402-0389 Physics of the Very Early Universe 2019
Problem Set 8 - for Weeks 8/9

This week's homework set is special. I would like you to work out fluctuations in the *New Ekpyrotic Scenario* of hep-th/0702154. In this model there are two very weakly coupled scalar fields, both canonically normalized and with negative exponential potentials. The first one (ϕ) dominates the background density, the second one (ψ) acts as a spectator scalar field. The potential is

$$V(\phi,\psi) \;=\; -V_0 exp(-\sqrt{2/p}\frac{\phi}{m_{pl}}) - U_0 exp(-\sqrt{2/q}\frac{\psi}{m_{pl}})$$

1. Show that there is an exact solution of the homogeneous field equations in an expanding background with

$$a(t) \;\sim\; (-t)^{p+q}$$

$$\phi(t) \;=\; \sqrt{2p}m_{pl}\log(-\sqrt{\frac{V_0}{m_{pl}^2 p(1-3(p+q))}}t)$$

$$\psi(t) \;=\; \sqrt{2q}m_{pl}\log(-\sqrt{\frac{U_0}{m_{pl}^2 q(1-3(p+q))}}t)$$

2. Using the spatially flat gauge derive the equations of motion for the fluctuations of the scalar fields ϕ and ψ.

3. Show that the fluctuations of the ψ field acquire an almost scale-invariant spectrum. The approximation I want to use is that the background is dominated by ϕ. If it is not dominated by ϕ alone, then there will be an adiabatic direction σ in field space given by

$$\dot{\sigma} = \cos(\theta)\dot{\phi} + \sin(\theta)\dot{\psi}$$

with

$$\tan(\theta) \;=\; \frac{\dot{\psi}}{\dot{\phi}}$$

and an entropic direction s given by

$$\delta s \;=\; -\sin(\theta)\delta\phi + \cos(\theta)\delta\psi$$

and it is the δs flutuations which get an almost scale-invariant spectrum.

4. Show how the scale-invariant fluctuations of ψ induce a scale-invariant spectrum of curvature fluctuations.