Our Evolving Universe
Assignemnt 3 Solutions

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I'll present ***brief*** solutions to the wonderful questions for "Our Evolving Universe".

Problem 1

In the center of our Milky Way galaxy there is a compact object of mass $10^6\ M_0$, where M_0 is the solar mass. This object does not emit any light. How could we know it is actually present?

Sol: We can infer its existence due to the motion of luminous objects (eg other stars) near the galactic centre. Since using Newton's gravitational law and the centripetal force expressions, the velocity of an object is proportional to the mass enclosed by its radius. This can be seen by

$$F_c = \frac{mv^2}{r} \qquad\qquad F_g = \frac{GmM}{r^2}$$

Here G is Newton's constant, r is the radius or distance from the centre of the galaxy, m is the mass of a luminous star and M is the mass enclosed by the circular orbit. Equating them you can rearrange to find

$$v = \sqrt{\frac{GM}{r}}$$

So the velocity depends on the mass inside the orbit of the star. If it moves faster than expected then we can imply some extra mass that is non-luminous (for example, a black hole or dark matter).

Problem 2

If the object described in the previous problem is a black hole, then what is its maximal radius? Hint: In class I discussed the Newtonian argument for the existence of a black hole based on the escape velocity from the surface of the object not being allowed to be larger than the speed of light. For the sun, this gave a radius of about 10km

Sol: Lets consider the gravitational potential energy and kinetic energies of an object

$$\text{PE}_g = \frac{GMm}{r} \qquad\qquad \text{KE} = \frac{1}{2}mv^2$$

Now imagine we are at a point where we start at no velocity, and we want to escape to an infinite distance from the black hole. The energy is conserved, and so the potential energy near the black hole must be equal to the kinetic energy very far away from it at a later time. Note that at large distances (large r) the potential energy gets very small and so its approximately 0. Mathematically we can equate and find

$$\text{PE}_{before} = \text{KE}_{after} \qquad \longrightarrow \qquad \frac{GMm}{r} = \frac{1}{2}mv^2$$

Now since the velocity can never be more than the speed of light, to find the radius at which things are trapped forever we can set $v \to c$ where c is the speed of light. This radius (the size of the black hole) is then

$$r = \frac{2GM}{c^2}$$

Now, plugging in numbers

$$M = 10^6 M_0 \qquad\qquad M_0 = 1.989 \cdot 10^{30} \text{ kg}$$
$$c = 3 \cdot 10^8 \text{ m/s} \qquad\qquad G = 6.67 \cdot 10^{-11} \text{ m}^3/\text{kg s}^2$$

Plugging these in yields

$$r \approx 3 \cdot 10^9 \text{ m}$$

Problem 3

An astronomer observes a bright star close to the center of the Milky Way and takes the spectrum of the light. The spectrum is similar to the spectrum of the sun, except that there are deep absorption lines at the frequency of a Level 1 hydrogen transition line. How would you interpret this result? Why is there this absorption line?

Sol: The properties of absorption lines tell you something about what material is between you and the source. Stars create a thermal and continuous spectrum of radiation (following a so-called blackbody distribution). An absorption line implies that hydrogen in the ground state has absorbed these photons in the spectrum with energy equal to that of hydrogen in the ground state and the first excited state.

This line can be interpreted as a photosphere that contains neutral hydrogen (the photosphere is a plasma that exists in the outer layers of a stellar atmosphere). It could also be interpreted as the existence of some type of hydrogen gas that lies between us on earth and the star we are observing.

Problem 4/5

An astronomer observes a quasar (a quasar is a compact object which emits an amount of light comparable to that of all of the stars in a galaxy), takes its spectrum and observes that there are emission line peaks in the spectrum whose frequencies oscillate about the mean value by 1%. What could the source of these oscillations be? Give both a qualitative and a quantitative answer.

Sol: Consider a Quasar as an object with a high-velocity rotational disk. The variations of the frequencies about the mean would correspond to the Doppler shift of the frequencies due to the relative velocities of the particles emitting the peaked spectrum. A Doppler shift can be understood as the compression and rarefaction (or stacking vs stretching) of the wavelength of light emitted by a source that is moving either toward or away us. An object moving quickly away from us will appear redder (the wavelengths are stretched), while one moving towards us will be blueshifted (compressed wavelengths). This would cause us to detect a variable peak of emission (also known as Doppler broadening) from a Quasar (or any object with high speed rotations).

Quantitatively, consider a rest wavelength emitted from some particle of 600 nm. A 1 % fluctuation about this mean would correspond to us observing wavelengths ranging from

$$594 \text{ nm} \leq \lambda \leq 606 \text{ nm}$$

The Doppler shift is given by the expression

$$\lambda_{obs} = \lambda_e \left(1 + \frac{v}{c}\right)$$

Where λ_{obs} is the observed wavelength of light and λ_e is the rest wavelength or emitted wavelength of light from the particle. From this we can calculate the velocity of this outer rotating disk! Solving for v we find

$$v = 3 \cdot 10^6 \text{ m/s}$$

This answer is of course true regardless of the rest wavelength we choose, and is just dependent on the oscillations about the mean.

Problem 6

Much of the Orion Nebula looks like a glowing cloud of gas. What type of spectrum would you expect to see from the glowing nebula and why?

Sol: The Orion Nebula would be emitting an emission spectra, or individual lines corresponding the the atomic excitations of whatever particles make up the gas. This would look like a bunch of discrete lines. The temperature of the gas would dictate the specific lines observed, as a hotter gas can potentially excited electrons to higher energy levels (producing more low frequency radiation the hotter it gets). This happens as long as the gas itself doesn't become ionized (since there are no discrete lines when the electrons are not bound to protons).

Problem 7

What is the energy (in Joules) of an ultraviolet photon with wavelength of 120nm?

Sol: Recall the expressions

$$E = h\nu \qquad \nu = \frac{c}{\lambda}$$

Where E is the energy, h is Planck's Constant, and ν is the wavelength of light. Plugging in

$$h = 6.63 \cdot 10^{-34} \text{ m}^2 \text{ kg/s} \qquad \lambda = 1.2 \cdot 10^{-7} \text{ m}$$

Plugging in we find

$$E = 1.66 \cdot 10^{-18} \text{ J}$$

Problem 8

Suppose that all the energy from a 100 Watt light bulb came in the form of photons with wavelength 600nm. a) Calculate the energy of a single such photon. b) How many of such photons must be emitted each second in order to account for all of the energy released? c) Based on your answer to b), explain why we do not observe the particle nature of this light.

Sol: A 100 Watt light bulb is the same as saying that the lightbulb emits 100 Joules per second (J/s)

a) The energy of a single such photon is

$$E = h\nu = \frac{hc}{\lambda}$$

Plugging in the the numbers from the problem yields

$$E = 3.3 \cdot 10^{-19} \text{ J}$$

b) The number of photons emitted per second can be found by the formula

$$N = P \cdot 1 \text{ s } \cdot \text{E}^{-1}$$

The units all cancel and the number of photons created per second is

$$N = 3 \cdot 10^{20} \text{ Photons}$$

c) With this many particles being created each second, you would see a continuous stream of photons, like a wave. The discrete nature of each photon would be hidden by the sheer number of particles.

Problem 9

A traditional incandescent light bulb uses a hot tungsten coil to produce a thermal radiation spectrum. The temperature of this coil is typically about 3000K. a) What is the wavelength of maximal intensity? Compare with the 500nm wavelength of maximal intensity for the sun. b) Do you expect the light from the lightbulb to be redder or more blue compared to the light from the sun? c) Do these light bulbs emit all of their energy as visible light? d) Fluorescent light bulbs emit emission primarily at fixed wavelengths as a line spectrum. Why? If the wavelength of such a bulb is in the visible range, then does a fluorescent light bulb emit more or less light than a traditional light bulb with the same voltage?

Sol: The radiation spectrum is thermal which means that it follows a blackbody distribution with a temperature of 3000 K.

a) The wavelength of maximal intensity is given by Wien's law, which is

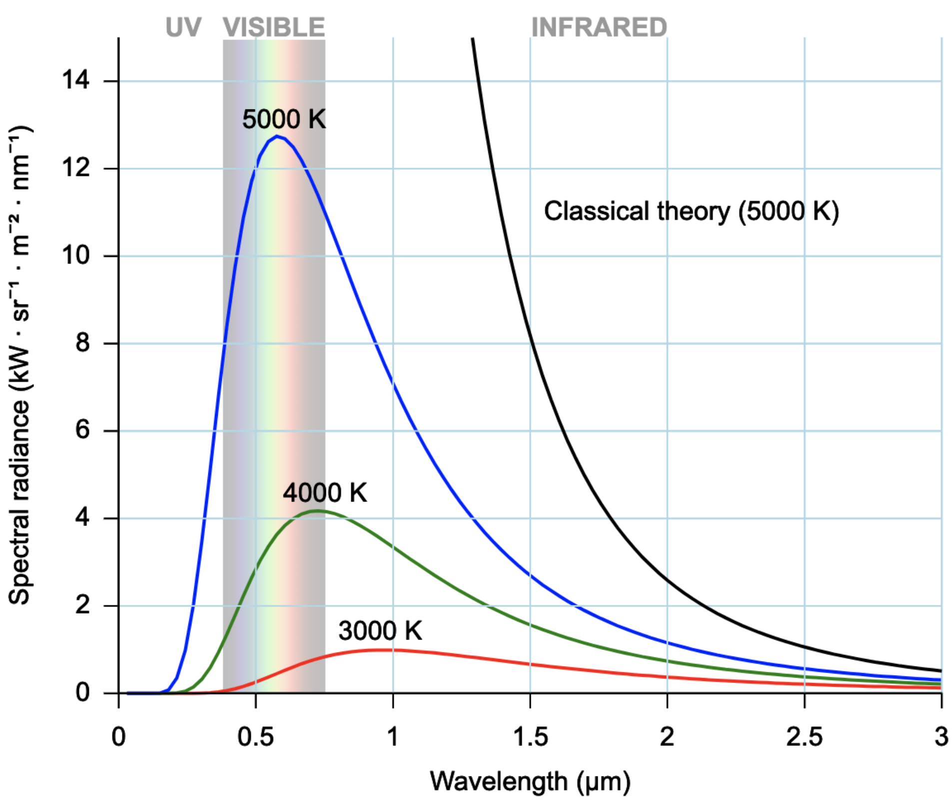
$$\lambda_{max} = \frac{2.897 \cdot 10^{-3}\ \text{m} \cdot \text{K}}{T}$$

Where the m and K in the numerator are the units (not input variables) of meters and Kelvin. With our temperature we get

$$\lambda_{max} \approx 966\ \text{nm}$$

This is a *longer* wavelength than that of the sun

b) Radiation that is thermal follows a blackbody distribution. The temperature of the object sets the radiation spectrum itself. See the below figure (credit to wikipedia)



The peak of the radiation is more red compared to the sun, and so we expect the light from the lightbulb to be redder than that of the sun.

c) No! From the above figure we can see that only a small fraction of the light is emitted in the visible spectrum! In fact, most of its light is emitted in the infrared.

d) Fluorescent lamps emit light primarily via the excitation of a mercury vapour contained within the bulb. The relaxation of the mercury back to its ground state causes the discrete line spectrum. If we have this discrete spectrum within the visible range, then for a same voltage and current bulb, the fluorescent lamp will be much brighter since all of the photons will be emitted in the visible range (as opposed to the small fraction in the scenario with an incandescent bulb)

Problem 10

. Suppose you were looking at our own solar system from a distance of 10 lightyears. a) What angular resolution would you need to see the sun and Jupiter as distinct points of light? b) What angular resolution to see the Earth and the sun as distinct points of light? c) How does the angular resolution you obtain in parts a) and b) compare with the resolution of the Hubble Space Telescope? d) Comment on the challenge of making images of planets around other stars.

Sol: The angular separation of two objects is given by the formula

$$\theta = \frac{a}{D}$$

Where a is orbital distance between two objects and D is the distance from the observer to the source. The corresponding answer is given in radians, and the formula can be derived by drawing the corresponding triangle for the three points in the problem.

a) Jupiter has a semi-major axis of

$$a = 5.2 \text{ AU}$$

Where an AU is the distance between the earth and the sun (astronomical unit). The observation distance is 10 light years, which is 632411 AU. Therefore the angular separation is

$$\theta = \frac{5.2}{632411} = 8.22 \cdot 10^{-6} \text{ Rad}$$

We want this in Arcseconds since that is what telescopes generally measure their resolution in. There are 360 degrees in $2pi$ radians, 60 arcminutes (') in one degree, and 60 arcseconds (") in one arcminute. The conversion formula from radians to arcseconds is therefore

$$1 \text{ rad} = \frac{360}{2\pi} \text{ (deg)} \cdot \frac{60\ '}{1 \text{ deg}} \cdot \frac{60\ ''}{1\ '}$$
$$1 \text{ rad} = \frac{360 \cdot 3600}{2\pi}\ ''$$

So the angular separation (and as well, the angular resolution necessary) in arcesconds is

$$\theta = 8.22 \cdot 10^{-6} \text{ Rad} \cdot \frac{360 \cdot 3600}{2\pi} \frac{''}{\text{Rad}} = 1.7\ ''$$

b) Now we just replace the Jupiter parameters with that of earth. The angular resolution necessary is

$$\theta = \frac{1}{632411} \text{ Rad} \cdot \frac{360 \cdot 3600}{2\pi} \frac{''}{\text{Rad}} = 0.33\ ''$$

c) The Hubble telescope has an angular resolution of about 0.1 ", and so if we were observing a system 10 light years away, we could resolve both the earth and Jupiter using this device.

d) HST is a great instrument, but we must note that 10 light years is still very near to us. For example, the Milky Way is 105,700 light years in diameter! By looking at the angular resolution formula, we can note that we would stop directly resolving earthlike orbiting planets at a distance of about 50-100 light years, and Jupiterlike orbiting planets at a stellar distance of about 500-1000 light years away. This is a main reason as to why astronomers use other techniques (such as searching for transits, or 'eclipses' of the star by the planet) when searching for exoplanets very far away.