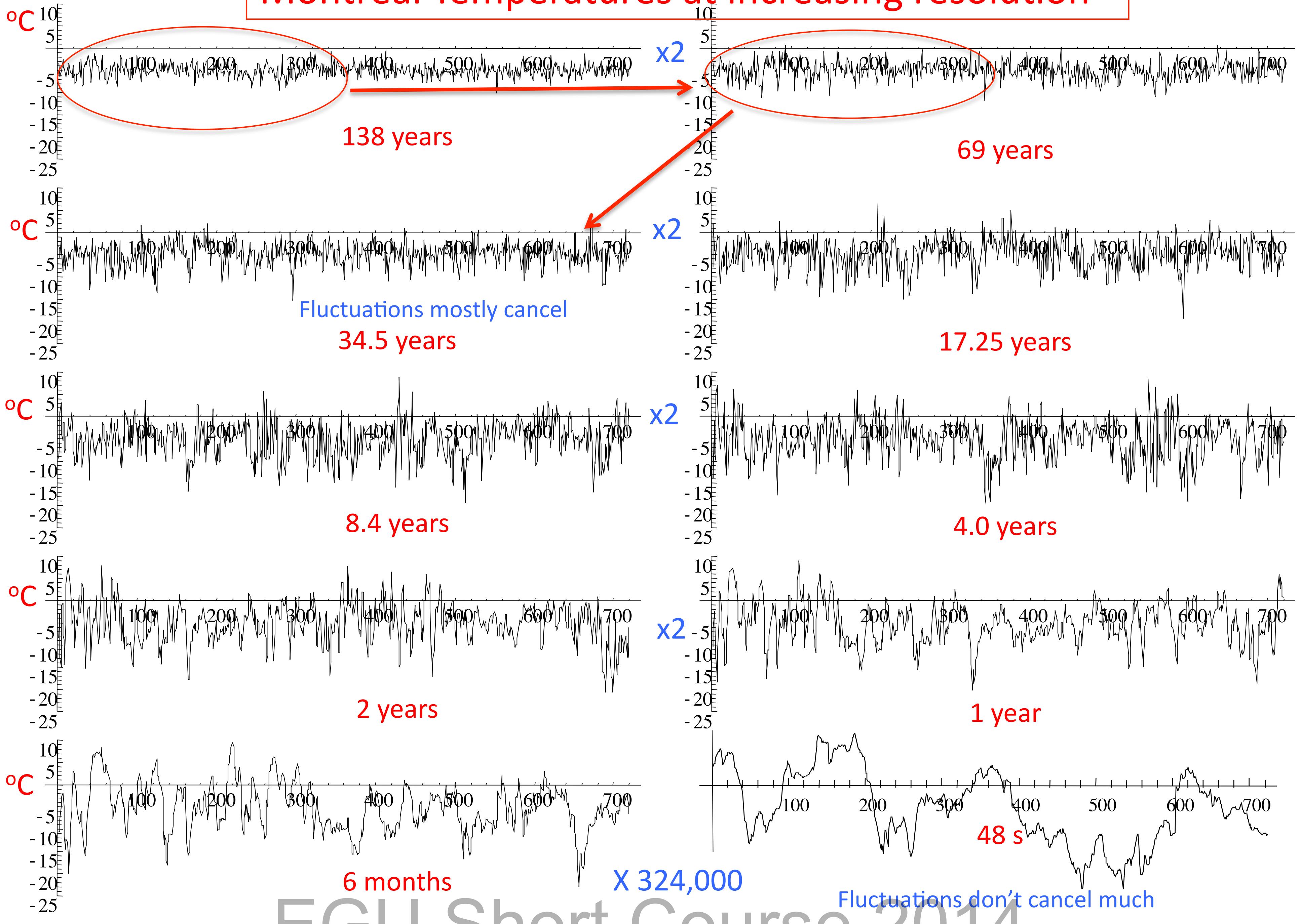
A voyage through scale…


4
-5
-10
$\delta^{18}O‰$
100
200
300
400
500
Phanerazoic eon
50 K
X8.3
Age (Myr BP)
4
3
2
1
-1
10
20
30
40
50
60
Cenozoic era
10 K
X5.8
Age (Myr BP)
0.5
-0.5
-1.0
0.5
1.0
1.5
2.0
2.5
Quaternary Period
10 K
X3.2


Middle and Late stages Pleistocene epoch (EPICA, Antarctica)
5
T
-5
-10
present
200
400
10 K
600
Age (kyr BP)
800
"ice ages"
X8.8
$\delta^{18}O$
Upper stage Pleistocene epoch (GRIP Summit 75°N)
6 K
-34
-36
-38
-40
-42
-44
interglacial
(Holocene)
last glaciation
Age (kyr BP)
20
40
60
80
X91
T
Last Millenium (Northern hemisphere)
X4.8
1.0
0.5
1 K
1200
1400
1600
1800
Date
2000

Anthropocene epoch (global, land 1753-2013)
T (K)
3
2
1
0
-1
-2
-3
-4
5 K
Date
1750
1800
1850
1900
1950
2000
Montreal 700 mb, 2°x2° 1871-2008
T (K)
10
5
- 5
- 10
- 15
25 K
0.85K
Annual cycle removed
T (K)
5
1871
2008
- 5
- 10
10 K
0.85K

Montreal Temperatures at increasing resolution



138 years

69 years

Fluctuations mostly cancel

34.5 years

17.25 years

8.4 years

4.0 years

2 years

1 year

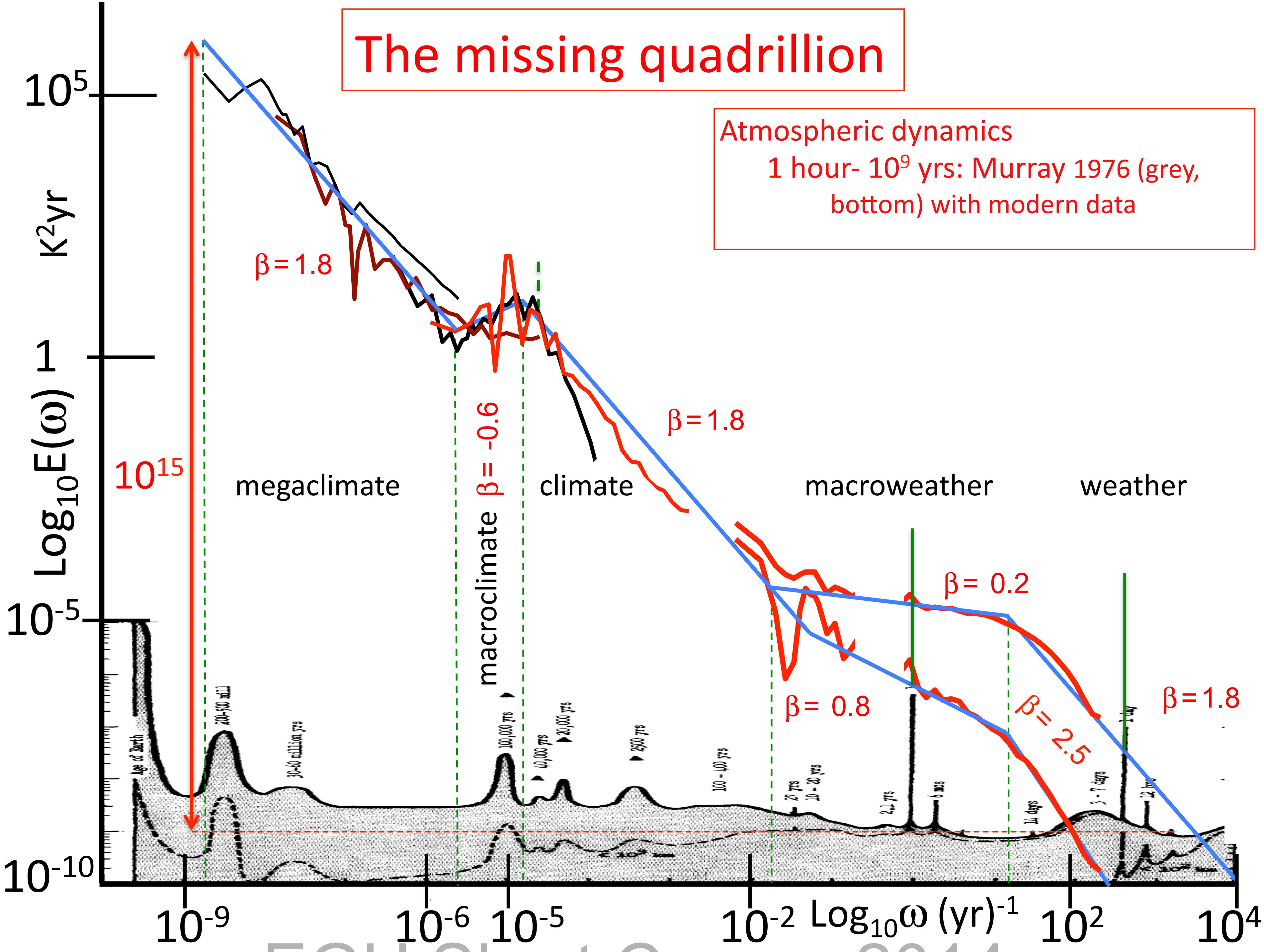
6 months

X 324,000

48 s

Fluctuations don't cancel much

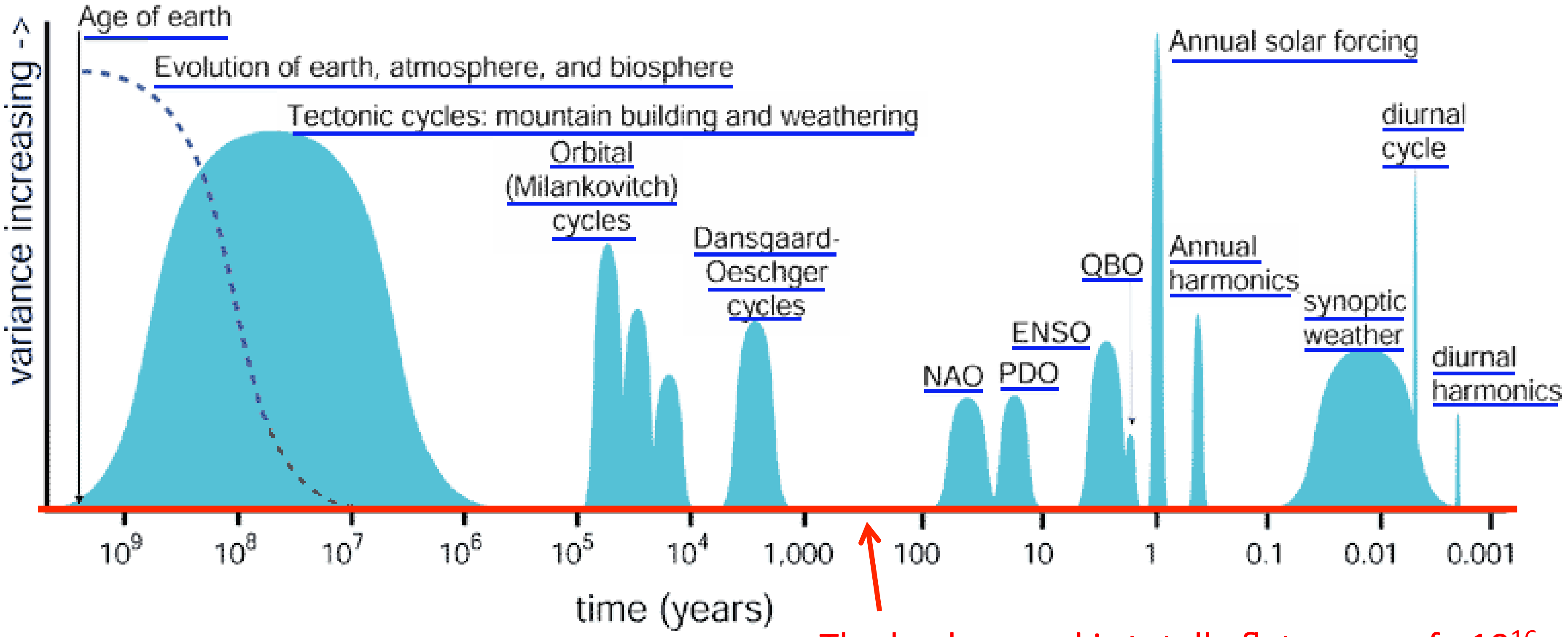
The missing quadrillion



Atmospheric dynamics

1 hour- 10^9 yrs: Murray 1976 (grey, bottom) with modern data

The NOAA NCDC Paleoclimate data site graph (inspired by Mitchell)



The background is totally flat: error of ≈10^{16}

The explanation of the figure:

"… figure is intended as a mental model to provide a general "powers of ten" overview of climate variability, and to convey the basic complexities of climate dynamics for a general science savvy audience."

The site assures us that just "because a particular phenomenon is called an oscillation, it does not necessarily mean there is a particular oscillator causing the pattern. Some prefer to refer to such processes as variability."

How to understand the variability?

Answer #1:

Scale bound thinking

Scale bound thinking

Antonie van Leeuwenhoek

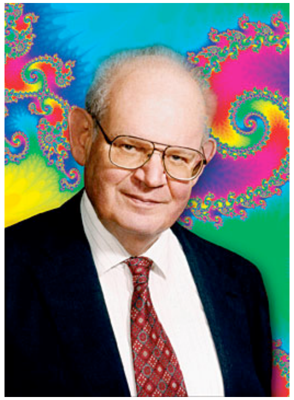
(1632–1723)

A new world in a drop of water

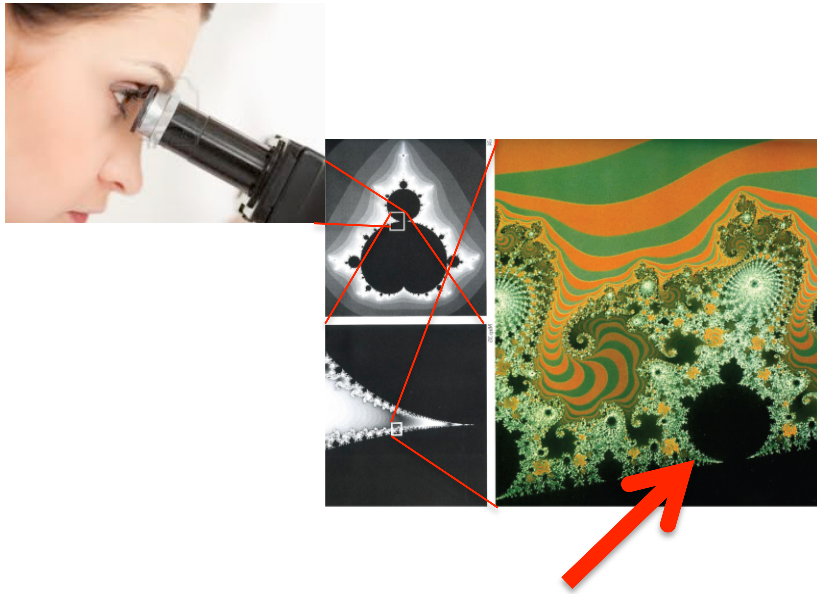
.....the discovery of micro-organisms

"Animalcules," described in depth by Leeuwenhoek, c1695–1698. By Anton van Leeuwenhoek

Pure, (self-similar) Fractal thinking



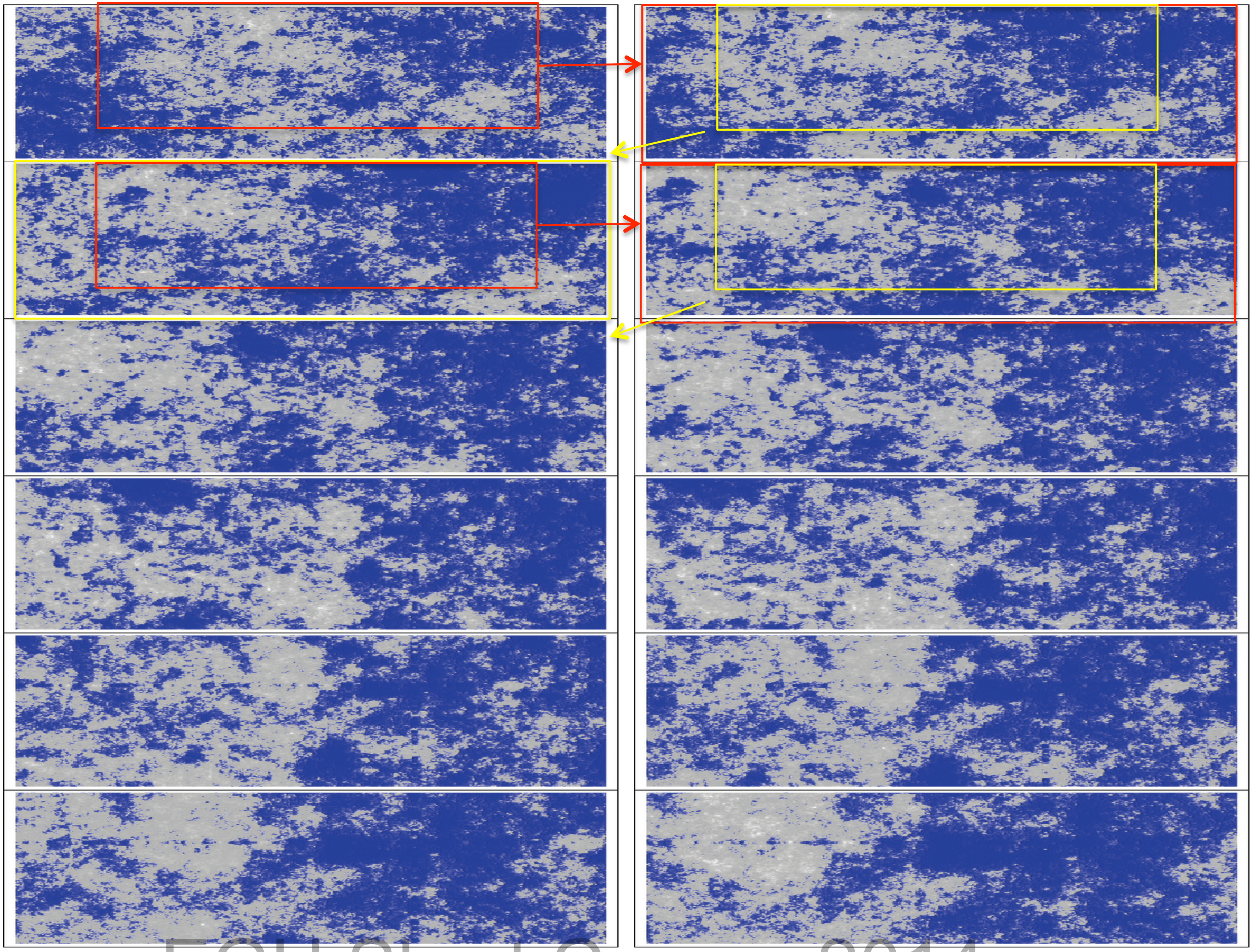
Mandelbrot 1924-2010



The same!!!

(the Mandelbrot set)

Self-similar Fractal thinking is OK here (Zooming in by factors of 1.7)

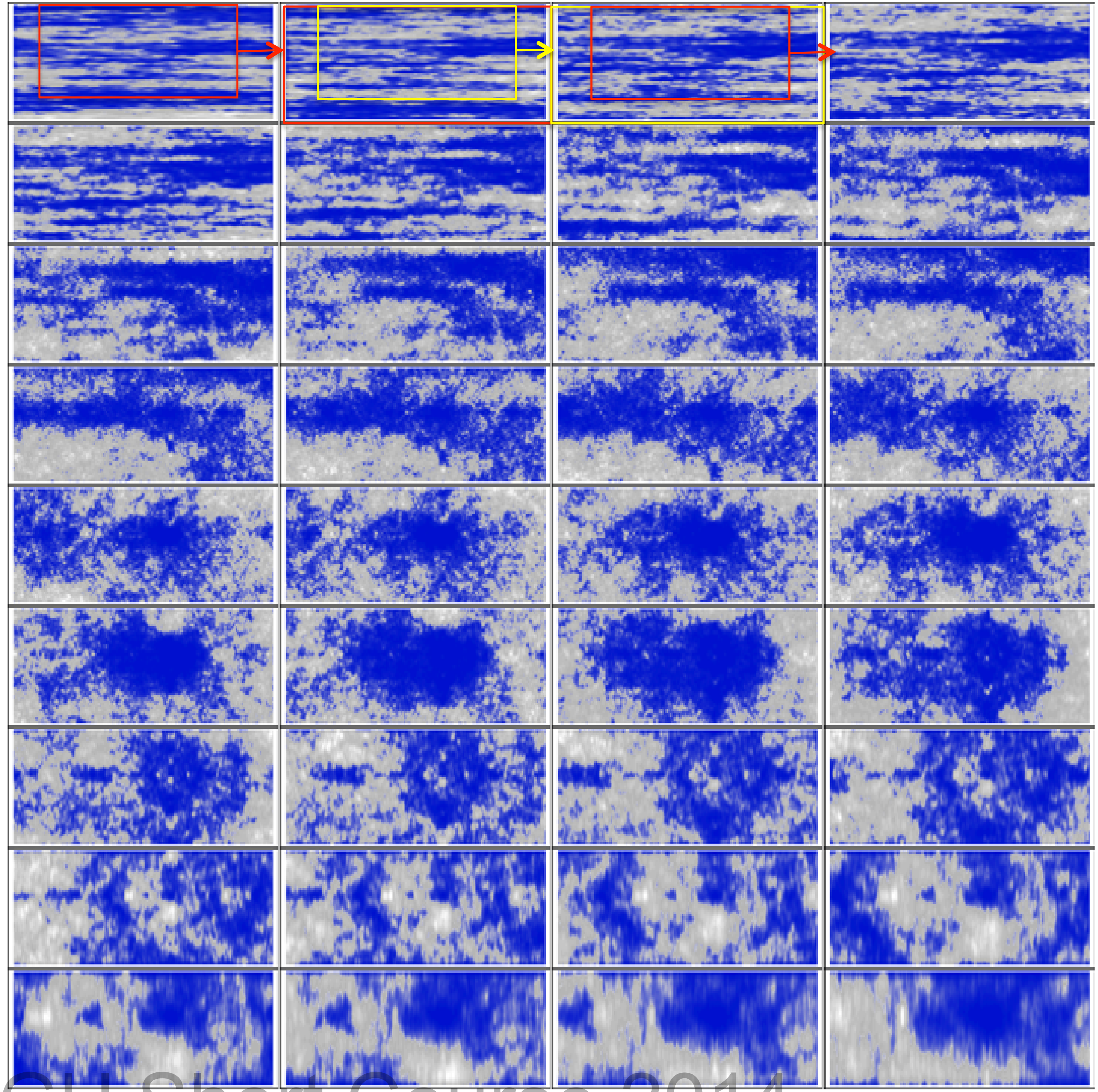


But not here!

Need Scale invariant thinking!

(Zoom factor 1000)

Vertical cross-section of the atmosphere

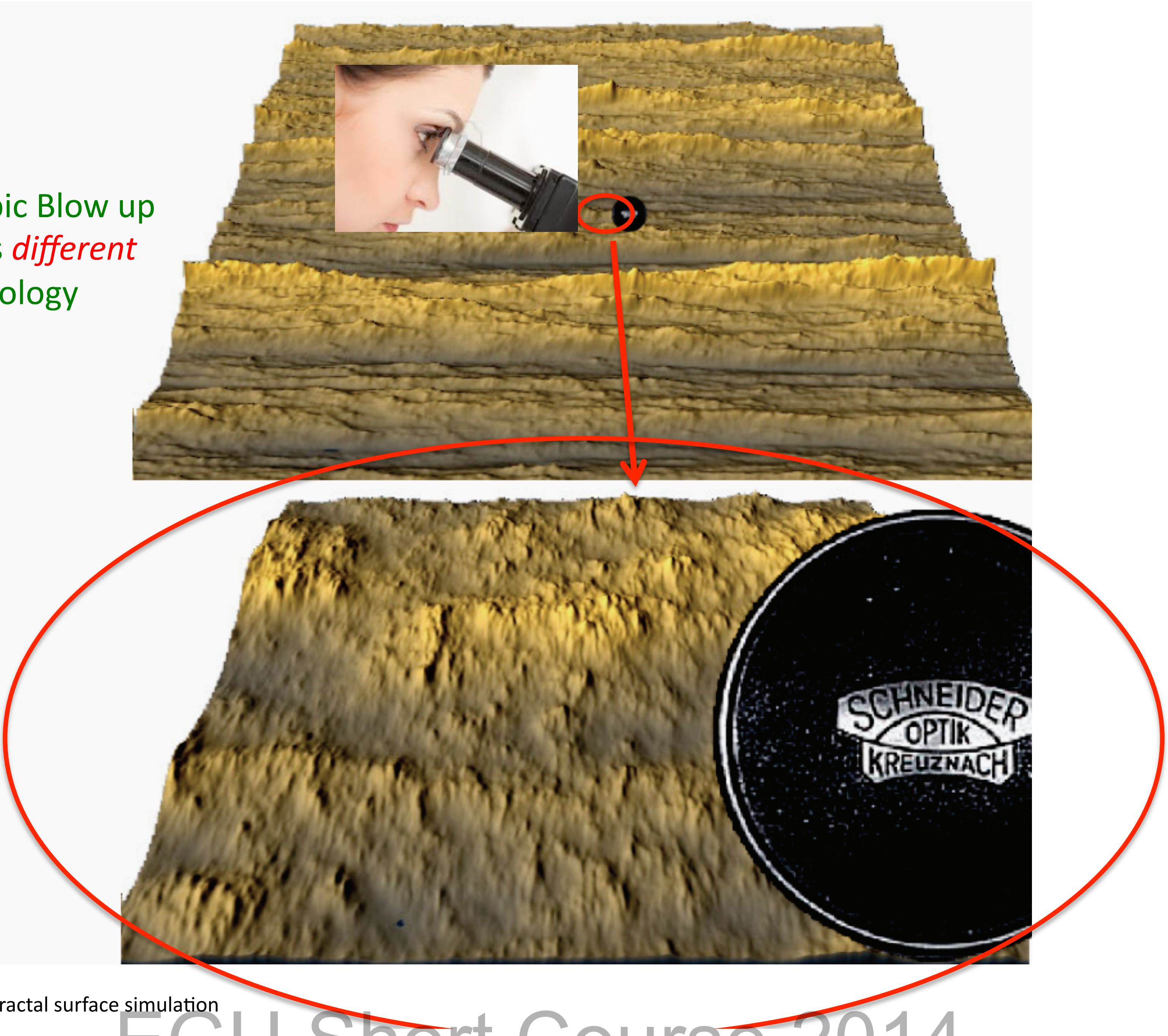


Scale invariance and the Phenomenological Fallacy

1) Morphology not dynamics is taken as fundamental

2) Scaling is reduced to the isotropic (self-similar) special case

Isotropic Blow up reveals *different* morphology



Anisotropic multifractal surface simulation

How to understand the variability?

Answer #2

- Scaling, scale invariance:

Fluctuation exponent

$$\Delta T\left(\Delta t\right)=\varphi\Delta t^{H}$$

Fluctuation

Time lag
(time scale)

Driving
dynamic flux
(cascade,
multifractal,
see later)

Difference, Tendency, Haar fluctuations

Differences: The difference in temperature between t and $t+\Delta t$

Tendency: The average of the temperature (with overall mean removed) between t and $t+\Delta t$

Haar: The difference between the average of the temperature from t and $t+\Delta t/2$ and from $t+\Delta t/2$ and $t+\Delta t$

Relations:

When $1 > H > 0$: Haar ≈ difference
When $0 > H > -1$: Haar ≈ tendency

Fluctuations and wavelets

In wavelet analysis, one defines fluctuations with the help of a basic “” $\Psi(x)$ and performs the convolution:

$$\Delta v(\Delta t) = \frac{1}{\Delta t}\int v(t')\Psi\left(\frac{t-t'}{\Delta t}\right)dt'$$

mother wavelet

Difference

$$(\Delta v)_{diff} = v(t+\Delta t/2) - v(t-\Delta t/2) \qquad \Psi(t) = \delta(t-1/2) - \delta(t+1/2)$$

Tendency

$$(\Delta v)_{tend} = \frac{1}{\Delta t}\int_{t}^{t+\Delta t} v'(t')dt'; \quad v'(t) = v(t) - \overline{v(t)} \qquad \Psi(t) = I_{[-1/2,1/2]}(t) - \frac{I_{[-\tau/2,\tau/2]}(t)}{\tau}; \quad \tau >> 1 \qquad I_{[a,b]}(t) = \begin{matrix} 1 & a \le t \le b \\ 0 & otherwise \end{matrix}$$

I is the indicator function

Haar

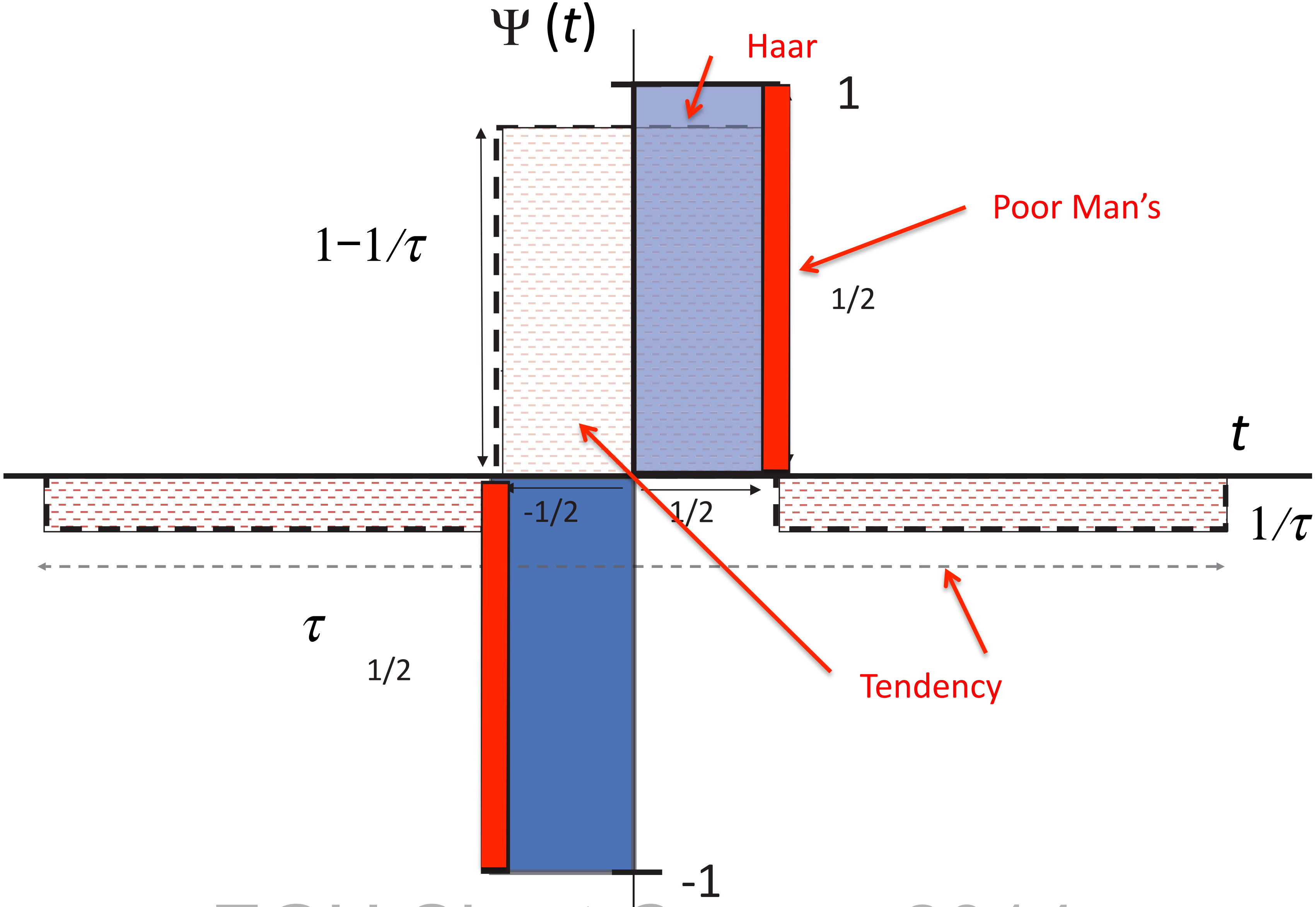
$$(\Delta v)_{Haar} = \frac{2}{\Delta t}\left[\int_{t}^{t+\Delta t/2} v(t')dt' - \int_{t-\Delta t/2}^{t} v(t')dt'\right] \qquad \Psi(t) = \begin{matrix} 1/2; & 0 \le t < 1/2 \\ -1/2; & -1/2 \le t < 0 \\ 0; & otherwise \end{matrix}$$

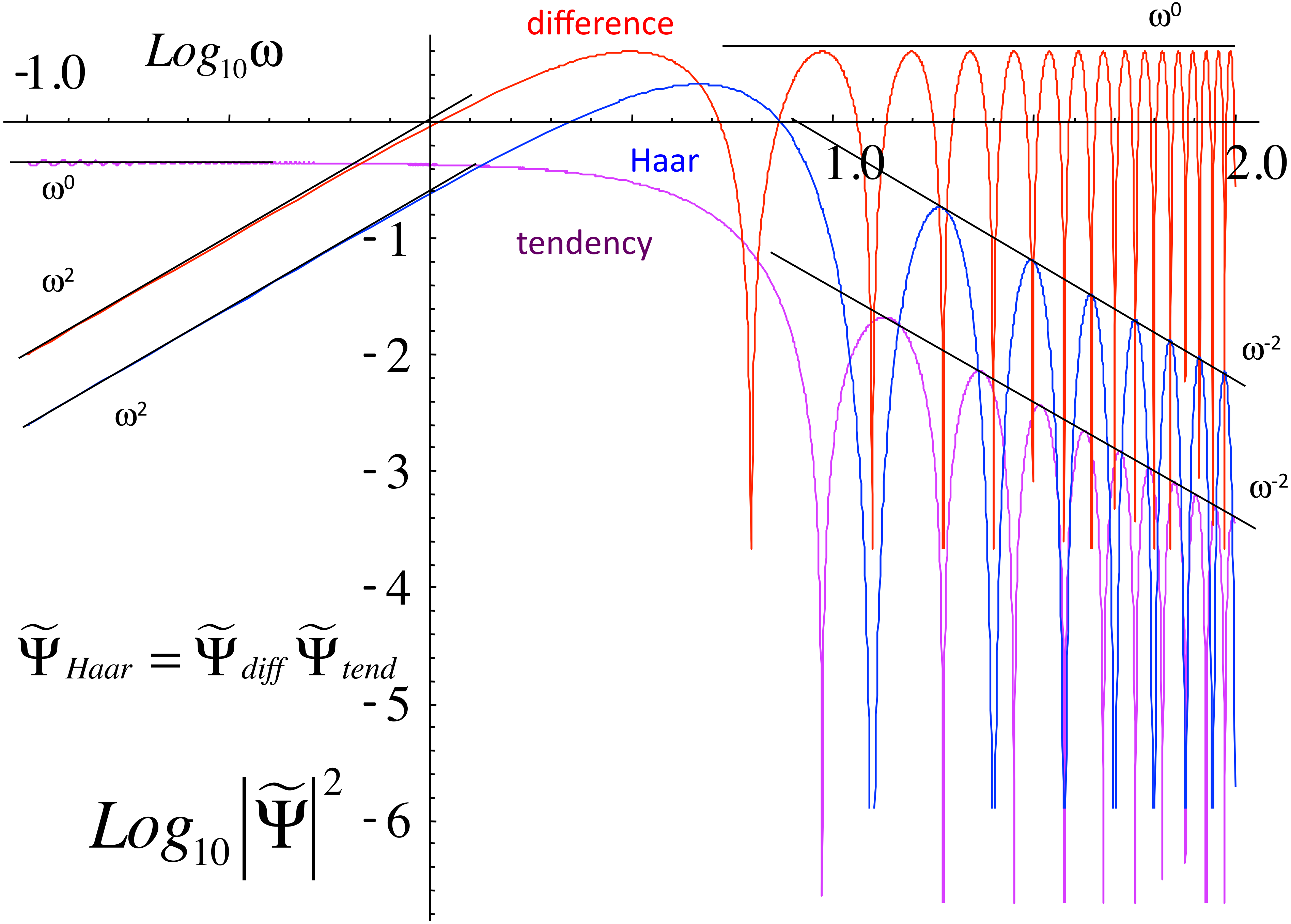
Relation between them:

$$(\Delta v)_{Haar} = \left(\Delta(\Delta v)_{tend}\right)_{diff}$$

Haar, tendency and poor man's wavelets

$$\Delta v(\Delta t) = \frac{1}{\Delta t}\int v(t')\Psi\left(\frac{t'-t}{\Delta t}\right)dt'$$





$$\widetilde{\Psi}_{Haar} = \widetilde{\Psi}_{diff}\widetilde{\Psi}_{tend}$$

Convergence of fluctuation variance

Fluctuations:

$$\Delta v(\Delta t) = \frac{1}{\Delta t}\int v(t')\Psi\left(\frac{t'-t}{\Delta t}\right)dt'$$

Fourier transforms

$$\widetilde{\Delta v}(\omega\Delta t) = \widetilde{v(\omega)}\widetilde{\Psi(\omega)}$$

Ensemble averaging of modulus squared:

$$\left\langle\left|\widetilde{\Delta v}(\omega\Delta t)\right|^2\right\rangle = \left\langle\left|\tilde{v}(\omega)\right|^2\right\rangle\left|\widetilde{\Psi(\omega)}\right|^2$$

Spectra

$$E_{\Delta v}(\omega) = E_v(\omega)\left|\widetilde{\Psi(\omega)}\right|^2$$

(Δt=1)

For scaling processes

$$E_v(\omega) \approx \omega^{-(1+2H')}$$

$$\left|\widetilde{\Psi(\omega)}\right|^2 \approx \begin{matrix} \omega^{2H_{low}}; & \omega \to 0 \\ \omega^{2H_{high}}; & \omega \to \infty \end{matrix}$$

$$\underbrace{\left\langle\Delta v^2\right\rangle = \int_{-\infty}^{\infty} E_{\Delta v}(\omega)\,d\omega}_{\text{Parseval's theorem}}$$

Converges only if:

$$\boxed{H_{low} > H' > H_{high}}$$

Various wavelets

Name	Wavelet	Frequency domain	low ω	high ω	H' range
Poor man's (first difference)	$\delta(t-1/2)-\delta(t+1/2)$	$2\sin(\omega/2)$	$\approx \omega$	≈ 0	$0 \le H' \le 1$
2^{nd} difference	$\frac{1}{2}(\delta(t+1/2)+\delta(t-1/2))-\delta(t)$	$\sin^2(\omega/4)$	$\approx \omega^2$	≈ 0	$0 \le H' \le 1$
Tendency	$I_{[-1/2,1/2]}(t)-\frac{I_{[-\tau/2,\tau/2]}(t)}{\tau}; \quad \tau >> 1$	$\frac{2}{\omega}\left(\sin\left(\frac{\omega}{2}\right)-\tau^{-1}\sin\left(\frac{\omega\tau}{2}\right)\right)$	$\frac{2\sin\left(\frac{\omega\tau}{2}\right)}{\omega\tau} \approx 0; \quad \omega\tau >> 1$	$\approx \omega^{-1}$	$-1 \le H' \le 0$
Haar	$\psi(t)=\begin{array}{ll} 1/2; & 0 \le t < 1/2 \\ -1/2; & -1/2 \le t < 0 \\ 0; & otherwise \end{array}$	$2i\omega^{-1}\sin^2\left(\frac{\omega}{4}\right)$	$\approx \omega$	$\approx \omega^{-1}$	$-1 \le H' \le 1$
Quadratic Haar	$\psi(t)=\begin{array}{ll} -1/3 & 1/3 < t < 1 \\ 2/3; & -1/3 \le t \le 1/3 \\ -1/3; & -1 \le t < -1/3 \\ 0; & otherwise \end{array}$	$\frac{2}{3\omega}\left(3\sin\frac{\omega}{3}-\sin\omega\right)$	$\approx \omega^2$	$\approx \omega^{-1}$	$-1 \le H' \le 2$
First derivative Gaussian	$\Psi(t) \propto \frac{d}{dt}e^{-t^2/2}$	$\omega e^{-\omega^2/2}$	$\approx \omega$	$e^{-\omega^2/2}$	$-\infty \le H' \le 1$
Mexican Hat	$\Psi(t) \propto \frac{d^2}{dt^2}e^{-t^2/2}$	$\omega^2 e^{-\omega^2/2}$	$\approx \omega^2$	$e^{-\omega^2/2}$	$-\infty \le H' \le 2$

Range of exponents over which average fluctuations at scale Δt corresponds to frequency 1/Δt

Fluctuation

$$\langle \Delta I \rangle = \langle \varphi \rangle \Delta t^{H}$$

= constant

$$E(\omega) = \left\langle \left| \tilde{I}(\omega) \right|^{2} \right\rangle = \omega^{-\beta}$$

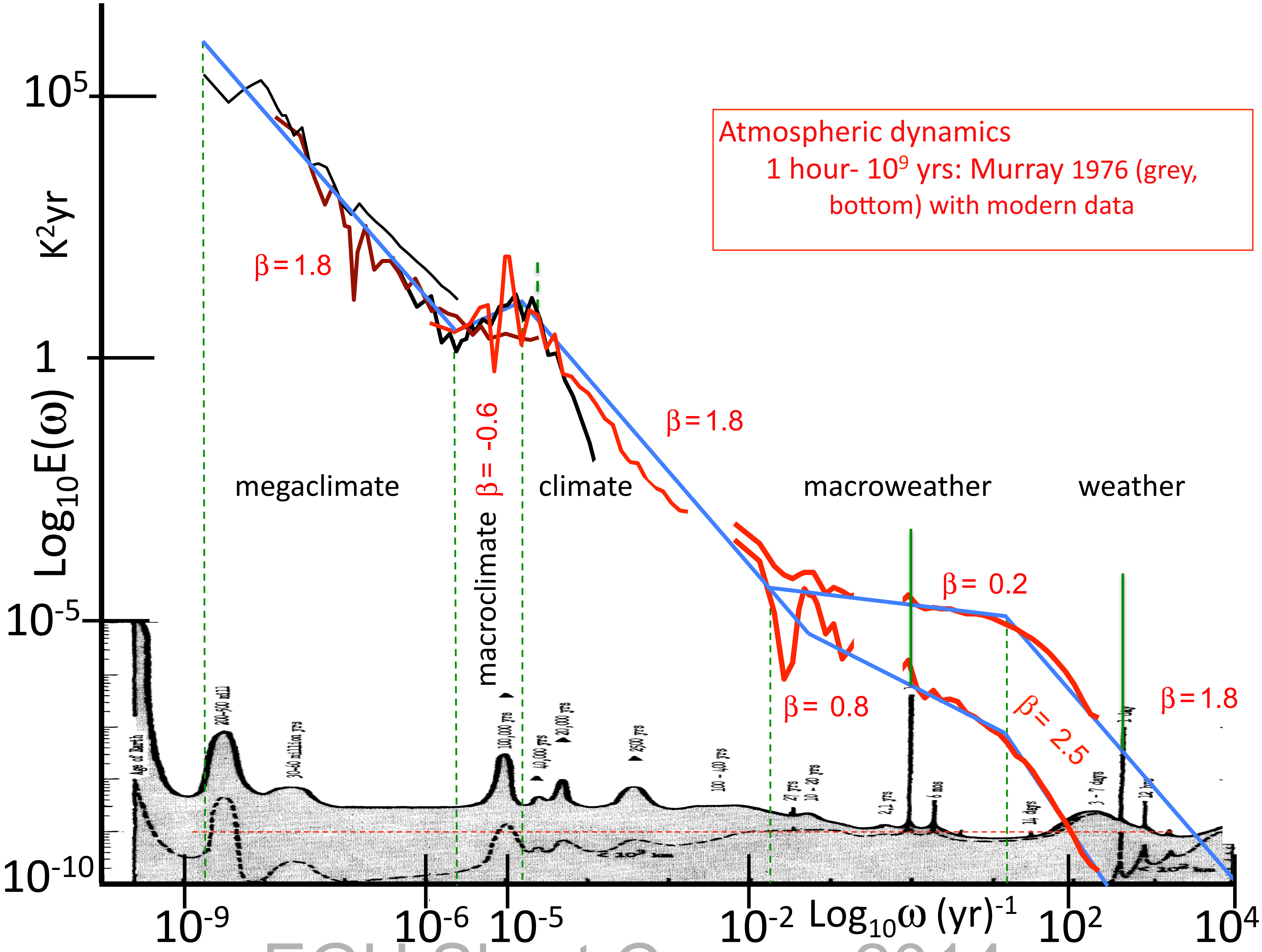
$$\beta = 1 + 2H - K(2)$$

Multifractal “correction”

$$H' = H - K(2)/2$$

Statistic	Range of H	Range of β	Comment
Spectrum	$-\infty < H < \infty$	$-\infty < \beta < \infty$	$E(\omega) \approx \omega^{-\beta}$
Difference	0<H<1	1<β+K(2)<3	“Poor man’s wavelet”
Tendency Fluctuation	-1<H<0	-1<β+K(2)<1	Average with overall mean removed (standard deviation= “Climactogram”, also called the “Aggregated Standard Deviation”)
Haar	-1<H<1	-1<β+K(2)<3	Difference of means of first and second halves of interval
Detrended Fluctuation Analysis (DFA, polynomial order n	-1<H<(n+1)	-1<β+K(2)<3+2n	Also multifractal extension (MFDFA), usually linear: n=1, Not a wavelet
Mexican Hat Wavelet	$-\infty < H < 2$	$-\infty < \beta + K(2) < 5$	2nd Derivative of a Gaussian
Generalized Haar	-m<H<n	1-2m<β+K(2)<3+2n	Interpretation not simple

Simple interpretation

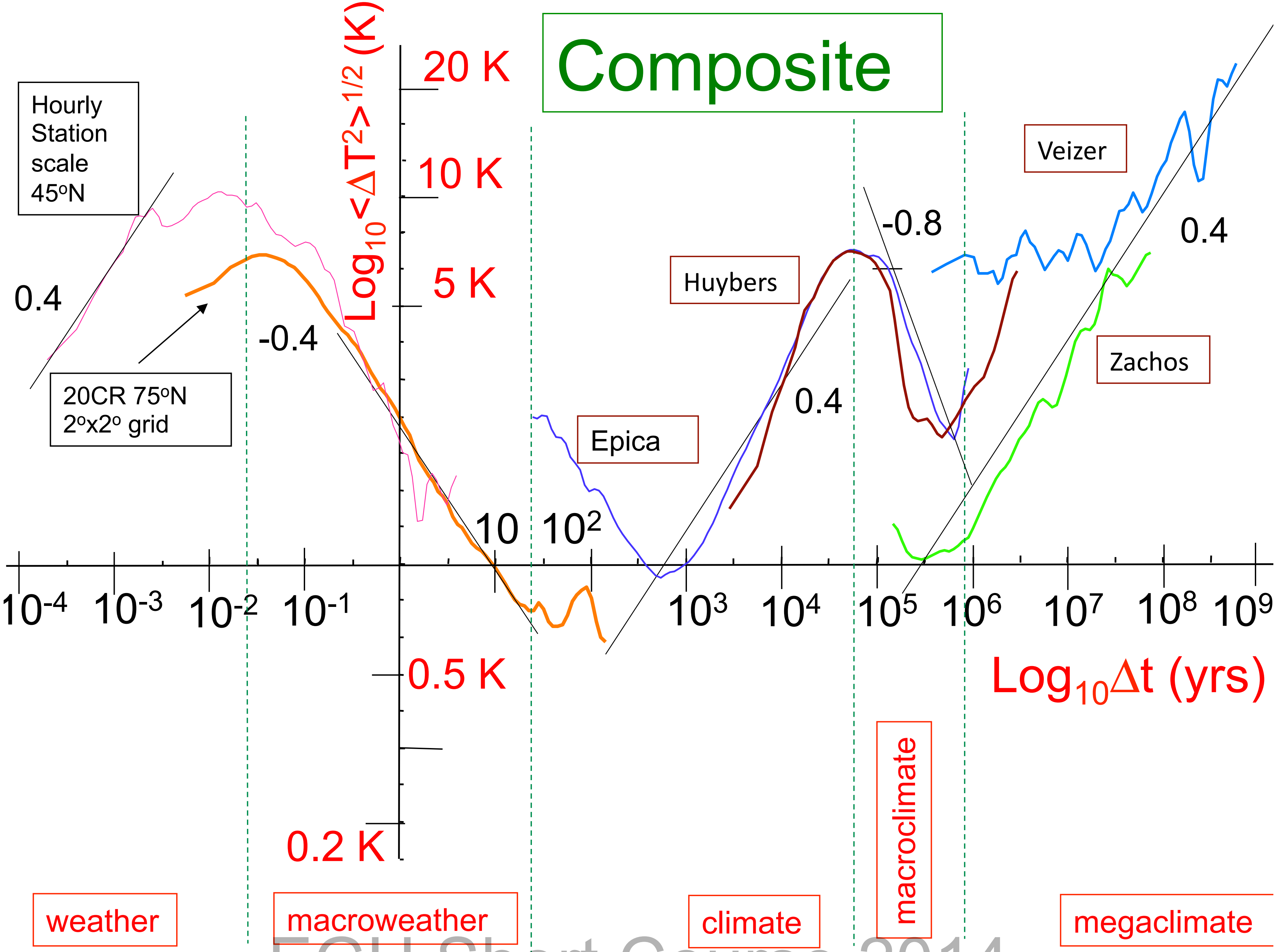


Atmospheric dynamics

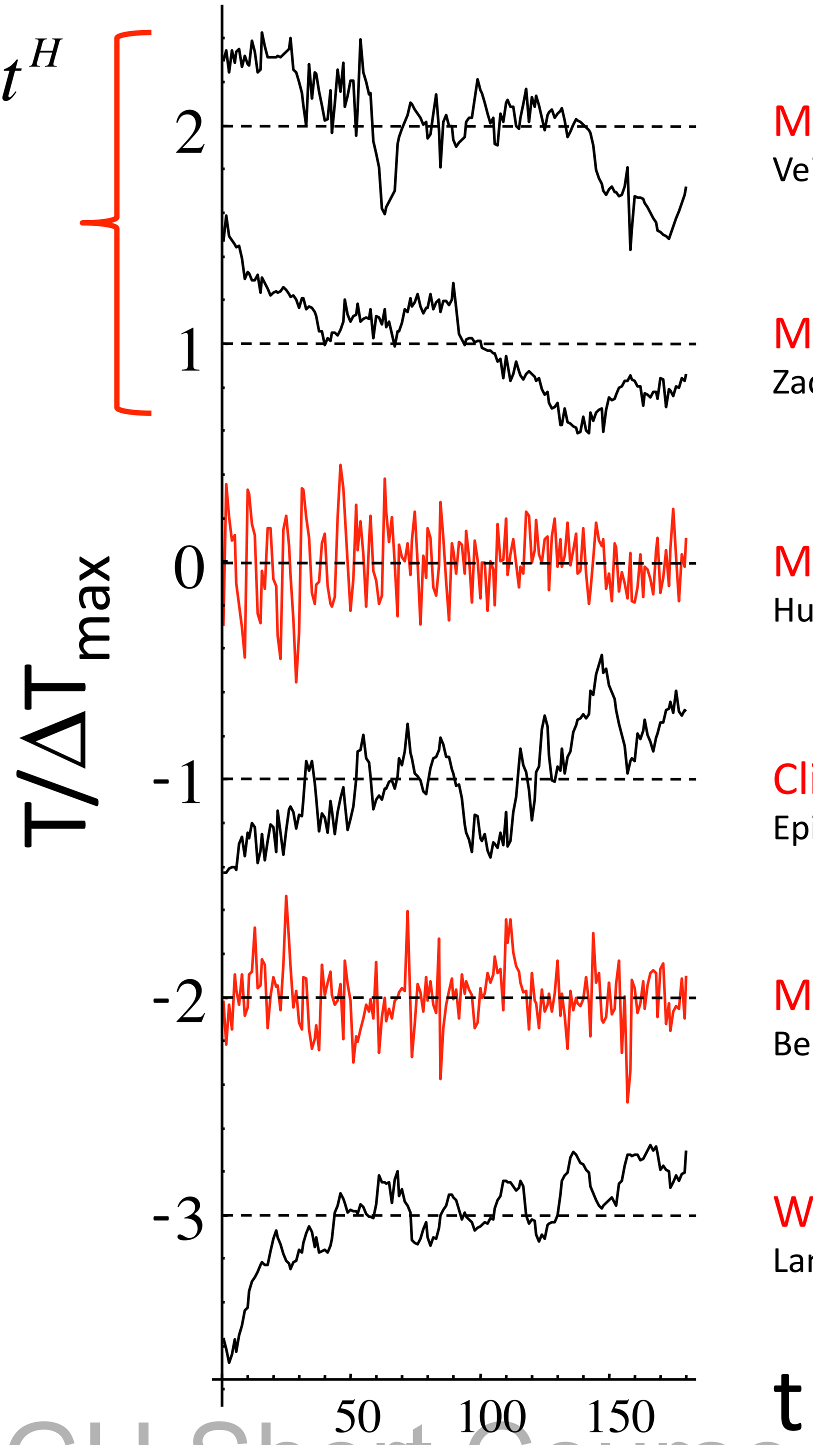
1 hour- 10^9 yrs: Murray 1976 (grey, bottom) with modern data

EGU Short Course 2014

Composite


Hourly
Station
scale
45ºN
20CR 75ºN
2ºx2º grid
0.4
-0.4
0.4
-0.8
0.4
Epica
Huybers
Veizer
Zachos
$\text{Log}_{10}\langle\Delta T^2\rangle^{1/2}$ (K)
20 K
10 K
5 K
0.5 K
0.2 K
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10
10^2
10^3
10^4
10^5
10^6
10^7
10^8
10^9
$\text{Log}_{10}\Delta t$ (yrs)
weather
macroweather
climate
macroclimate
megaclimate

$$\left\langle \Delta T\left(\Delta t\right)\right\rangle \propto \Delta t^{H}$$



H ≈ 0.4

Megaclimate

Veizer: 290 Mys - 511Myrs BP (1.23Myr)

Megaclimate

Zachos: 0-67 Myrs (370 kyr)

H ≈ -0.8

Macroclimate

Huybers: 0-2.56 Myrs (14 kyrs)

H ≈ 0.4

Climate

Epica: 25-97 BP kyrs (400 yrs)

H ≈ - 0.4

Macroweather

Berkeley: 1880-1895 AD (1 month)

H ≈ 0.4

Weather

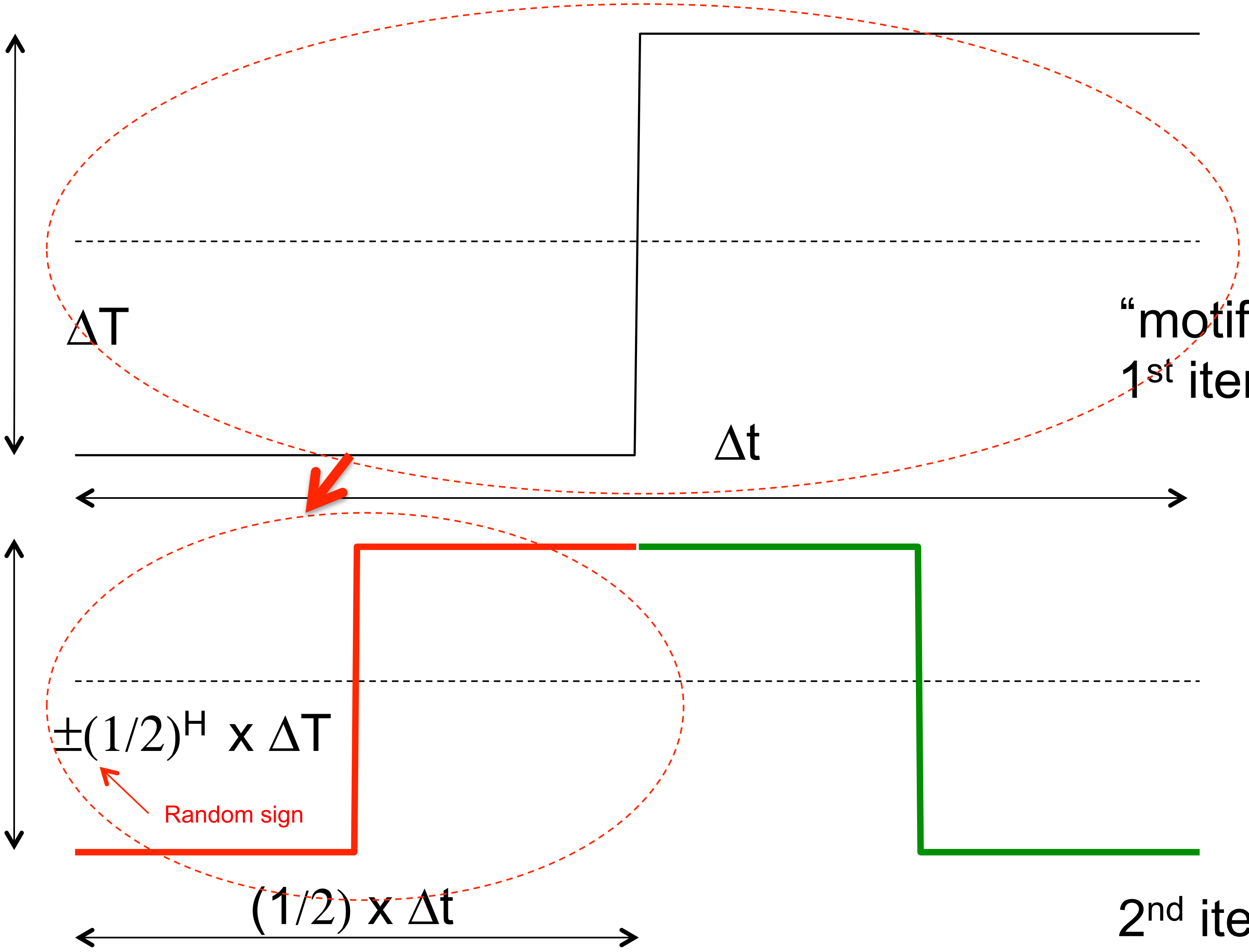
Lander Wy.: July 4-July 11, 2005 (1 hour)

Understanding the Fluctuation exponent H

The fractal H model

(Lovejoy 2013)

(fractal dimenension= 2-H)



ΔT

“motif”= 1st iteration

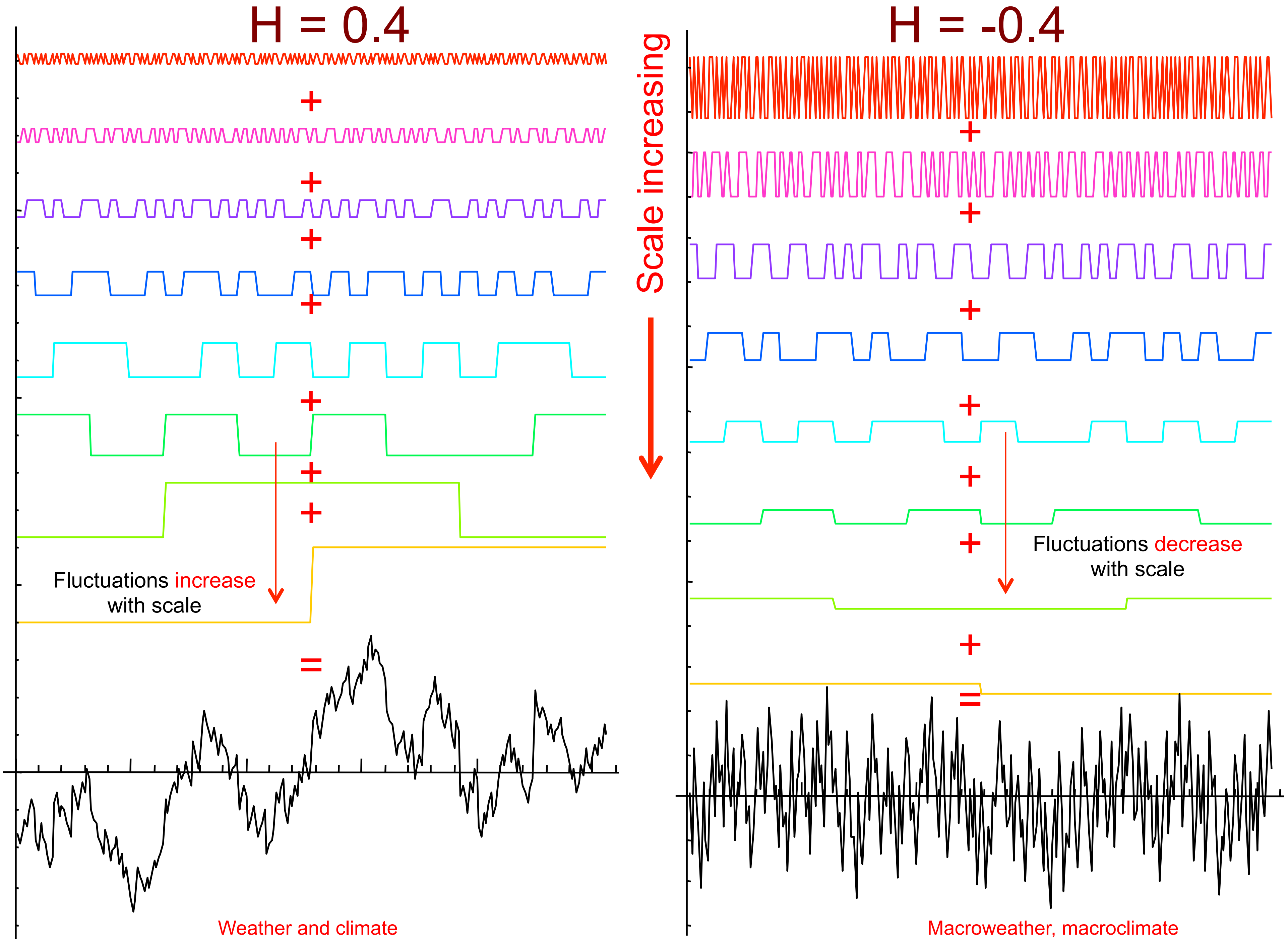
Δt

$\pm(1/2)^H$ x ΔT

Random sign

$(1/2)$ x Δt

2nd iteration


H = 0.4
H = -0.4
Scale increasing
+
=
Fluctuations increase with scale
Fluctuations decrease with scale
Weather and climate
Macroweather, macroclimate

End Part 1