

Problem Set 5 - for Week 5

1. Consider a cosmological model in which there is a period of primordial inflation after which the universe is dominated by radiation. In class I computed the spectrum of cosmological perturbations on super-Hubble scales at the end of inflation. How does this spectrum evolve in the radiation phase? How would the spectrum evolve if the universe were dominated by cold matter after inflation?

2.. Consider a contracting, matter dominated universe, conformal time decreasing from $-\infty$ to 0. At $\eta = 0$ there is a Big Crunch singularity. Assume that fluctuations originate as quantum vacuum perturbations at very early times in the contracting phase. Compute the power spectrum of curvature perturbations shortly before the Big Crunch singularity on super-Hubble scales.

3. In class I discussed the quantum theory of cosmological perturbations and applied the analysis to inflationary cosmology. Compute the Bogoliubov coefficients α_k and β_k at time η for modes whose wavelength is larger than the Hubble radius.

4. Verify that the Bogoliubov coefficients obey the relation

$$|\alpha_k|^2 - |\beta_k|^2 = 1$$

5. Consider the contracting model of Problem 2. Assume that both cosmological fluctuations and gravitational waves start out in their vacuum state at very early times. Compute the tensor to scalar ratio r close to the Big Crunch on super-Hubble scales? Discuss why the initial conditions which I am discussing are well defined.

6. Consider a contracting universe in which matter is dominated by a substance with equation of state parameter $w \gg 1$. Compute the spectra of cosmological perturbations and gravitational waves at late times on super-Hubble scales, assuming vacuum initial conditions at very early times.