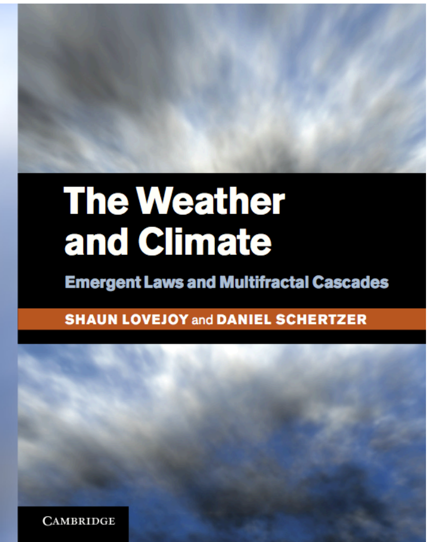


Emergent space-time scaling laws in precipitation: Weather, macroweather and climate regimes

Hydrofractals 2013, Kos
18 October, 2013

S. Lovejoy McGill, Montreal

The Emergence of physical laws

Quantum mechanics → Classical Mechanics

Classical Statistical mechanics
(Boltzman equation) → Continuum mechanics, thermodynamics

The emergence of atmospheric dynamics

(Classical)

Continuum mechanics

deterministic

Low level
(fundamental)

Large Re

Laws of turbulence

Classical:

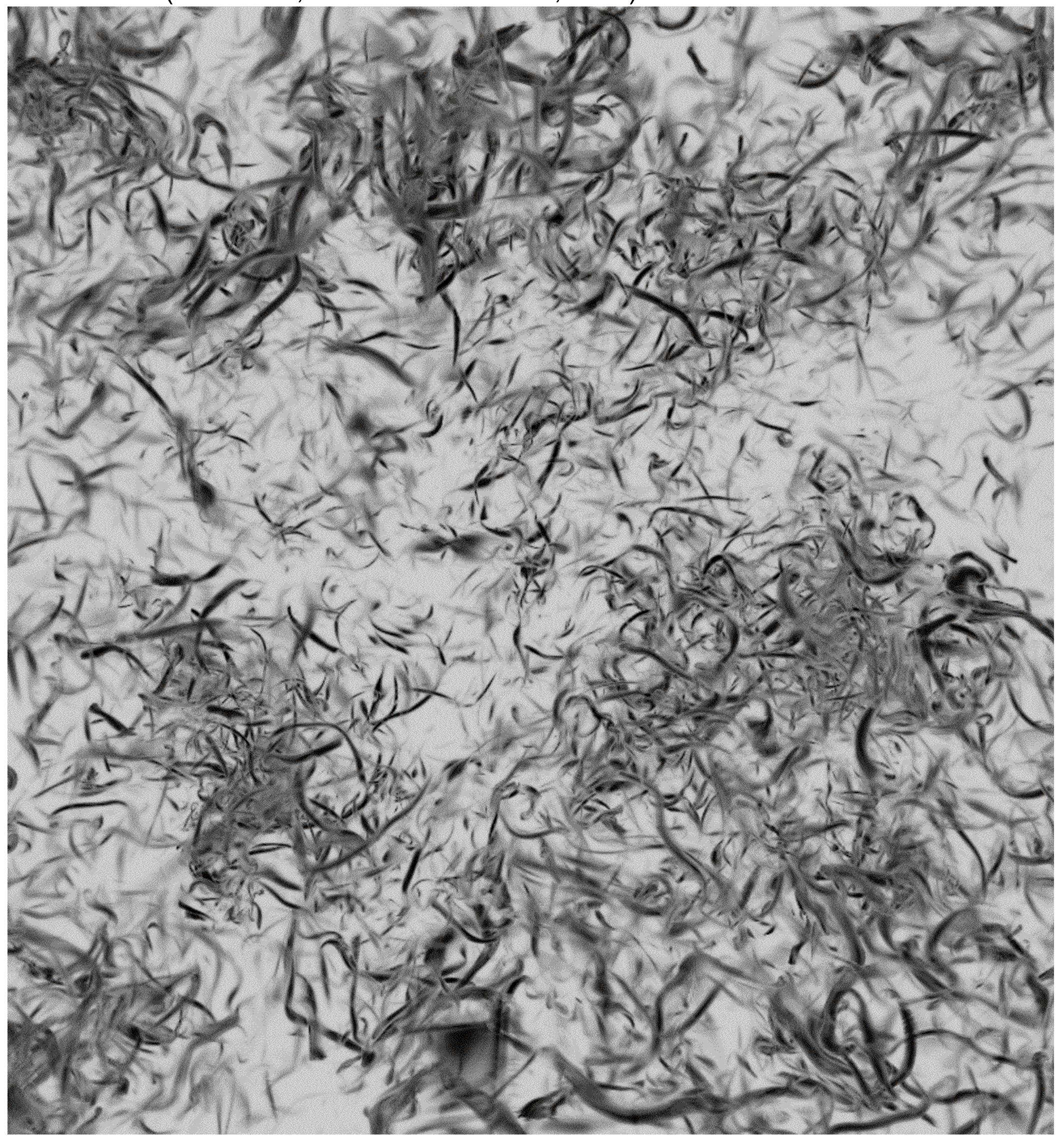
Richardson, Kolmogorov, Corrsin, Obukhov, Bolgiano

High level

stochastic

Vortices in strongly turbulent fluid

(M. Wiczek, numerical simulation, 2010)



$$\Delta v(\underline{\Delta r}) = \varphi \left|\underline{\Delta r}\right|^{H}$$

e.g. Kolmogorov $\varphi=\varepsilon^{1/3}$, $H=1/3$

a) $\left|\underline{\Delta r}\right| <\approx 100m$ b) isotropic

c) $\varphi \approx$ constant, quasi Gaussian

Emergence of Atmospheric laws

(Modern)

Fluctuations ≈ (turbulent flux) x $(\text{scale})^H$

- Fluctuations: Differences, tendencies, wavelet coefficients
- (turbulent flux): Cascading Turbulent flux
- (scale): Anisotropic Space-time Scale function
- H: Fluctuation /conservation exponent

Fourier domain:

$$\left(\frac{Variance_{observables}}{wavenumber}\right) = \left(\frac{Variance_{flux}}{wavenumber}\right)(wavenumber)^{-2H}$$

$$= (wavenumber)^{-\beta}$$

Space: $E(k) \approx k^{-\beta}$

Time: $E(\omega) \approx \omega^{-\beta}$

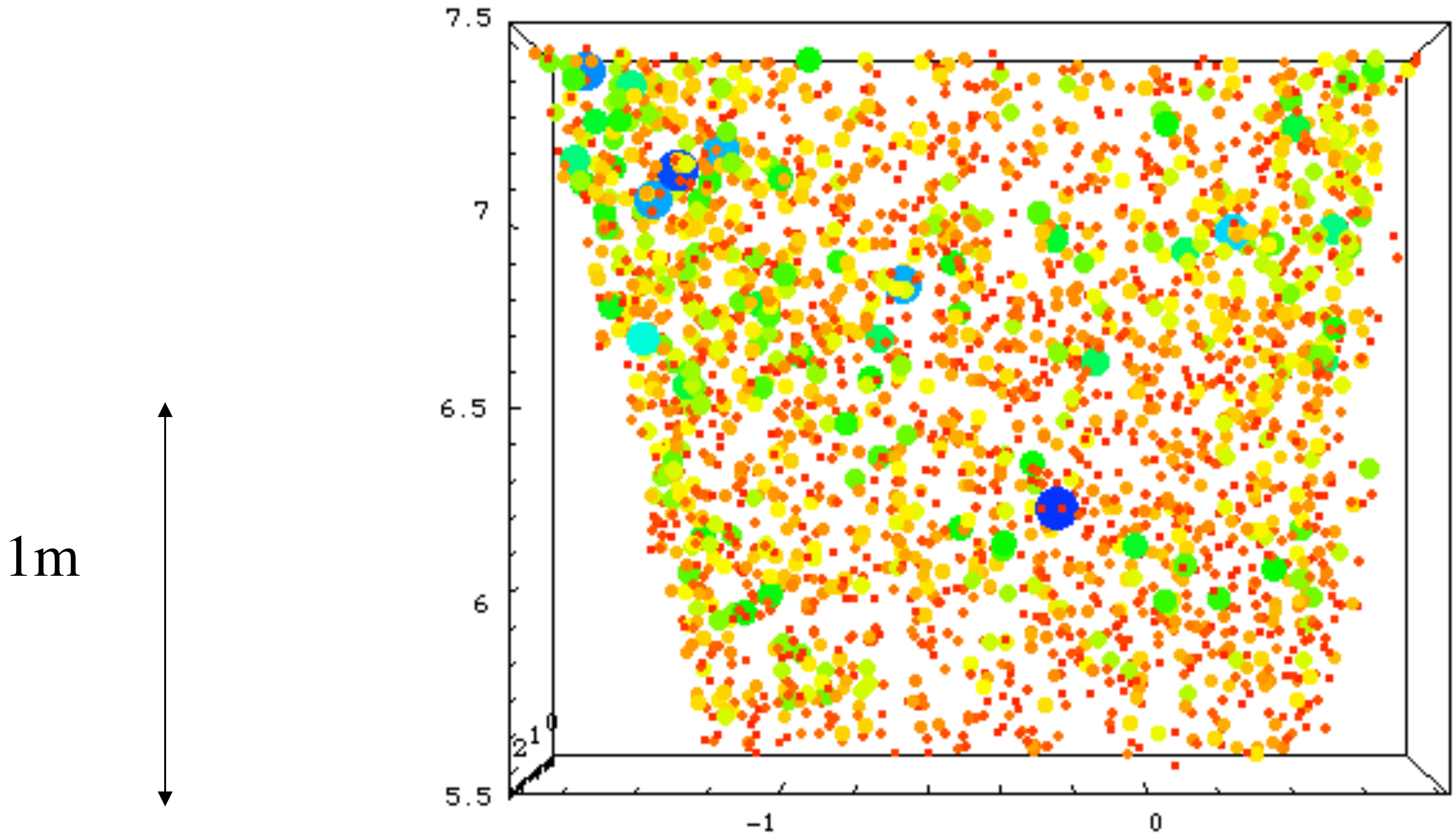
The emergent laws hold up to planetary scales (Horizontal scaling)

$$E(k) = k^{-\beta}$$

From small scales

Stereophotography of drops

(HYDROP experiment) (storm 295 no. 2)

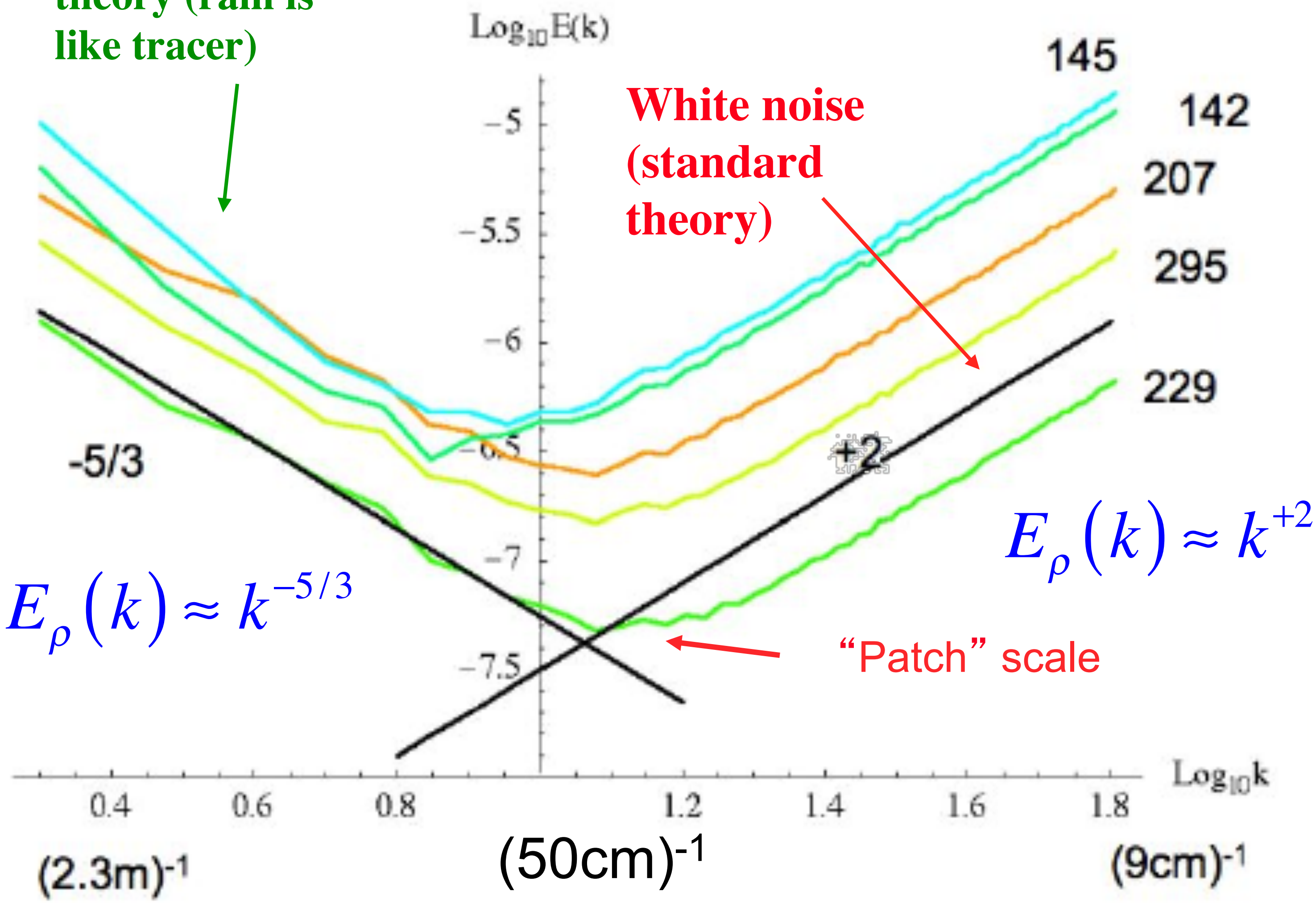


The angle averaged drop spectra

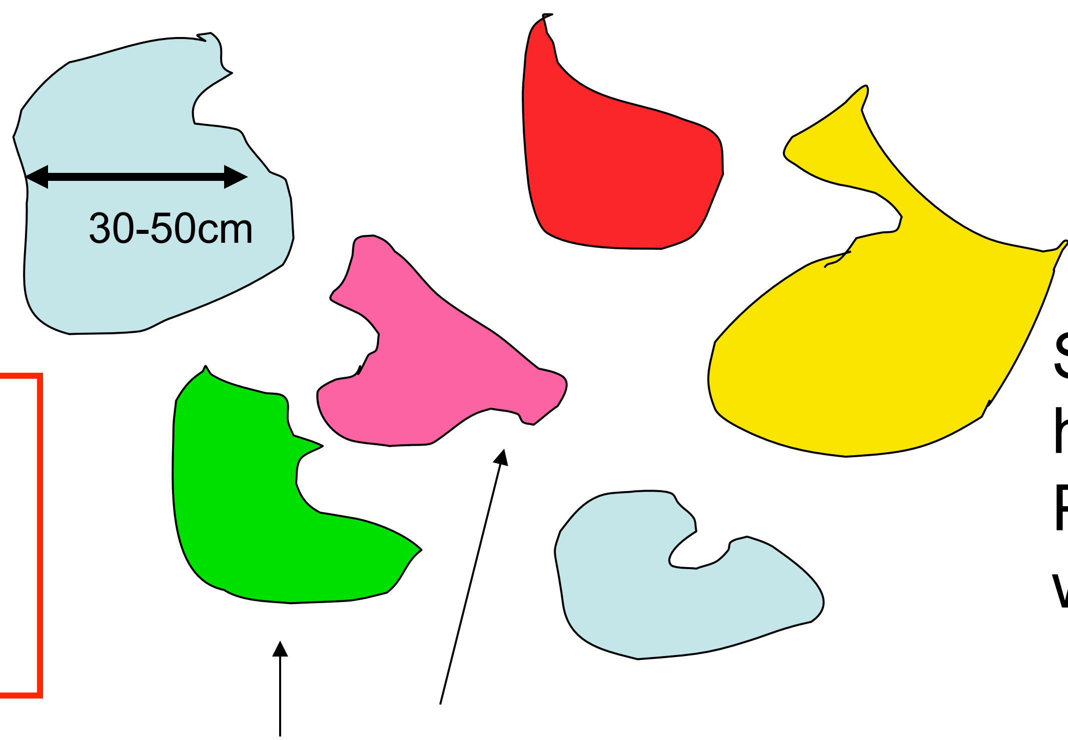
5 storms, 18 triplets

Corrsin-Obukov passive scalar theory (rain is like tracer)

Direct evidence that rain behaves as a passive scalar at large enough scales



Interpretation of HYDROP spectra, moment analysis: Patches

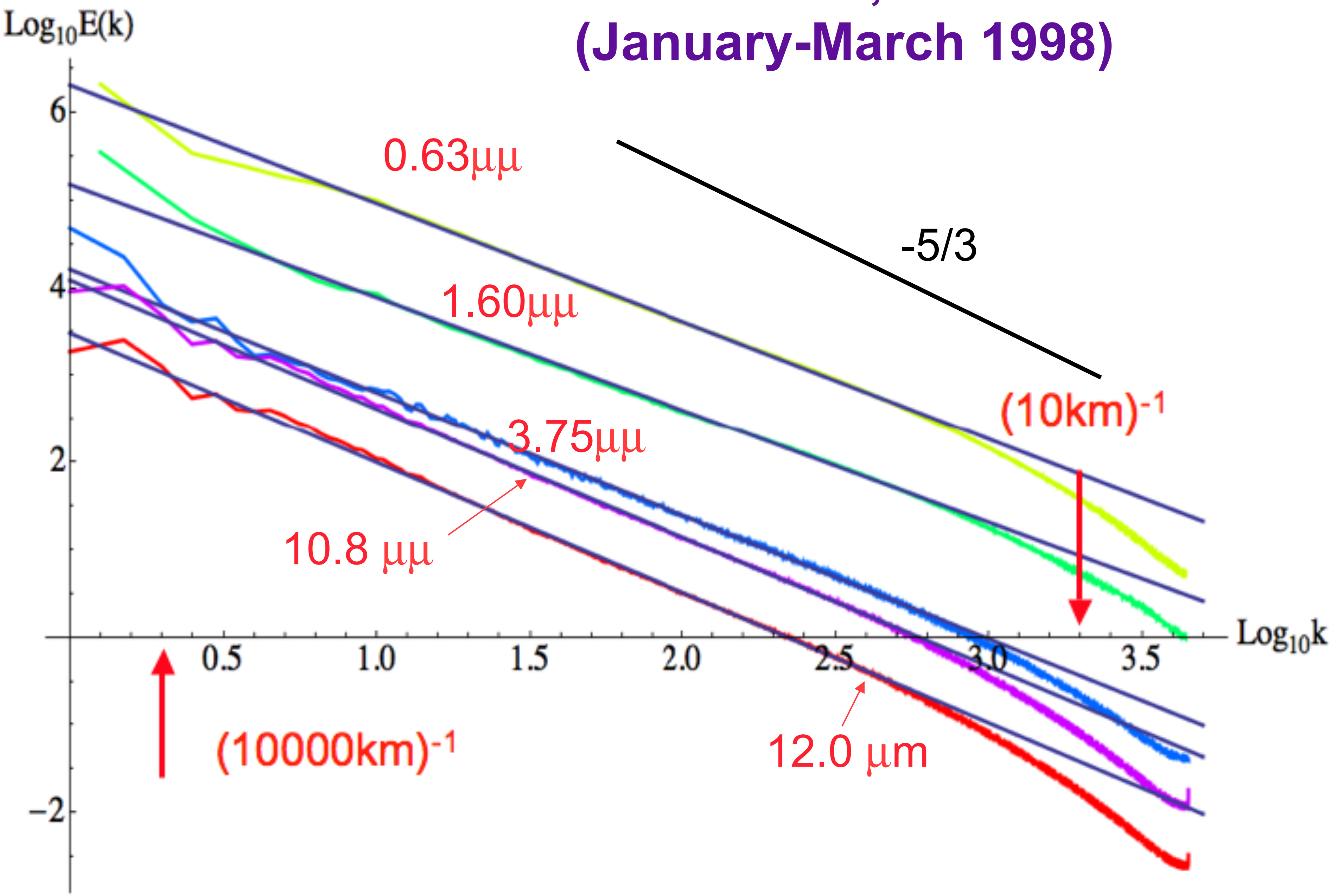


Patch scale = scale at which Stokes number =1

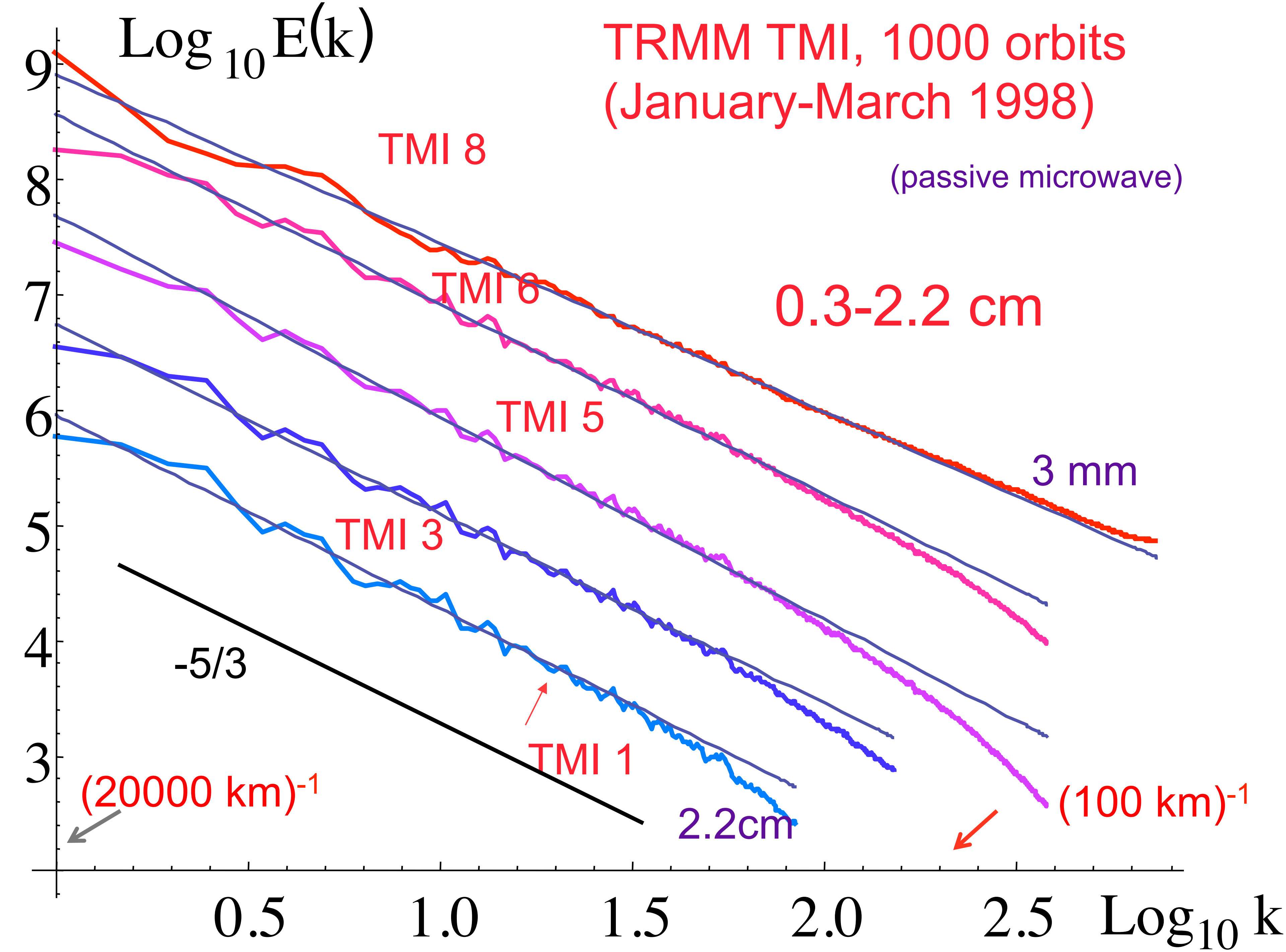
Highly variable multifractal density

Satisfies usual homogenous Poisson statistics - within patches

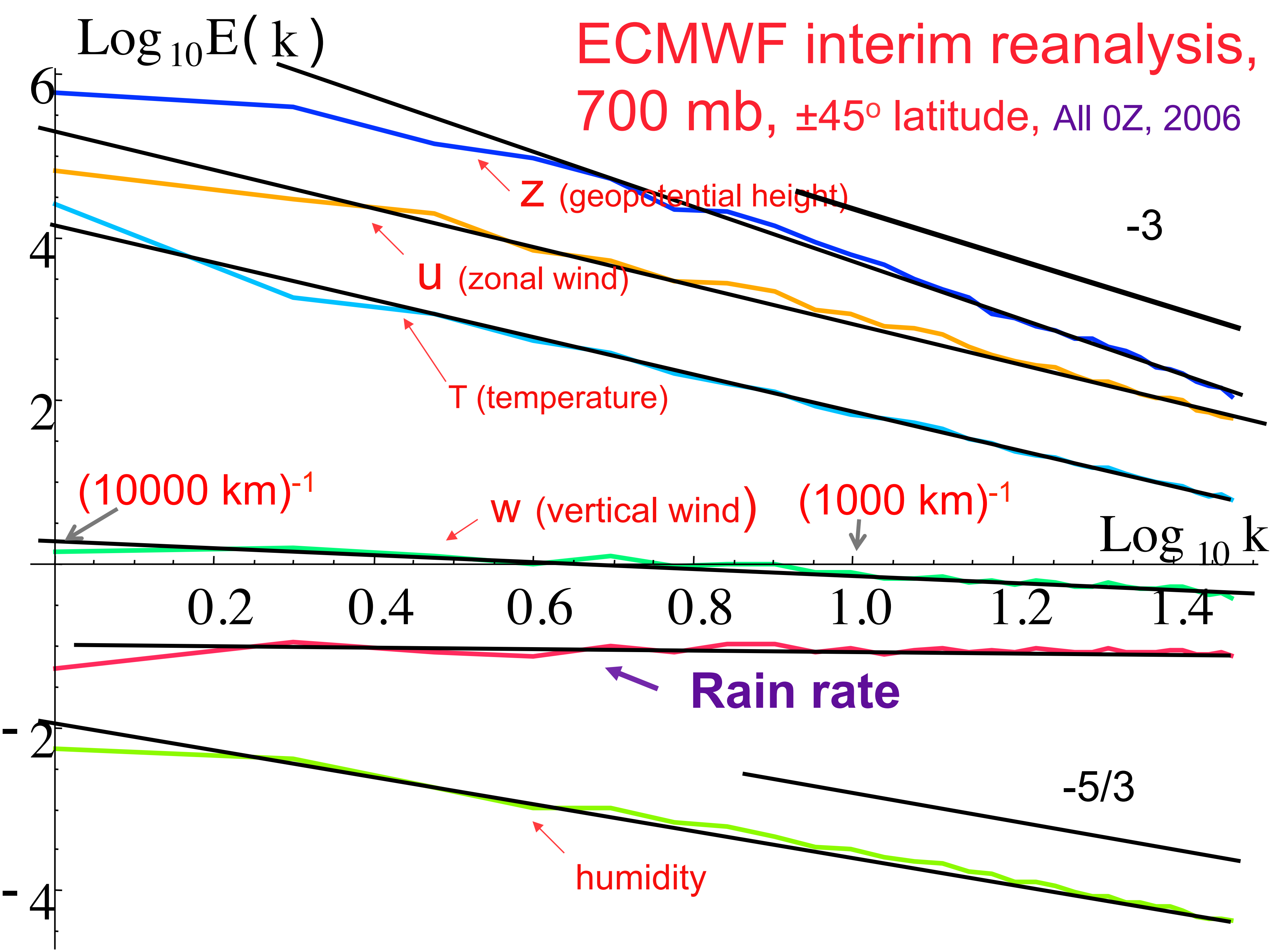
TRMM VIRS, 1000 orbits (January-March 1998)



Visible, near infra red, thermal infra red



Log 10 E(k)
TRMM TMI, 1000 orbits
(January-March 1998)
(passive microwave)
TMI 8
TMI 6
0.3-2.2 cm
TMI 5
3 mm
TMI 3
-5/3
TMI 1
(20000 km)-1
2.2cm
(100 km)-1
9
8
7
6
5
4
3
0.5
1.0
1.5
2.0
2.5
Log10 k



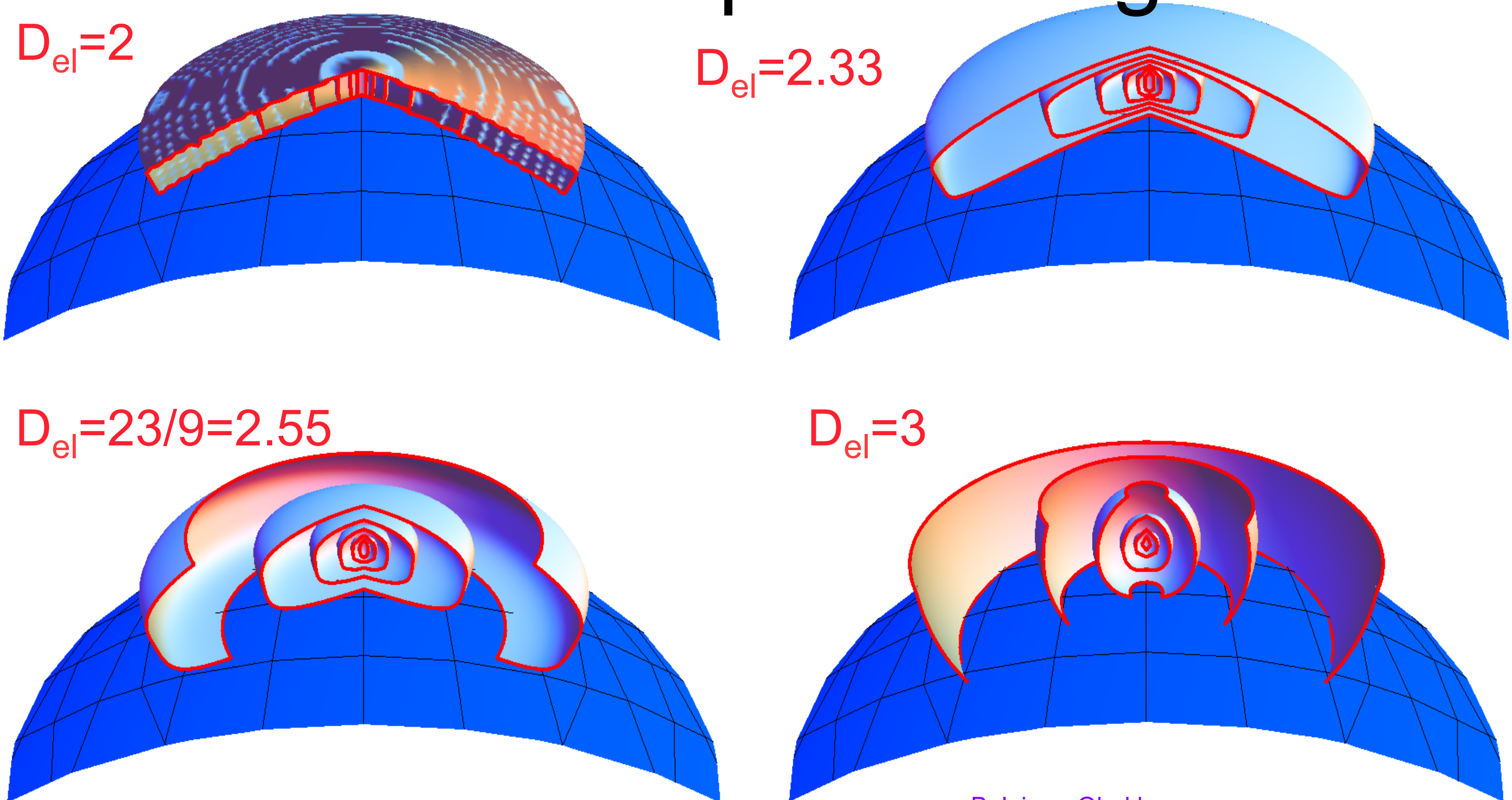
Log 10 E(k)
ECMWF interim reanalysis,
700 mb, ±45° latitude, All 0Z, 2006
Z (geopotential height)
-3
u (zonal wind)
T (temperature)
(10000 km)-1
w (vertical wind)
(1000 km)-1
Log 10 k
Rain rate
-5/3
humidity
6
4
2
-2
-4
0.2
0.4
0.6
0.8
1.0
1.2
1.4

These huge scaling ranges are possible because the scaling is *anisotropic*

Isotropic turbulence - including Geostrophic turbulence - is irrelevant in the atmosphere!

$$\left|\underline{\Delta r}\right| \to \left\|\underline{\Delta r}\right\|$$

Anisotropic Scaling



The 23/9D model:

Bolgiano-Obukhov

$$\underbrace{\Delta v(\Delta x) = \varepsilon^{1/3}\Delta x^{1/3}}_{\text{Kolmogorov}}; \quad \overbrace{\Delta v(\Delta z) = \phi^{1/5}\Delta z^{3/5}}$$

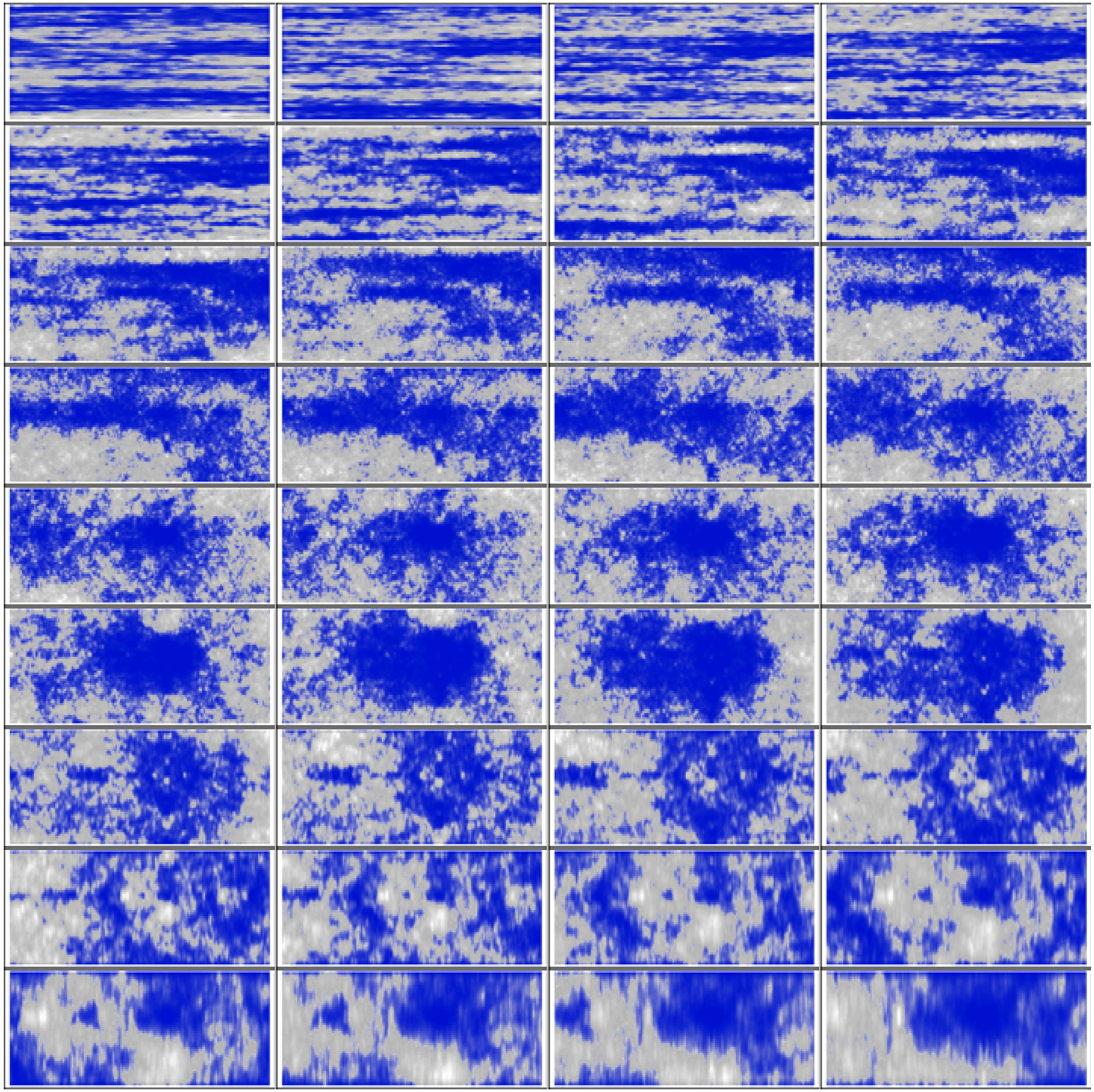
H_z=(1/3)/(3/5)=5/9

Kolmogorov

Volume≈LxLxL^{Hz}≈L^{Del}

D_{el}=2+H_z=23/9

Zoom factor 1000



Vertical cross-section

The turbulent fluxes follow multiplicative cascades, multifractal behaviour

$$\Delta I = \varphi \left[\!\left[\left(\Delta x, \Delta y, \Delta z, \Delta t \right) \right]\!\right]^{H}$$

$$\frac{\varphi}{\langle \varphi \rangle} = \frac{\Delta I}{\langle \Delta I \rangle}$$

Multiplicative Cascades

Generic statistical behaviour:

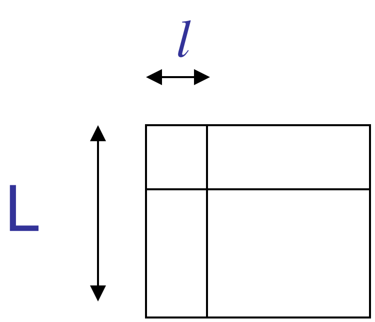
scaling

Scale invariant

$$\left\langle \varphi_\lambda^q \right\rangle \approx \lambda^{K(q)}$$

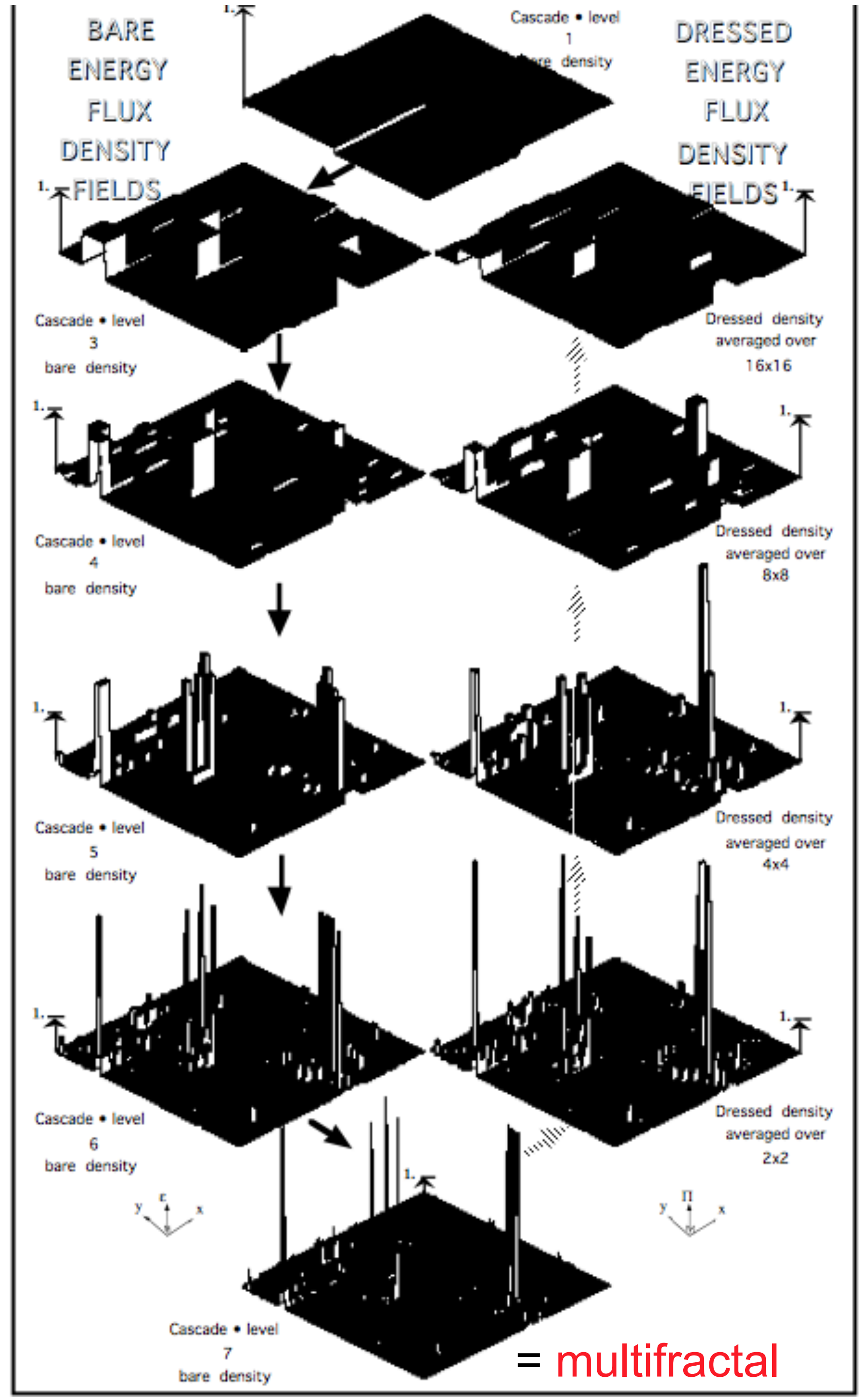
Statistical averaging

Resolution: ratio $\lambda = L/l$



Probabilities:

$$\Pr\left(\varphi_\lambda > \lambda^\gamma\right) \approx \lambda^{-c(\lambda)}$$



= multifractal

Early evidence of cascades:

Precipitation 1987

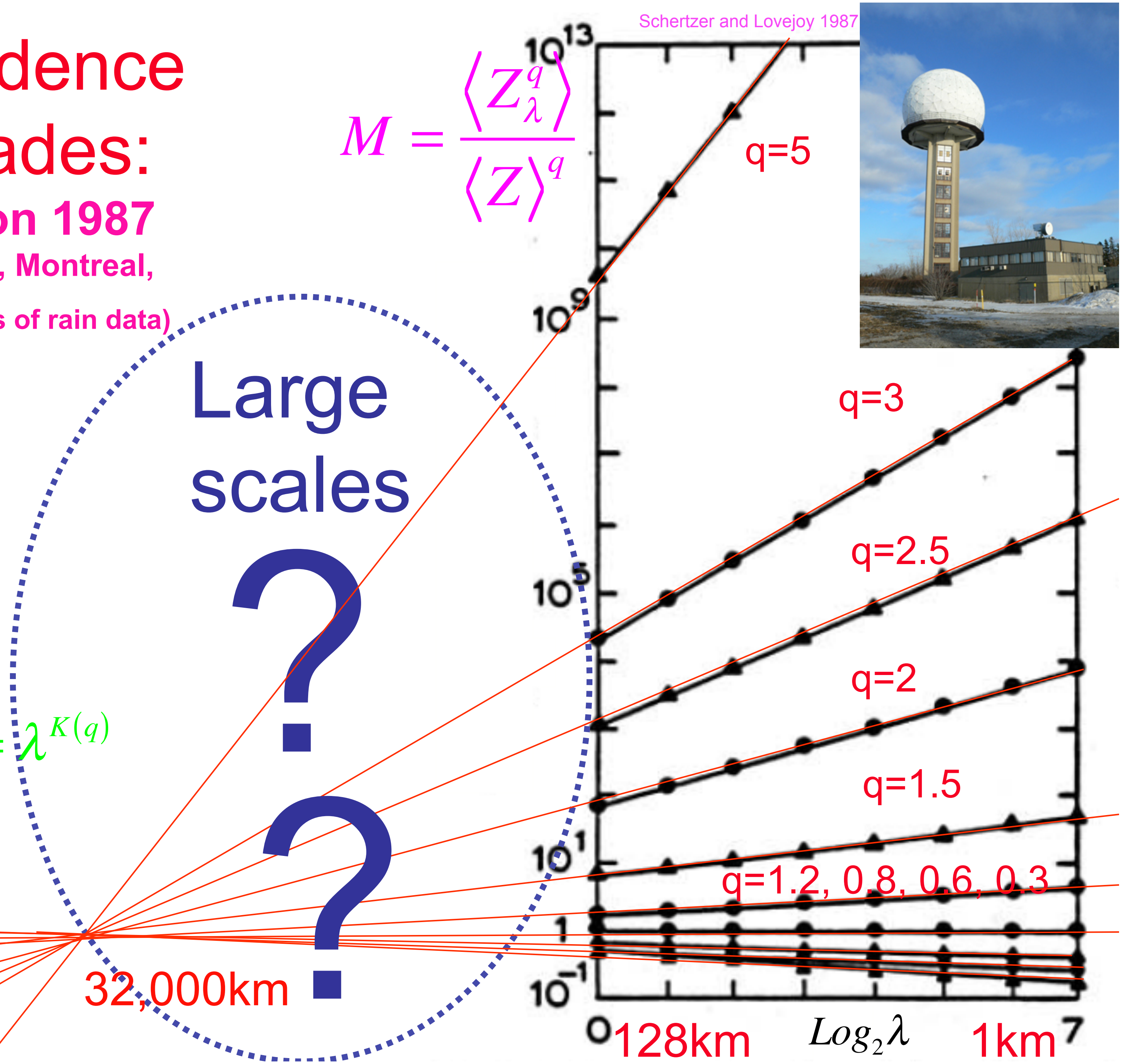
(70 Radar Scans, Montreal,

horizontal 3 weeks of rain data)

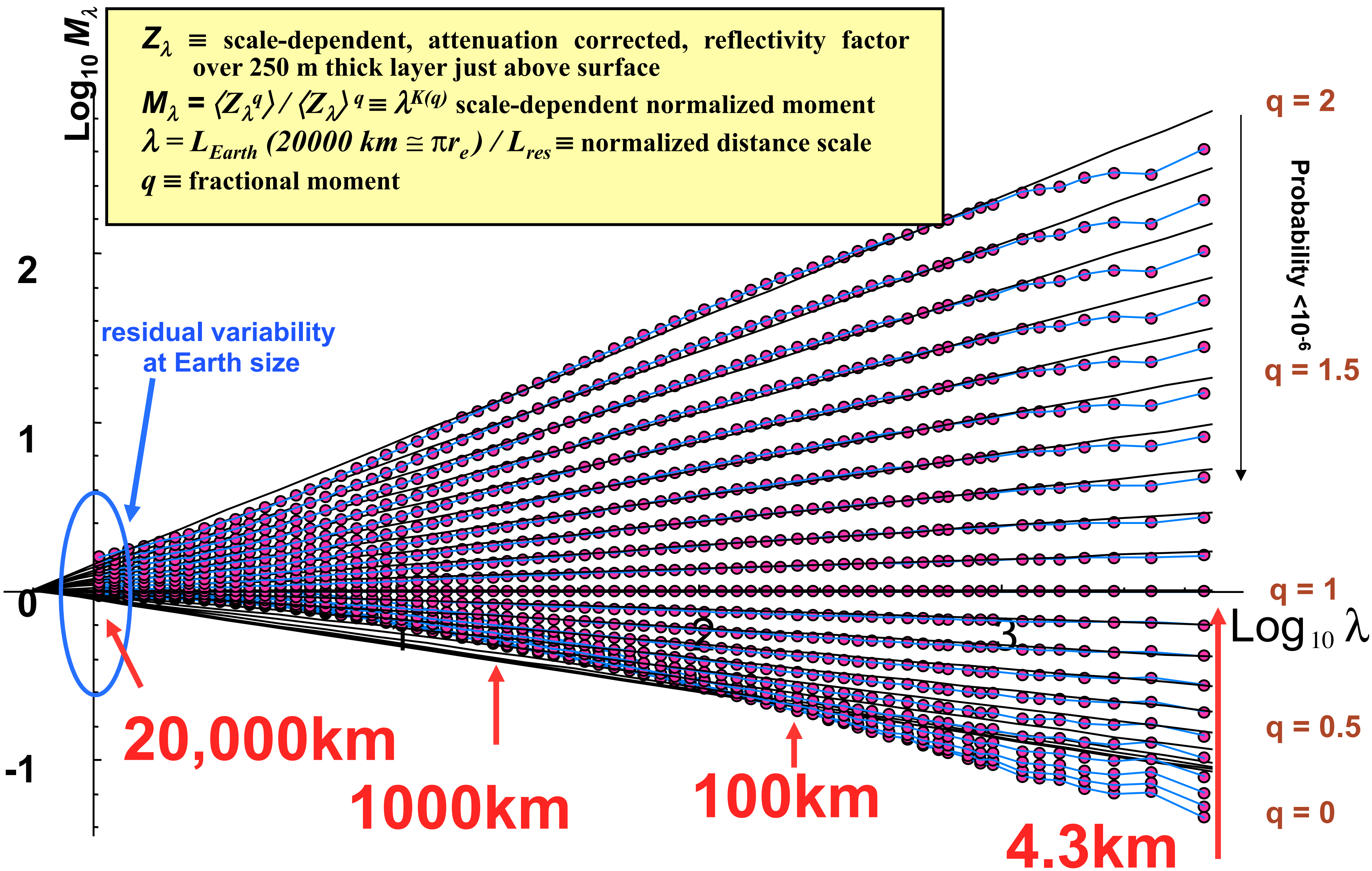
Cascade prediction:

$$\left\langle Z_\lambda^q \right\rangle / \left\langle Z_1 \right\rangle^q = \lambda^{K(q)}$$

$$\lambda = L_{eff} / L_{res}$$

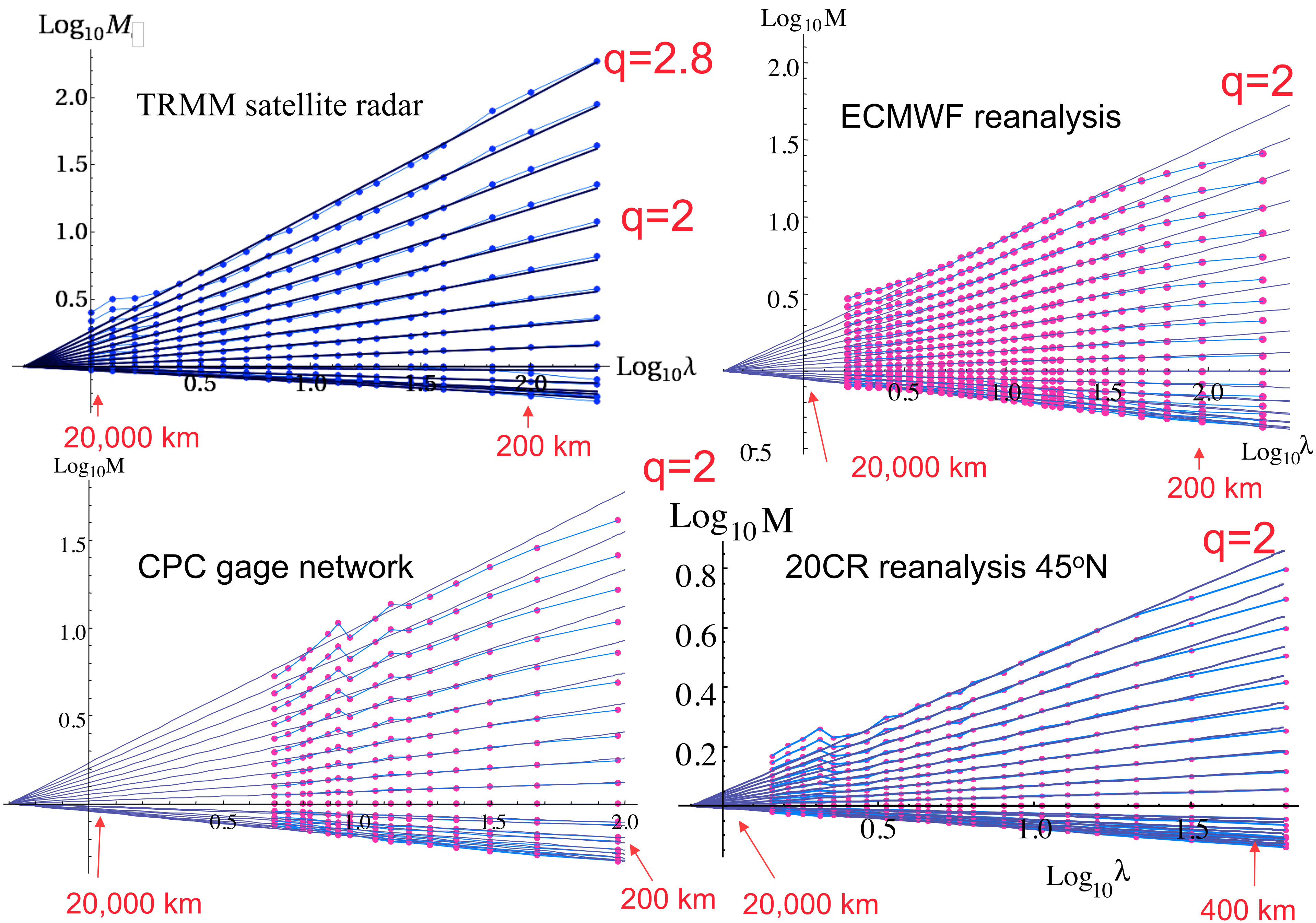


Scale-dependent TRMM PR Attenuation Corrected Reflectivity Factor [Z_λ] (1176 consecutive orbits -- ~70 days)



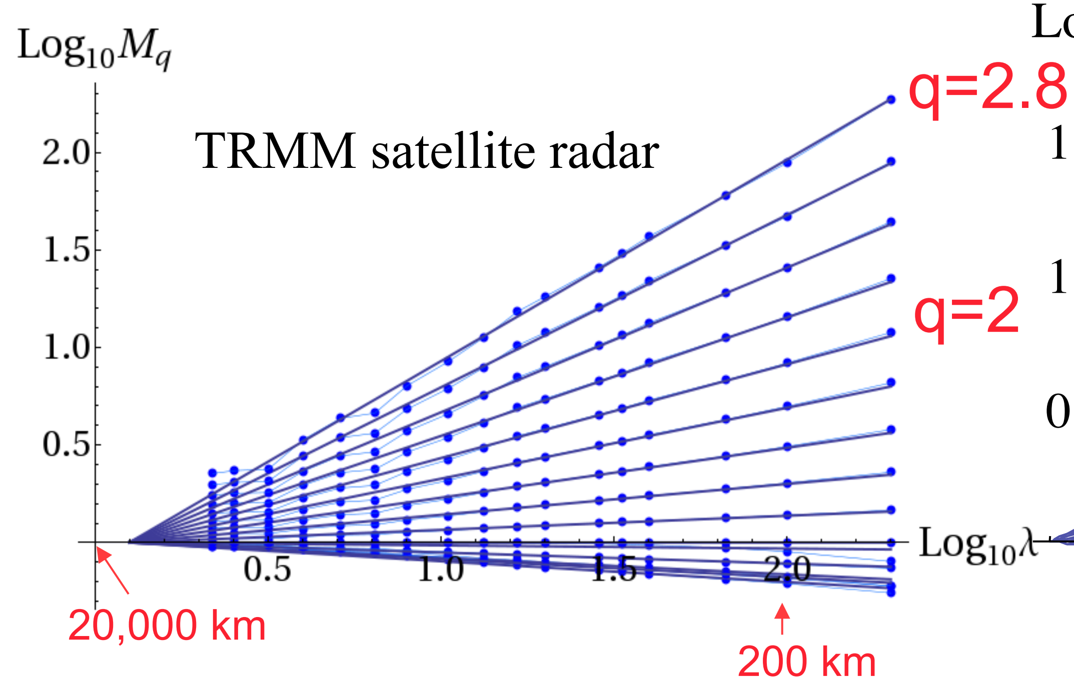
$M = \langle \phi_\lambda^q \rangle / \langle \phi \rangle^q$

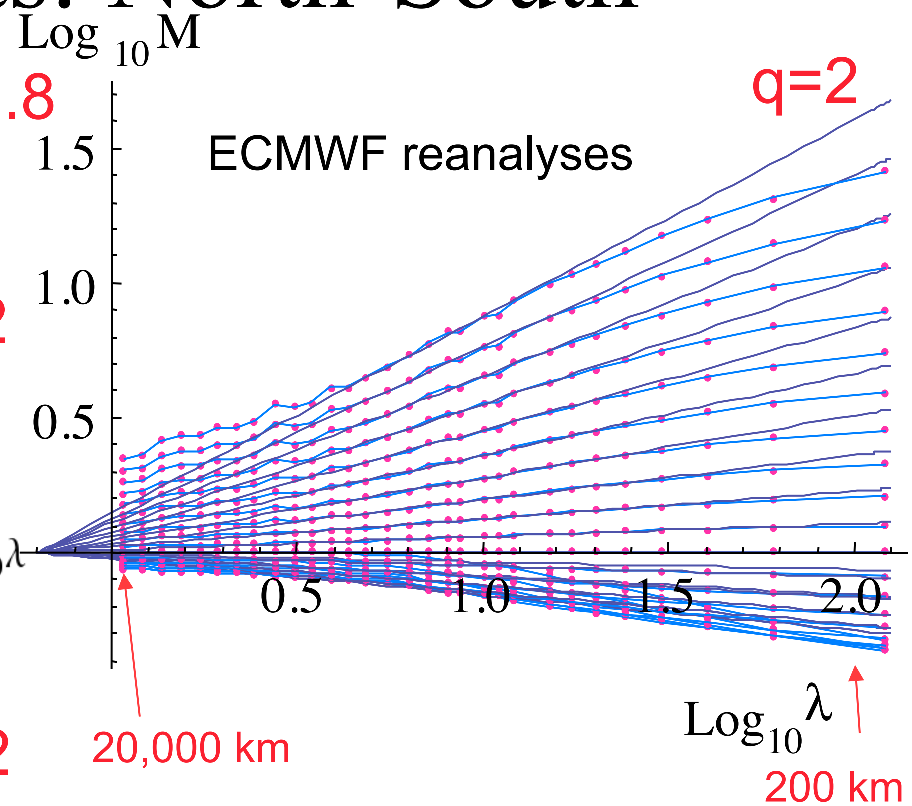
Rainrate Moments: East-West

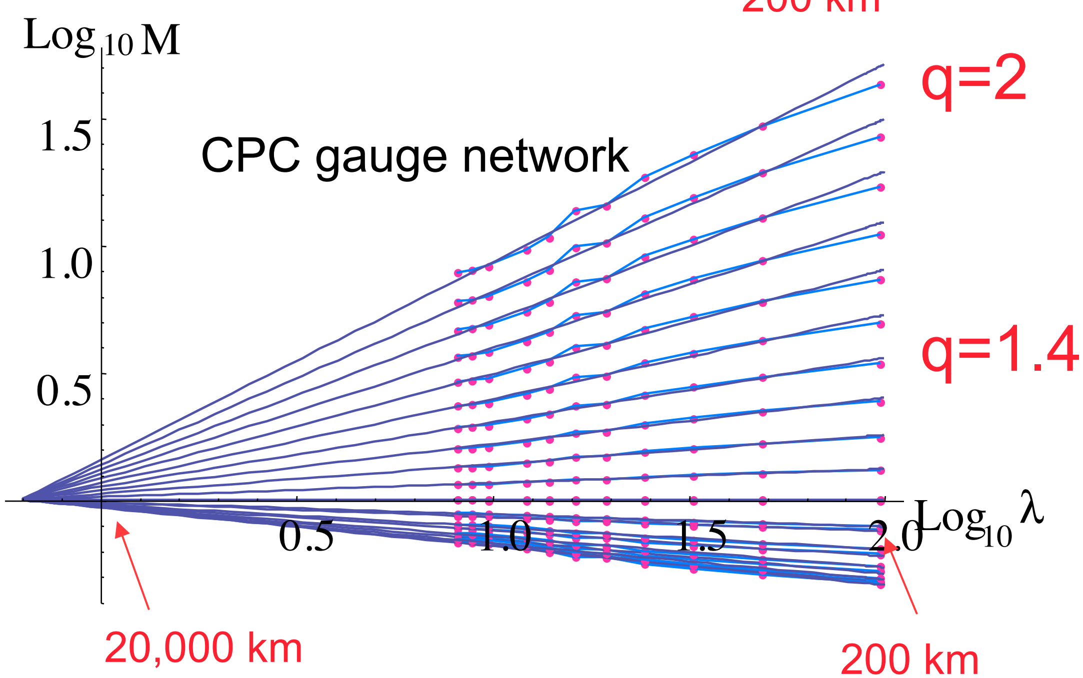


$M = \langle \phi_\lambda^q \rangle / \langle \phi \rangle^q$

Rainrate Moments: North-South

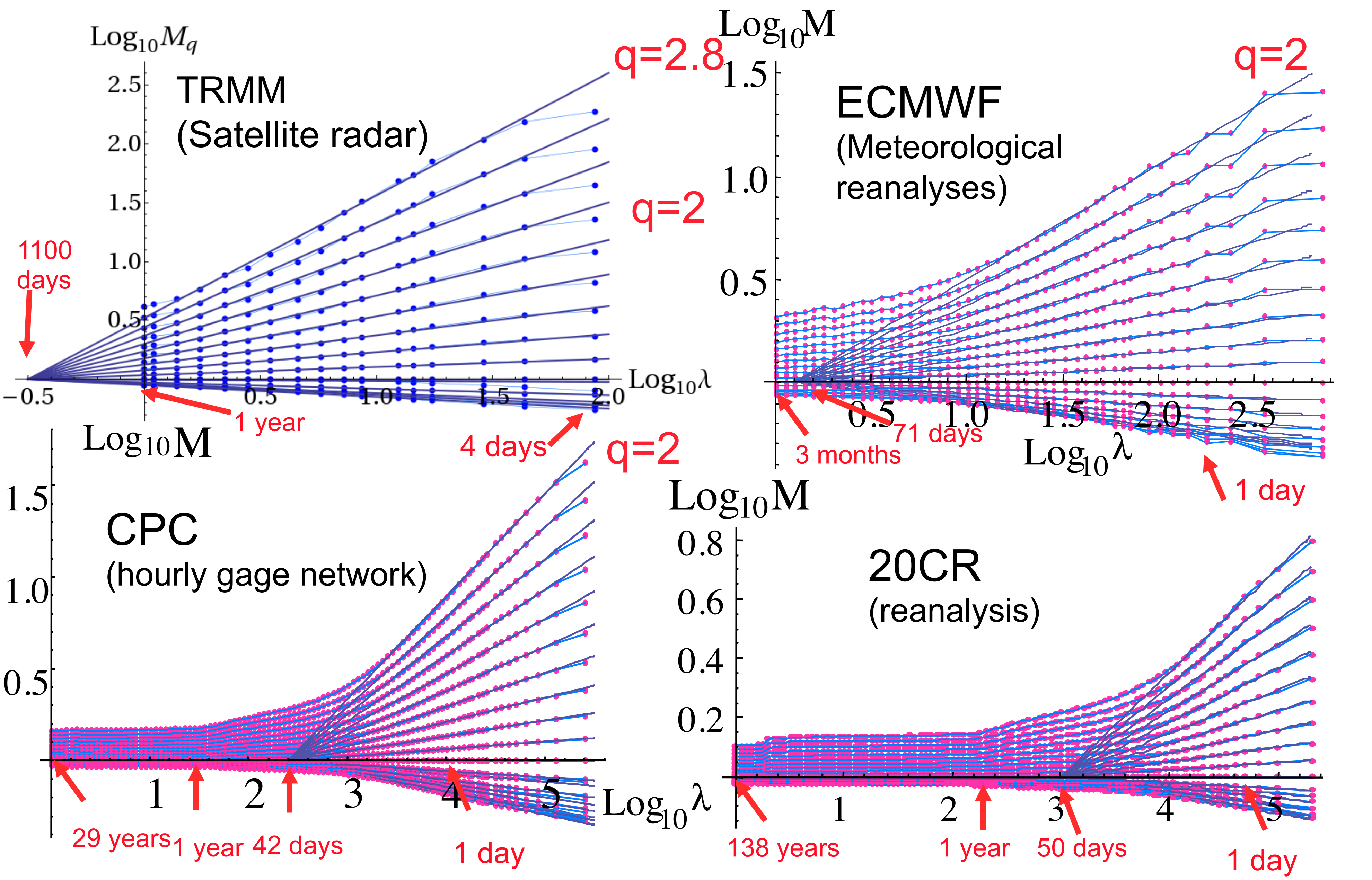






$M = \langle \phi_\lambda^q \rangle / \langle \phi \rangle^q$

Rainrate Moments: (time)



Conclusion:

Qualitatively:
gauges, radar, reanalyses have similar space-time cascade structures

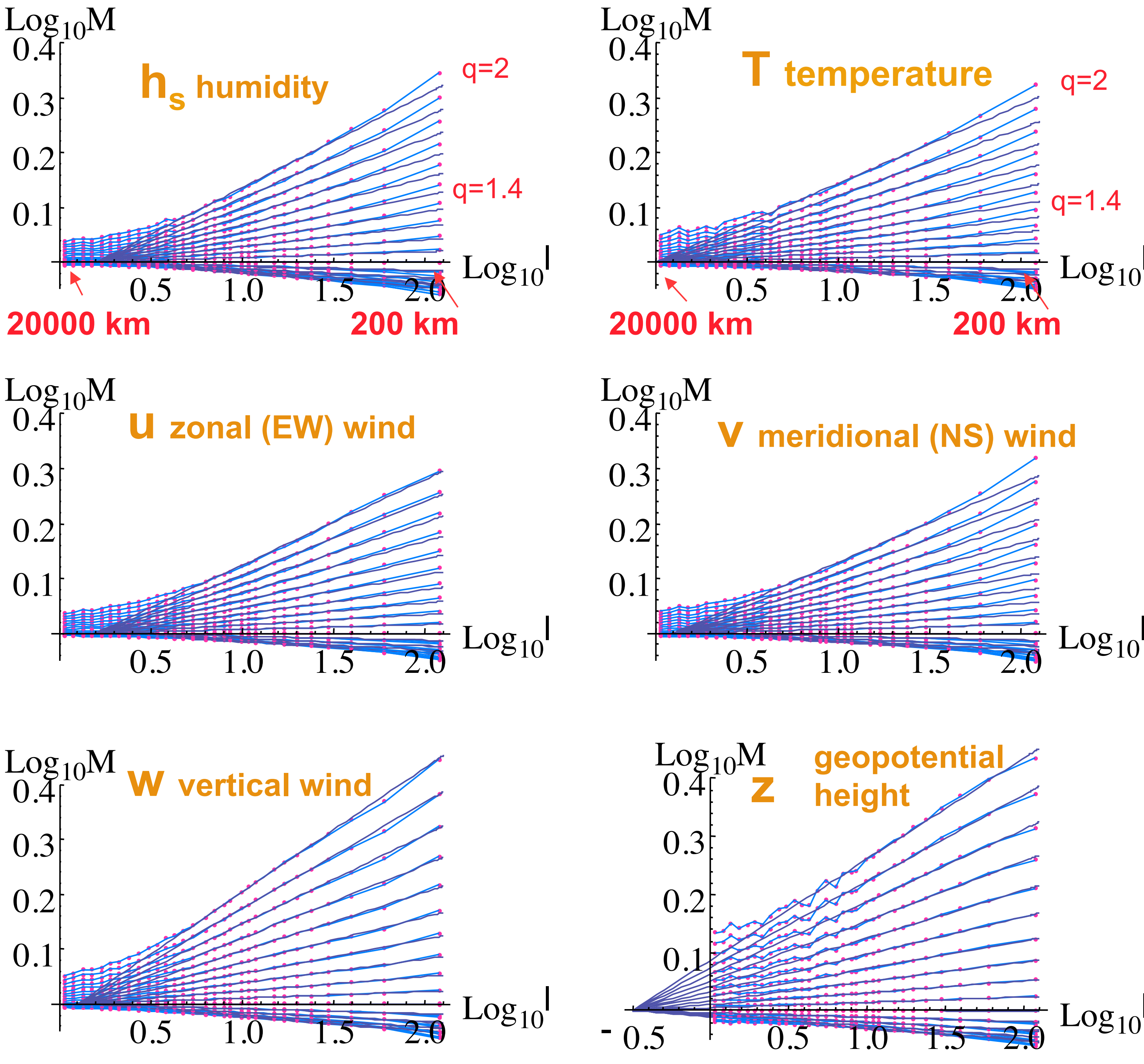
Quantitatively:
They are all different: which one is right?

$M = \langle \phi_\lambda^q \rangle / \langle \phi \rangle^q$

ECMWF reanalysis

East-West

(2006, 0Z, 700 mb)

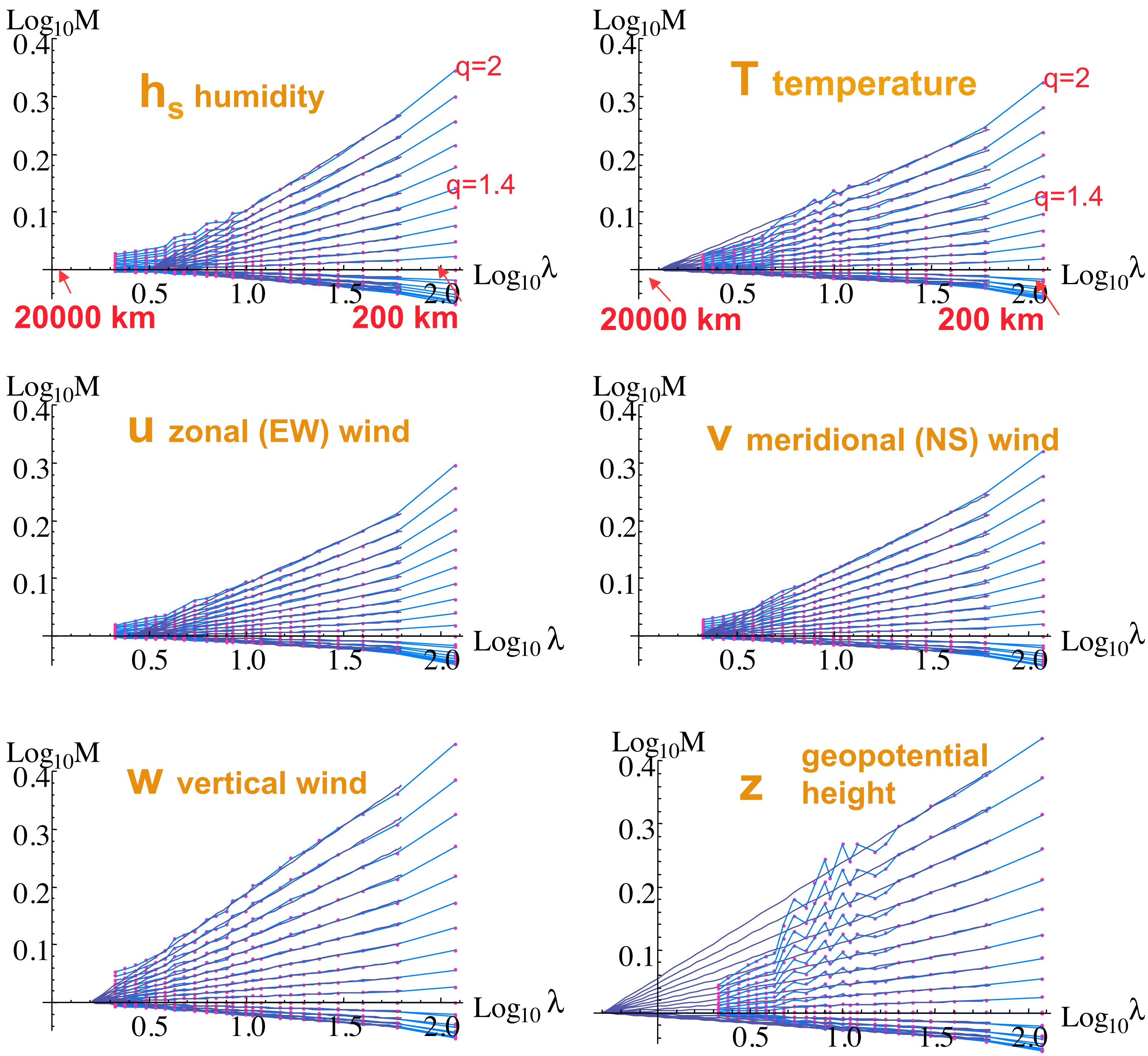


$M = \langle \phi_\lambda^q \rangle / \langle \phi \rangle^q$

ECMWF reanalysis

North-South

(2006, 0Z, 700 mb , ±45°)



Spatial Scaling: Comparison other geofields

		C_1	α	H	β	L_{eff} (km)
State variables	*u*, *v*	0.09	1.9	1/3, (0.77)	1.6, (2.4)	(14000)
	w	(0.12)	(1.9)	(-0.14)	(0.4)	(15000)
	T	0.11, (0.08)	1.8	0.50, (0.77)	1.9, (2.4)	5000 (19000)
	h	0.09	1.8	0.51	1.9	10000
	z	(0.09)	(1.9)	(1.26)	(3.3)	(60000)
Precipitation	*R*	0.4	1.5	0.00	0.2	32000
Radiances	Infra Red	0.08	1.5	0.3	1.5	15000
	visible	0.08	1.5	0.2	1.5	10000
	Passive microwave	0.1-0.26	1.5	0.25-0.5	1.3-1.6	5000-15000
Topography	Altitude	0.12	1.8	0.7	2.1	20000

C_1: Sparseness of mean

α: Index of multi-fractality

H: Scale by scale conservation

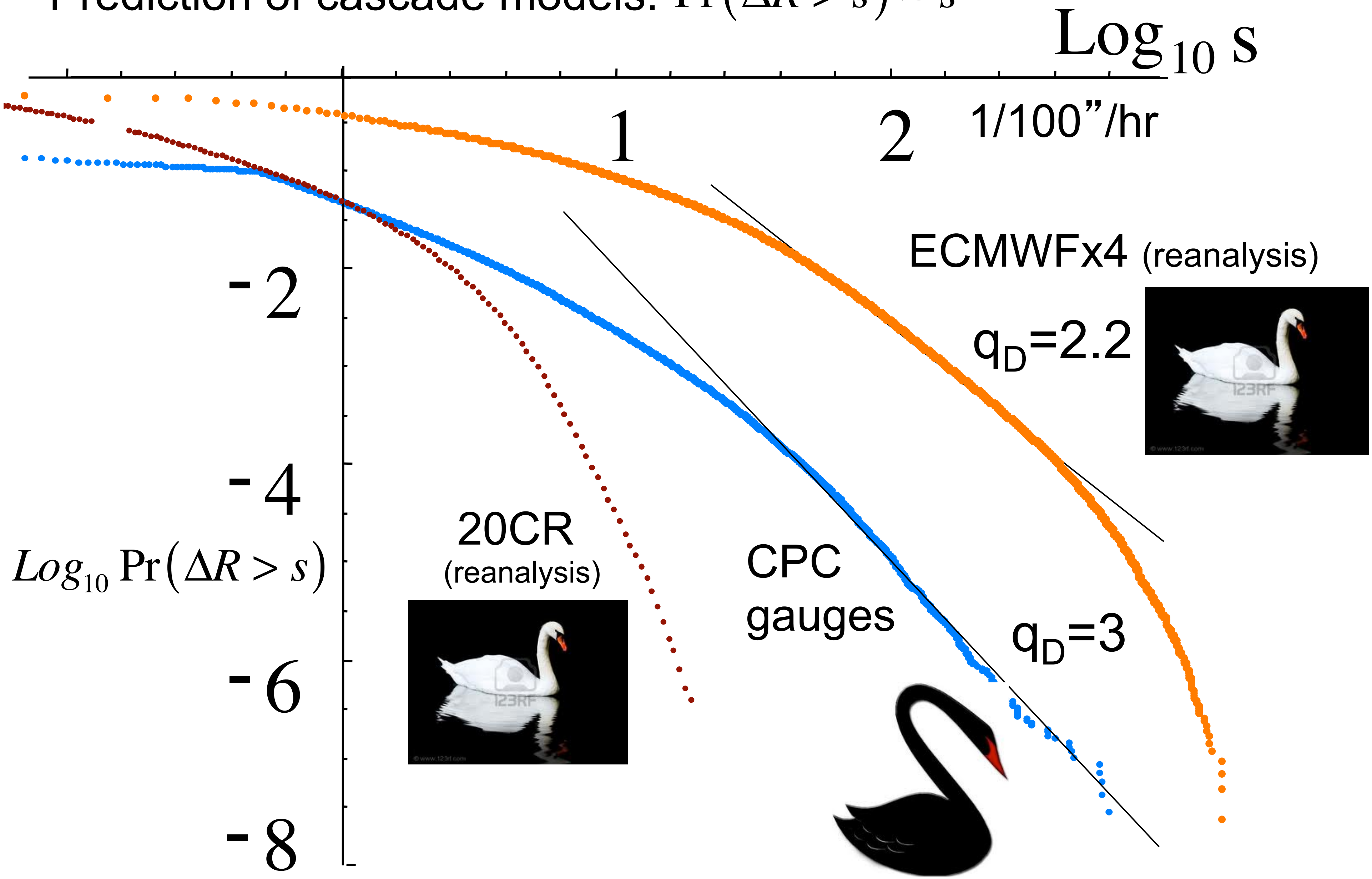
β: Spectral exponent

L_{eff}: Effective External scale

Parentheses = reanalysis values

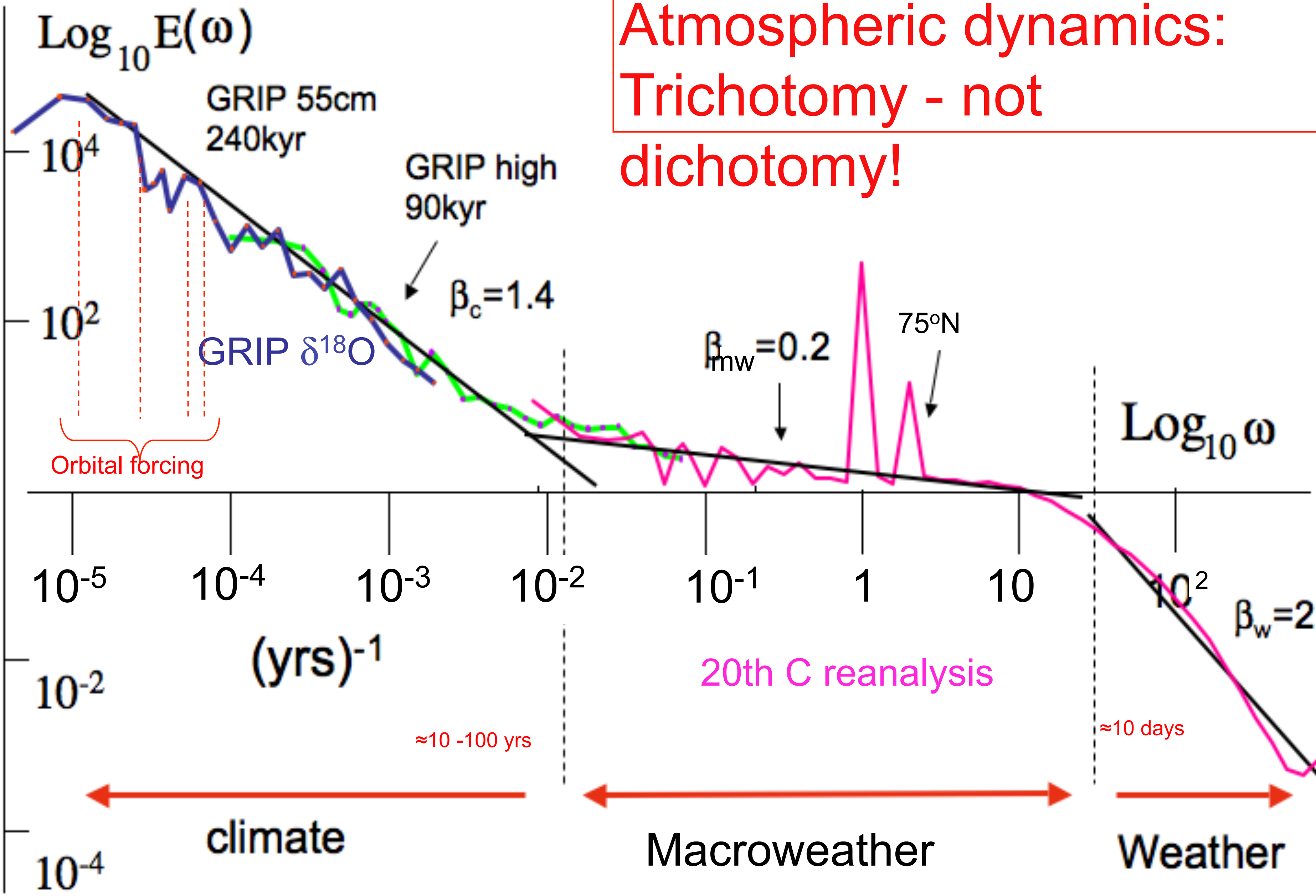
Extremes

Prediction of cascade models: $\Pr(\Delta R > s) \approx s^{-q_D}$



Temporal structure

Atmospheric dynamics: Trichotomy - not dichotomy!

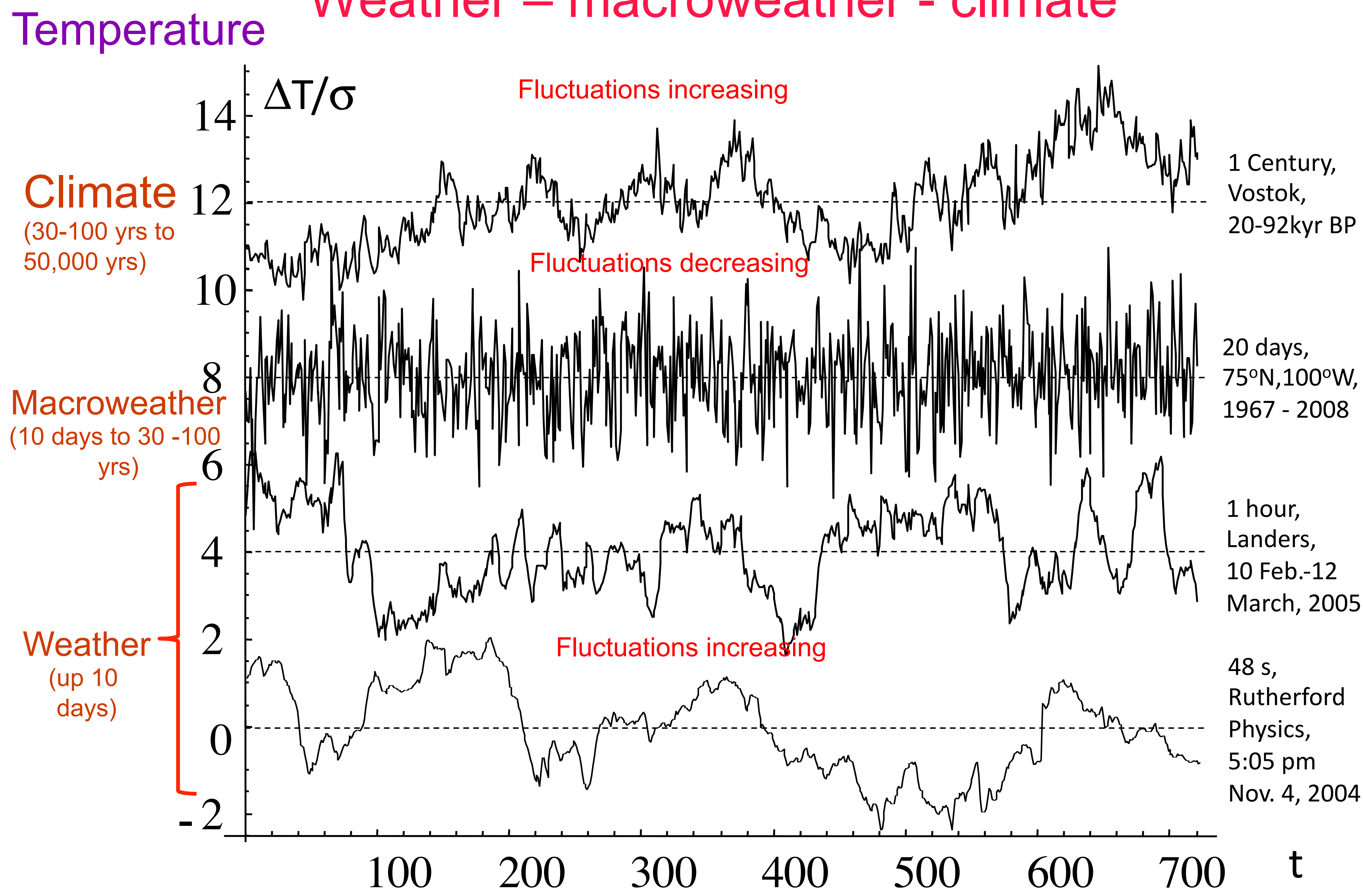


Two data sources only GRIP, 20CR

Lovejoy and Schertzer 2011

Trichotomy:

Weather – macroweather - climate



Basic characteristics of the three regimes

$$\langle \Delta I \rangle = \langle \varphi \rangle \Delta t^{H}$$

Fluctuation

= constant

"Climate is what you expect, weather is what you get."
-Lazarus Long, **character in R. Heinlein 1971**

Weather:

$\Delta t < \tau_w$ ($\approx$ 10 days): H>0,

Fluctuations grow with scale "unstable"

"...Weather is what you get"

Macroweather:

(10 days $\approx$) $\tau_w < \Delta t < \tau_c$ ($\approx$ 10- 100 yrs): H<0,

Fluctuations diminish with scale;
atmospheric states are "stable".

"Macroweather is what you expect..."

Climate:

(10- 100 yrs $\approx$) $\tau_c <\approx \Delta t <\approx$ 100 kyrs: H>0,

Fluctuations grow with scale; atmospheric states are "unstable",
subject to "climate change".

"The climate is not what you expect..."

Real space analysis

Range of exponents over which average fluctuations at scale Δt corresponds to frequency 1/Δt

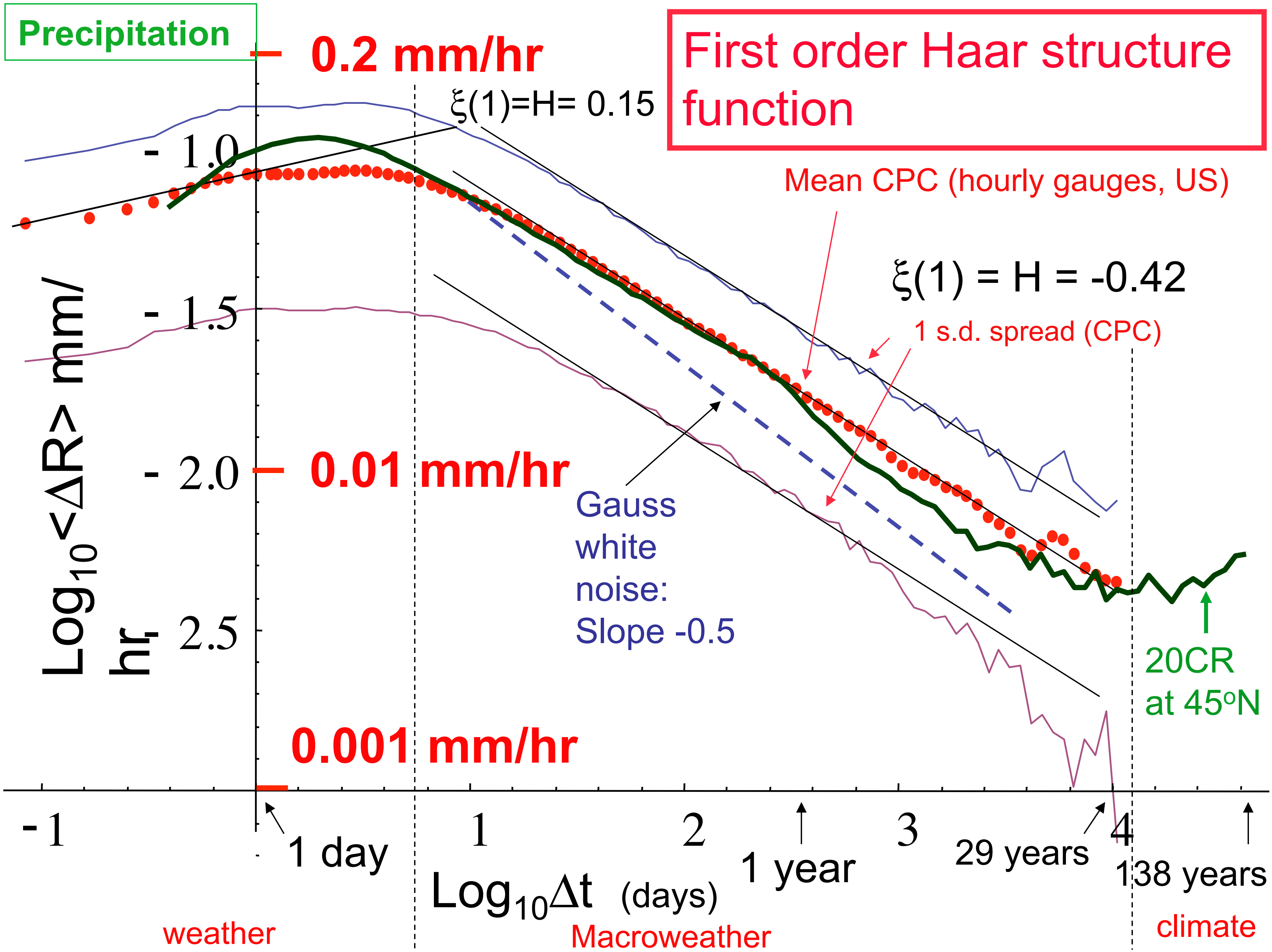
Fluctuation → $\langle \Delta I \rangle = \langle \varphi \rangle \Delta t^{H}$ ← = constant

$E(\omega) = \left\langle \left| \tilde{I}(\omega) \right|^2 \right\rangle = \omega^{-\beta}$

$\beta = 1 + 2H - K(2)$ ← Multifractal "correction"

Statistic	Range of H	Range of β	Comment
Spectrum	$-\infty < H < \infty$	$-\infty < \beta < \infty$	$E(\omega) \approx \omega^{-\beta}$
Difference	0<H<1	1<β+K(2)<3	"Poor man's wavelet"
Tendency Fluctuation	-1<H<0	-1<β+K(2)<1	Average with overall mean removed (standard deviation= "Climactogram", also called the "Aggregated Standard Deviation")
Haar	-1<H<1	-1<β+K(2)<3	Difference of means of first and second halves of interval
Detrended Fluctuation Analysis (DFA, polynomial order n	-1<H<(n+1)	-1<β+K(2)<3+2n	Also multifractal extension (MFDFA), usually linear: n=1, Not a wavelet
Mexican Hat Wavelet	-1<H<2	-1<β+K(2)<5	2nd Derivative of a Gaussian
Generalized Haar	-m<H<n	1-2m<β+K(2)<3+2n	Interpretation not simple

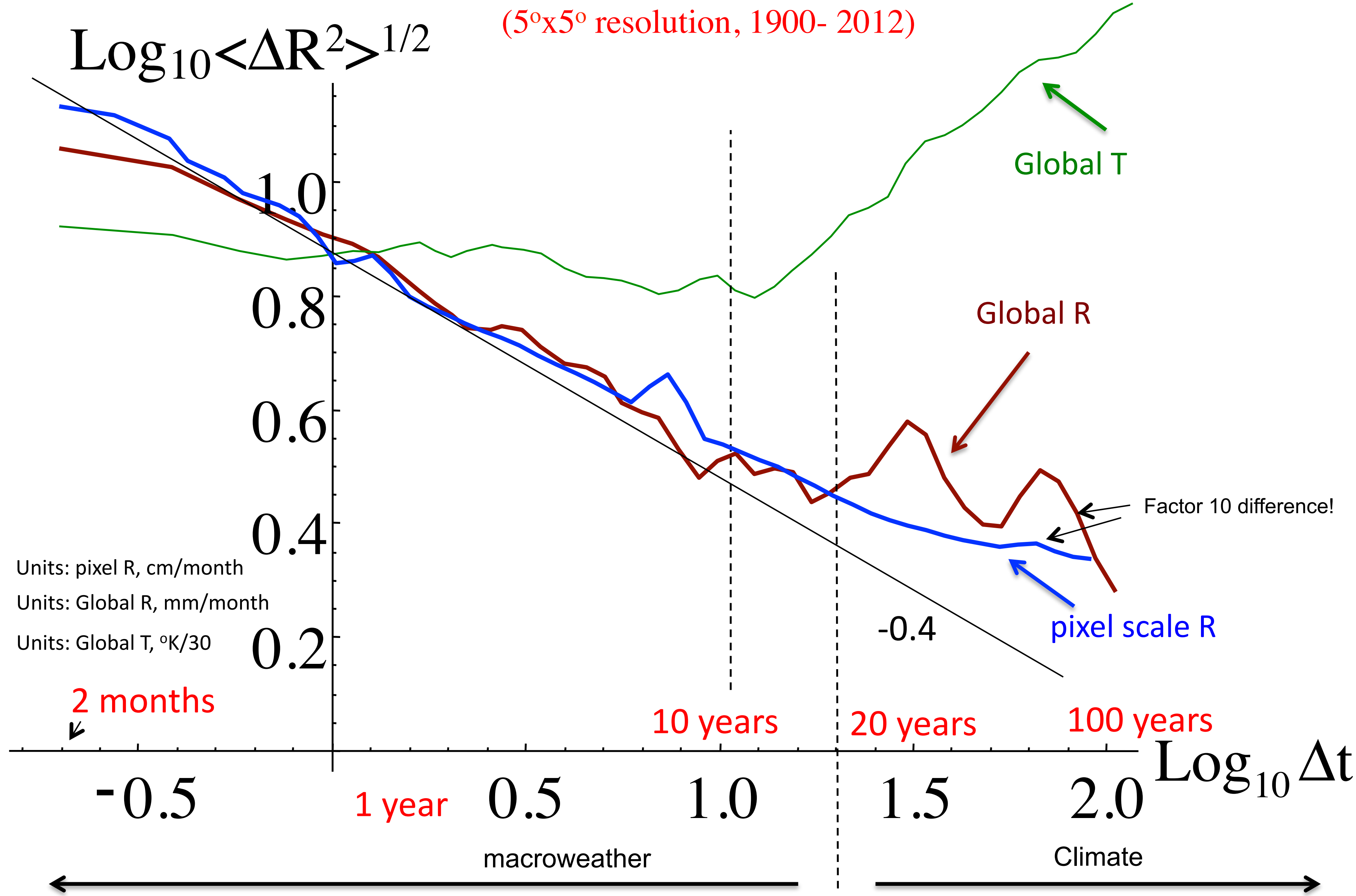
Simple interpretation (Difference, Tendency Fluctuation, Haar)

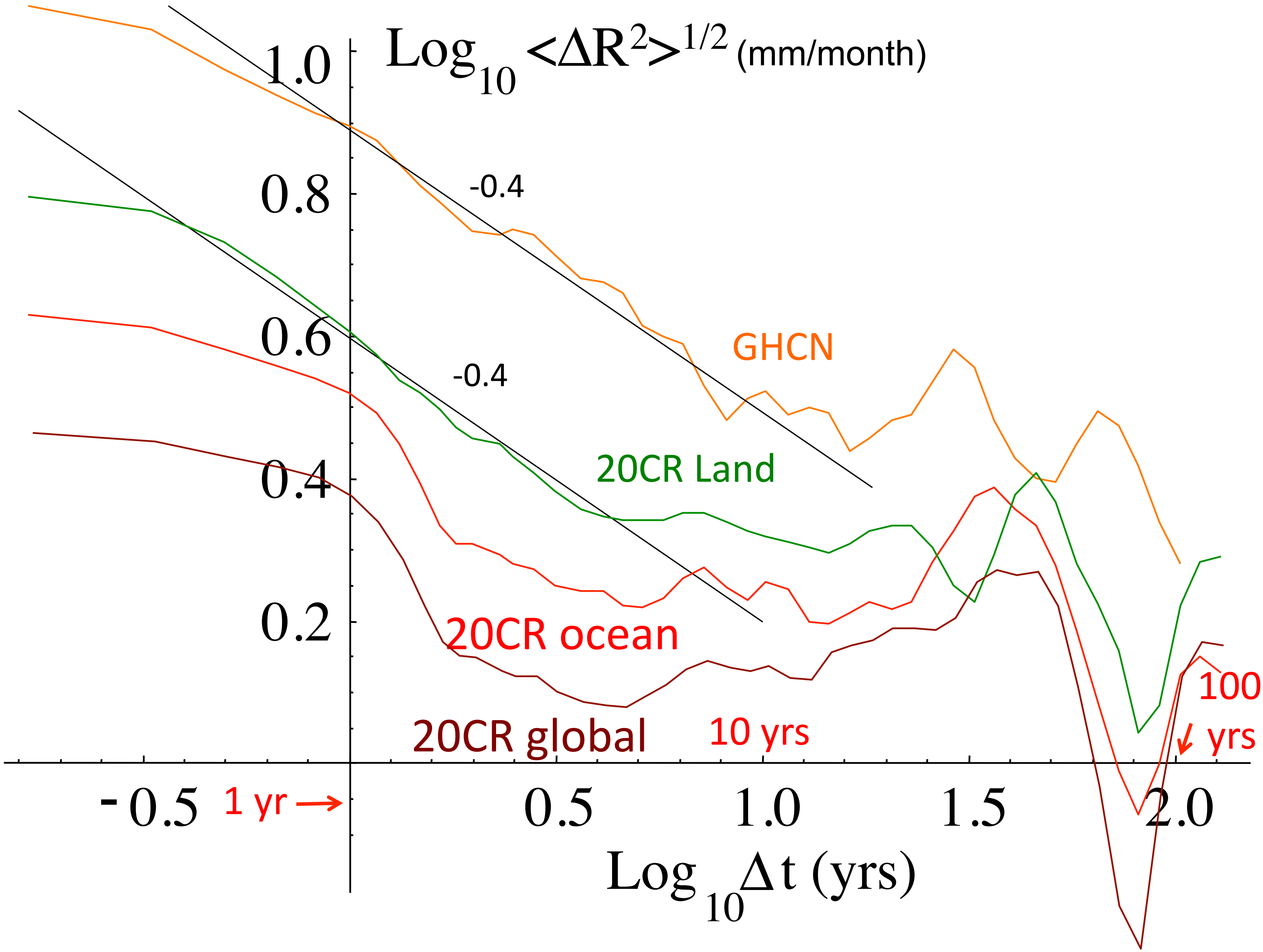


Precipitation
First order Haar structure function
0.2 mm/hr
0.01 mm/hr
0.001 mm/hr
ξ(1)=H= 0.15
ξ(1) = H = -0.42
Mean CPC (hourly gauges, US)
1 s.d. spread (CPC)
Gauss white noise: Slope -0.5
20CR at 45°N
- 1.0
- 1.5
- 2.0
- 2.5
Log10<ΔR> mm/hr
-1
1
2
3
4
1 day
1 year
29 years
138 years
Log10Δt (days)
weather
Macroweather
climate

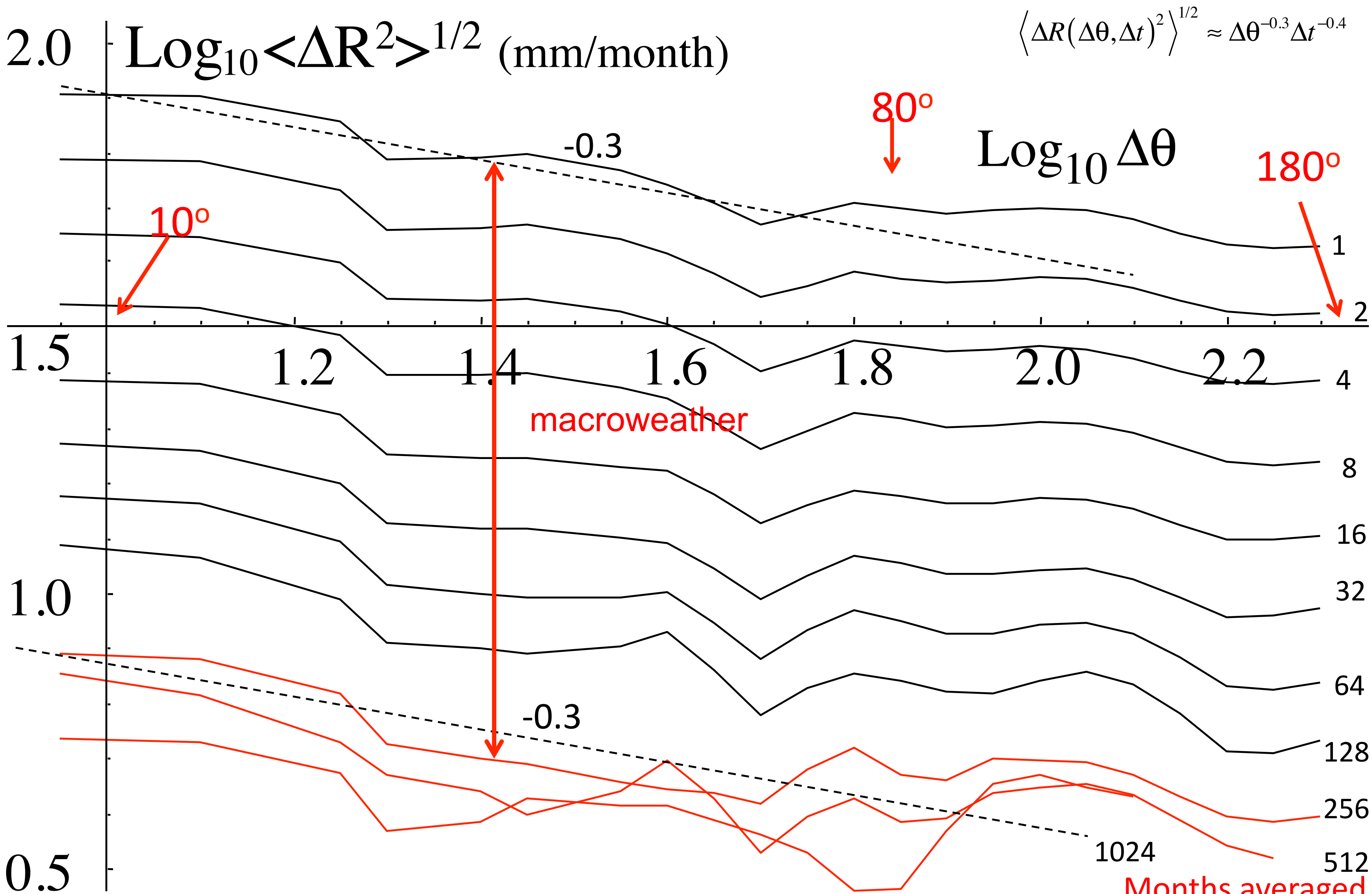
Precipitation and temperature:

GHCN=Global Historical Climate Network
(5°x5° resolution, 1900- 2012)




Log10 <ΔR2>1/2 (mm/month)
1.0
0.8
0.6
0.4
0.2
-0.4
-0.4
GHCN
20CR Land
20CR ocean
20CR global
10 yrs
100
yrs
-0.5
1 yr
0.5
1.0
1.5
2.0
Log10 Δt (yrs)

GHCN in EW direction for increasing averaging times (anomalies)



Conclusions

High level (emergent) turbulent space-time laws

Precipitation as a turbulent process

Cascades:

-Multiplicative Cascades in space-time, data, models, reanalyses

-Cascades are Anisotropic: vertical and horizontal cascades are different.

-Power law extremes

Temporal scaling trichotomy:
weather-macroweather-climate

Applications

-Stochastic space-time precipitation modelling
-Solving the problem of measuring areal precipitation
-Improving numerical models (of atmosphere and hydrology)
-climate, climate change, anthropogenic effects