402-0389 Physics of the Very Early Universe 2019
Problem Set 6 - for Week 6

1/2. Consider the Mathieu equation discussed in class

$$\ddot{\chi}_k + (k^2 + g^2\sigma^2 \cos(mt))\chi_k \, = \, 0 \, ,$$

where σ is a mass scale, $g \ll 1$ is a dimensionless coupling constant and m is a frequency. Find the resonance bands of the system, i.e. the values of k for which the equation has exponentially increasing solutions.

3.. For an oscillating inflatonφ background (in the context of large field inflation), the equation of motion for a massless field χ coupled to φ as discussed in class is an equation of Mathieu type with a very large coupling constant. In this case there is broad parametric resonance. Using the adiabaticity condition discussed in class, determine the range of k values for which there is resonance.

4. In the above case, verify that there is a range of k values for which the expansion of space can be neglected.

5/6. In class I mentioned the tachyonic resonance which appears if the field χ is coupled to φ with a negative coupling constant, i.e. the interaction Lagrangian is

$$\mathcal{L}_I \, = \, \tfrac{1}{2} g \varphi^2 \chi^2 \, ,$$

where g is a positive constant. In order that the system is stable, one needs to assume the presence of a nonlinear term $\lambda\chi^4$ in the potential for χ. Assume that λ is a very small positive constant. Study the growth of fluctuations of χ in this model (neglecting the expansion of space), discuss what back-reaction effects need to be considered, and estimate how long the resonance of χ persists until back-reaction effects become important.