PHYS 514 GENERAL RELATIVITY AND COSMOLOGY 2018
READING and PROBLEM SET 9

READING: Textbook, Chapter 8, Sections 8.5 - 8.8, Chapter 9 (optional)

PROBLEMS, due TUESDAY, April 10 2018 (in class):

1. In class I discussed the Shapiro time delay. Starting point was a sketch showing the trajectories of light in flat space-time and in curved space-time. I then sketched the computation of the time delay. As I mentioned, there is an inconsistency between the computation and the sketch. What is this inconsistency? Try to fix the error, and show that to leading order in m, the results of the improved calculation agree with what was obtained in class.

2. Our current universe appears to be dominated by a cosmological constant. Compute the lifetime of our universe assuming that today (when the Hubble expansion rate is $h \times 100 \mathrm{km} s^{-1} \mathrm{Mpc}^{-1}$ with $h \simeq 0.7$) 70% of the energy is in the form of the cosmological constant and 30% is in the form of cold matter.

3. In class I wrote down the equation of motion for a scalar field in an expanding flat FRW Universe. I also mentioned what is meant by the *slow rolling approximation*. Consider now a homogeneous scalar field in the chaotic inflation model with potential

$$V(\phi) \; = \; \frac{1}{4}\lambda\phi^4.$$

Show that for sufficiently large values of ϕ, the slow rolling equation is self consistent and find the limiting value of ϕ for which this ceases to be true.

4. Textbook, Problem 8.5

5/6. Consider a contracting matter-dominated universe which tends to a Big Crunch singularity at time $t = 0$.
a) Plot the time evolution of the Hubble radius, the particle horizon and the wavelength of a fixed comoving scale.
b) Consider vacuum initial conditions for cosmological fluctuations and compute the power spectrum of the fluctuations on super-Hubble scales close to the bounce point.
c) Compare with the predictions of inflationary cosmology.
NB: Attempt only after the lecture of April 3.