

Readings and Homework Set 10

Readings: Chapter 10.

Problems, due Dec. 1, in my mail box:

1. Consider a string of length L with constant mass per unit length and constant tension. Determine the allowed values of the wave number k for the following boundary conditions:

- a) Fixed boundary conditions at $x = 0$ and free boundary condition at $x = L$.
- b) Free boundary conditions at both ends.

In both cases, write down standing wave solutions.

2. Consider a string of length L whose spatial boundaries are not fixed. Consider the action principle derivation for the string done in class. What boundary conditions for the string follow from the action principle? Give the derivation.

3/4. (20 points) Consider a string of length L with constant mass per unit length ρ and a tension which very mildly depends on x :

$$\tau(x) = \tau_0 + \epsilon x$$

where τ is a constant and ϵ is a positive constant satisfying $\epsilon L \ll 1$

- a) What is the general equation of motion for the displacement $q(x)$?
- b) What approximation method would you suggest to solve this equations? Hint: to back to earlier chapters of the class.
- c) Write down a simple but non-vanishing solution of the zero'th order equation (the equation you get for $\epsilon = 0$).
- c) What is the equation of motion which yields the first order solution for the same initial conditions which your zero'th order solution satisfies?
- d) (extra credit) Find the first order solution.

5. Textbook, Exercise 10.20.

6. At a polar angle θ , a projectile is fired eastward with speed v_0 at an angle α above the ground. Show that the southward (in the northern hemisphere) and eastward deflections due to the Coriolis force are (to first order in ω)

$$d_{south} = (4\omega v_0^3/g^2)\cos(\theta)\cos(\alpha)\sin^2(\alpha),$$

$$d_{east} = (4\omega v_0^3/g^2)\sin(\theta)(\cos^2(\alpha)\sin(\alpha) - 1/3\sin^3(\alpha)).$$

Hint: The first term in d_{east} arises because the flight time is modified due to the vertical component of the Coriolis force.