Evaluation of conservative (H=0) multifractal simulations using Mathematica software:

Continuous in scale multifractal simulations have "finite size" issues for the smallest and largest scales, meaning that the scaling is not so well respected at the smallest and largest scales. Some of the small scale issues are dealt with in [*Lovejoy and Schertzer*, 2010a], [*Lovejoy and Schertzer*, 2010b] (http://www.physics.mcgill.ca/~gang/eprints/eprintLovejoy/neweprint/Continuous.multifractals.partI.3.9.10.pdf, http://www.physics.mcgill.ca/~gang/eprints/eprintLovejoy/neweprint/Continuous.multifractals.partII.3.9.10.pdf) where there are evaluations of the quality of the simulation statistics notably using spectra, but also trace moments. These publications describe the basis for the simulation software on this site.

As a further demonstration of the accuracy of the software, we provide the results of analyses of ensembles of 10 simulations of 256x256, causal 2-D. The simulations were analysed using Haar graphical method (see the Haar fluctuation analysis software), with the parameters estimated from regressions fit from 3rd smallest interval to 5th largest interval (scales 8 to 140 pixels). The simulations were in causal mode, the analyses were mostly in the space direction (considering each time step as a separate realization), although see the bottom table for the same $C_1 = 0.2$ analyses in the time direction (the parameters are a little better). The figures with error bars are for the exponents fit on each realization separately, the "ens" figure is for a single fit to the ensemble of 10 realizations.

In general, the H's are a little too high (by about 0.05), the C_1's a little too low (by 0.01 to 0.02), the α's are quite variable (± 0.1 to ± 0.3). It isn't clear whether the ensemble estimation method is better than the other individual realizations.

For other simulation evaluations (of the MatLab versions of the software – they should be equivalent – see: http://www.physics.mcgill.ca/~gang/software/doc/MultiFSimulation.software.pdf)

Results

	H	H_{ens}	C_1	C_{1ens}	α	α_{ens}
$C_1 = 0.1, \alpha=0.4$	0.072±0.095	0.085	0.11±0.049	0.089	0.58±0.11	0.22
$C_1 = 0.1, \alpha=0.8$	0.023±0.052	0.028	0.081±0.03	0.078	0.87±0.10	0.86
$C_1 = 0.1, \alpha=1.2$	0.009±0.039	0.011	0.081±0.028	0.077	1.13±0.27	0.011
$C_1 = 0.1, \alpha=1.5$	0.050±0.059	0.055	0.087±0.021	0.081	1.54±0.20	1.47
$C_1 = 0.1, \alpha=1.8$	0.041±0.033	0.061	0.093±0.028	0.084	1.85±0.24	1.81

Above has C_1 =0.1, 2-D causal simulations, analysis in the spatial direction. Note: the value of H should be zero in all cases.

	H	H_{ens}	C_1	C_{1ens}	α	α_{ens}
$C_1 = 0.2, \alpha=0.4$	0.113±0.085	0.12	0.195±0.05	0.181	0.365±0.087	0.29
$C_1 = 0.2, \alpha=0.8$	0.067±0.056	0.072	0.182±0.051	0.173	0.748±0.089	0.72
$C_1 = 0.2, \alpha=1.2$	0.017±0.020	0.018	0.157±0.021	0.153	1.16±0.021	1.13
$C_1 = 0.2, \alpha=1.5$	0.037±0.05	0.036	0.174±0.022	0.166	1.57±0.14	1.52
$C_1 = 0.2, \alpha=1.8$	0.045±0.034	0.047	0.190±0.027	0.185	1.81±0.17	1.79

Above has C_1 =0.2, 2-D causal simulations; the above table analyses the simulations in the spatial direction.

	H	Hens	C_1	C_1ens	α	α ens
$C_1 = 0.2, \alpha=0.4$	0.035±0.10	0.05	0.196±0.08	0.183	0.48±0.07	0.46
$C_1 = 0.2, \alpha=0.8$						
$C_1 = 0.2, \alpha=1.2$	-0.001±0.11	0.016	0.199±0.088	0.184	1.31±0.36	1.18
$C_1 = 0.2, \alpha=1.5$						
$C_1 = 0.2, \alpha=1.8$	0.018±0.06	0.024	0.20±0.032	0.197	1.72±0.17	1.74

The same causal simulations as in the table above (with C_1 =0.2) but for analysis in the time direction. Notice that in the time direction, the C_1 and H are more accurately reproduced than in the space direction.

References:

Lovejoy, S., and D. Schertzer (2010a), On the simulation of continuous in scale universal multifractals, part I: spatially continuous processes, *Computers and Geoscience*, *36* doi: 10.1016/j.cageo.2010.04.010.

Lovejoy, S., and D. Schertzer (2010b), On the simulation of continuous in scale universal multifractals, part II: space-time processes and finite size corrections,, *Computers and Geoscience*, *36*, 1404-1413 doi: 10.1016/j.cageo.2010.07.001.