Testing String Theory with Cosmological Observations?

CAP-CRM Prize Lecture

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McGill University

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Thanks to:

- Klaus Hepp (ETH Zürich)
- Konrad Osterwalder (ETH Zürich)
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- Jennie Traschen
- Raul Bott, Sidney Coleman (Harvard)
- Bill Press, Doug Eardley (Harvard)
- Neil Turok, Andy Albrecht
- Anne-Christine Davis (DAMTP), Slava Mukhanov (LMU), Cumrun Vafa (Harvard), Xinmin Zhang (IHEP Beijing)

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- Hume Feldman, Leandros Perivolaropoulos, Jong Kung, Jinwu Ye, Tomislav Prokopec, Andrew Sornborger, Richhild Moessner, Mark Trodden, Raul Abramo, Matthew Parry, Martin Götz, Stephon Alexander, Damien Easson, Antonio Guimaraes, Zeeya Merali, Ghazal Geshnizjani, Scott Watson, Thorsten Battefeld.
- Subodh Patil, Natalia Shuhmaher, Patrick Martineau, Rebecca Danos, Nima Lashkari, Andrew Stewart, Francis Cyr-Racine, Jean Lachapelle, Johanna Karouby, Wei Xue, Laurence Perreault-Levasseur
- My colleagues at McGill

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Main Messages

- There are alternatives to inflationary cosmology.
- String Theory leads to a new paradigm for early universe cosmology.
- String Theory can be tested with cosmological observations.

Outline

Plan

Current Paradigm for Early Universe Cosmology

The Inflationary Universe Scenario is the current paradigm of early universe cosmology.

Successes:

- Solves horizon problem
- Solves flatness problem
- Solves size/entropy problem
- Provides a causal mechanism of generating primordial cosmological perturbations (Chibisov & Mukhanov, 1981).

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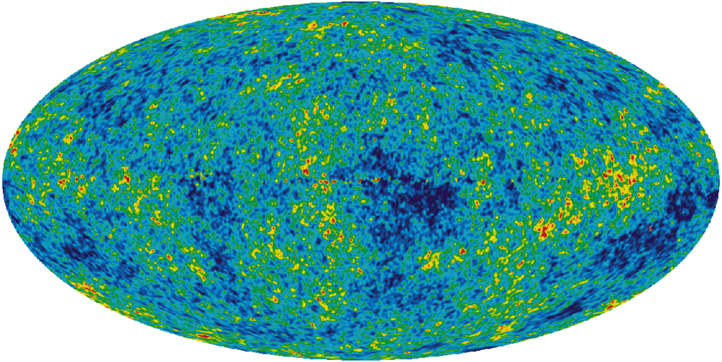
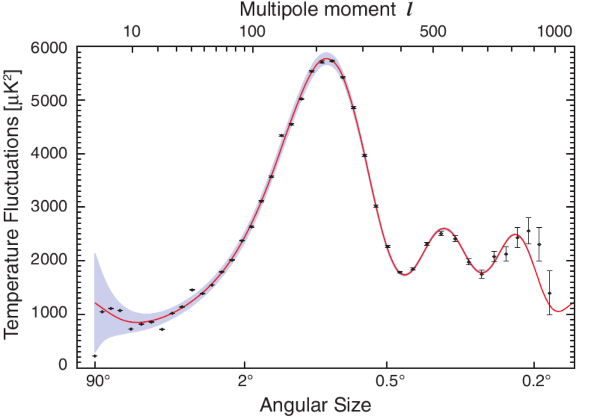


Credit: NASA/WMAP Science Team



Credit: NASA/WMAP Science Team

Challenges for the Current Paradigm

- In spite of the phenomenological successes, current realizations of the inflationary scenario suffer from several conceptual problems.
- In light of these problems we need to look for input from new fundamental physics to construct a new theory which will overcome these problems.
- Question: Can Superstring theory lead to a new and improved paradigm?
- Question: Can this new paradigm be tested in cosmological observations?

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Review of Inflationary Cosmology

Context:

- General Relativity
- Scalar Field Matter

$$\text{Metric}: \; ds^2 = dt^2 - a(t)^2 d\mathbf{x}^2 \tag{1}$$

Inflation:

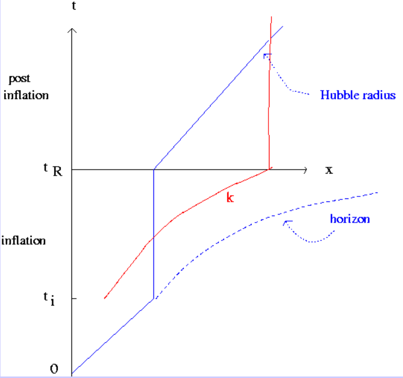
- phase with $a(t) \sim e^{tH}$
- requires matter with $p \sim -\rho$
- requires a slowly rolling scalar field φ
- - in order to have a potential energy term
- - in order that the potential energy term dominates sufficiently long

Time line of inflationary cosmology:

- t_i: inflation begins
- t_R: inflation ends, reheating

Review of Inflationary Cosmology II

Space-time sketch of inflationary cosmology:



Note:

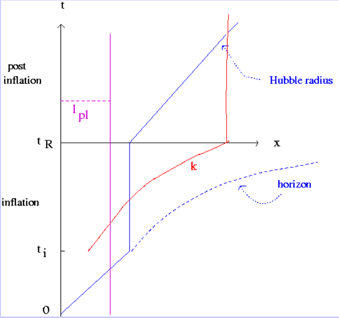
- $H = \frac{\dot{a}}{a}$
- curve labelled by k: wavelength of a fluctuation

- inflation renders the universe large, homogeneous and spatially flat
- classical matter redshifts → matter vacuum remains
- quantum vacuum fluctuations: seeds for the observed structure [Chibisov & Mukhanov, 1981]
- sub-Hubble → locally causal

Conceptual Problems of Inflationary Cosmology

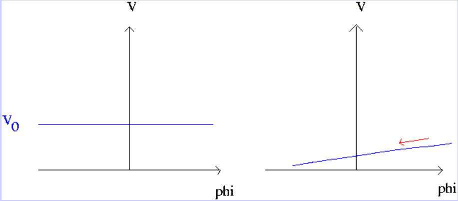
- Nature of the scalar field φ (the “inflaton")
- Conditions to obtain inflation (initial conditions, slow-roll conditions, graceful exit and reheating)
- Amplitude problem
- Trans-Planckian problem
- Singularity problem
- Cosmological constant problem
- Applicability of General Relativity

Trans-Planckian Problem



- **Success of inflation**: At early times scales are inside the Hubble radius → causal generation mechanism is possible.
- **Problem:** If time period of inflation is more than $70H^{-1}$, then $\lambda_p(t) < l_{pl}$ at the beginning of inflation
- → new physics MUST enter into the calculation of the fluctuations.

Cosmological Constant Problem



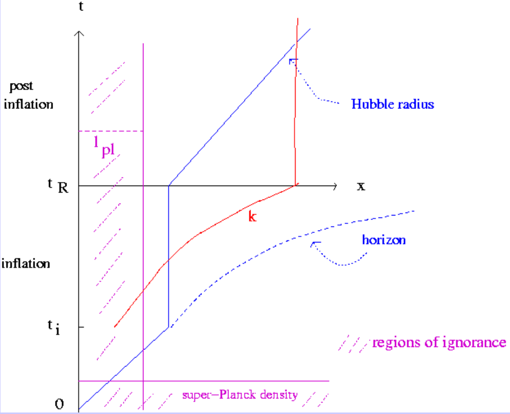
- Quantum vacuum energy does not gravitate.
- Why should the almost constant $V(\varphi)$ gravitate?

$$\frac{V_0}{\Lambda_{obs}} \sim 10^{120} \tag{2}$$

Applicability of GR

- In all approaches to quantum gravity, the Einstein action is only the leading term in a low curvature expansion.
- Correction terms may become dominant at much lower energies than the Planck scale.
- Correction terms will dominate the dynamics at high curvatures.
- The energy scale of inflation models is typically $\eta \sim 10^{16}$GeV.
- $\rightarrow \eta$ too close to m_{pl} to trust predictions made using GR.

Zones of Ignorance



Message

- Current realizations of inflation have conceptual problems.
- We need a new paradigm of very early universe cosmology based on new fundamental physics.
- Hypothesis: New paradigm based on Superstring Theory.
- New cosmological model motivated by superstring theory: String Gas Cosmology (SGC) [R.B. and C. Vafa, 1989]
- New structure formation scenario emerges from SGC [A. Nayeri, R.B. and C. Vafa, 2006].
- Testable prediction for cosmological observations: Blue tilt in the spectrum of gravitational waves [R.B., A. Nayeri, S. Patil and C. Vafa, 2006]

Plan

Principles

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Idea: make use of the new symmetries and new degrees of freedom which string theory provides to construct a new theory of the very early universe.

Assumption: Matter is a gas of fundamental strings

Assumption: Space is compact, e.g. a torus.

Key points:

- New degrees of freedom: string oscillatory modes
- Leads to a maximal temperature for a gas of strings, the Hagedorn temperature
- New degrees of freedom: string winding modes
- Leads to a new symmetry: physics at large R is equivalent to physics at small R

T-Duality

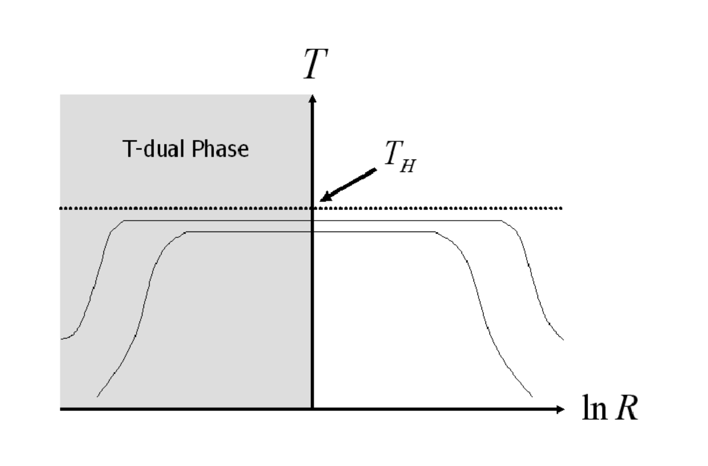
T-Duality

- Momentum modes: $E_n = n/R$
- Winding modes: $E_m = mR$
- Duality: $R \rightarrow 1/R \quad (n, m) \rightarrow (m, n)$
- Mass spectrum of string states unchanged
- Symmetry of vertex operators
- Symmetry at non-perturbative level $\rightarrow$ existence of D-branes

Adiabatic Considerations

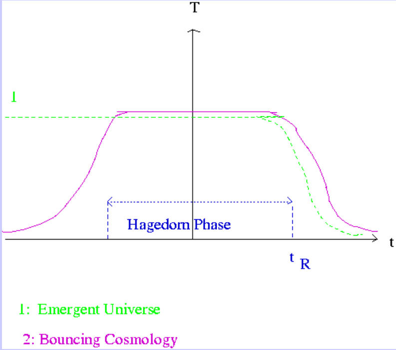
R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Temperature-size relation in string gas cosmology



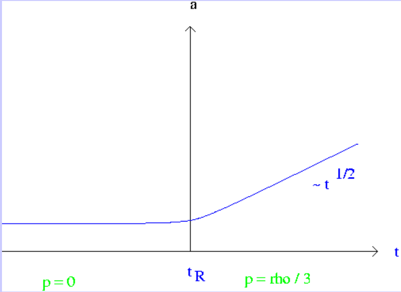
Dynamics

Assume some action gives us $R(t)$



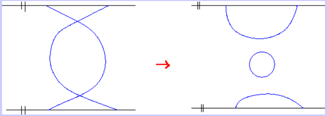
Dynamics II

We will thus consider the following background dynamics for the scale factor $a(t)$:



Dimensionality of Space in SGC

- Begin with all 9 spatial dimensions small, initial temperature close to T_H $\rightarrow$ winding modes about all spatial sections are excited.
- Expansion of any one spatial dimension requires the annihilation of the winding modes in that dimension.



- Decay only possible in three large spatial dimensions.
- $\rightarrow$ dynamical explanation of why there are exactly three large spatial dimensions.

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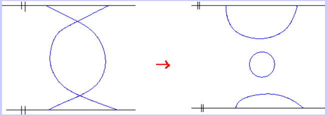

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Theory of Cosmological Perturbations: Basics

Cosmological fluctuations connect early universe theories with observations

- Fluctuations of matter $\rightarrow$ large-scale structure
- Fluctuations of metric $\rightarrow$ CMB anisotropies
- N.B.: Matter and metric fluctuations are coupled

Key facts:

- 1. Fluctuations are small today on large scales
- $\rightarrow$ fluctuations were very small in the early universe
- $\rightarrow$ can use linear perturbation theory
- 2. Sub-Hubble scales: matter fluctuations dominate
- Super-Hubble scales: metric fluctuations dominate

Quantum Theory of Linearized Fluctuations

V. Mukhanov, H. Feldman and R.B., *Phys. Rep. 215:203 (1992)*

Step 1: Metric and matter including fluctuations

$$ds^2 = a^2[(1+2\Phi)d\eta^2 - (1-2\Phi)d\mathbf{x}^2] \quad (3)$$

$$\varphi = \varphi_0 + \delta\varphi \quad (4)$$

Note: Φ and $\delta\varphi$ related by Einstein constraint equations
Step 2: Expand the action for matter and gravity to second order about the cosmological background:

$$S^{(2)} = \frac{1}{2}\int d^4x\left((v')^2 - v_{,i}v^{,i} + \frac{z''}{z}v^2\right) \quad (5)$$

$$v = a\left(\delta\varphi + \frac{z}{a}\Phi\right) \quad (6)$$

$$z = a\frac{\varphi_0'}{\mathcal{H}} \quad (7)$$

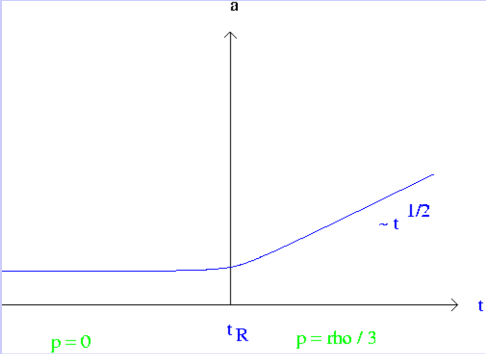
Step 3: Resulting equation of motion (Fourier space)

$$v_k'' + (k^2 - \frac{z''}{z}) v_k = 0 \tag{8}$$

Features:

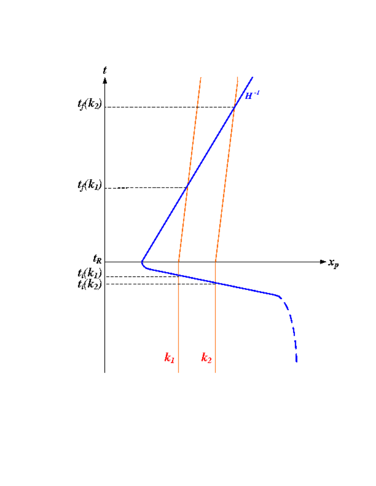
- oscillations on sub-Hubble scales
- squeezing on super-Hubble scales $v_k \sim z$

Background for string gas cosmology



Structure formation in string gas cosmology

A. Nayeri, R.B. and C. Vafa, *Phys. Rev. Lett. 97:021302 (2006)*



N.B. Perturbations originate as thermal string gas fluctuations.

Method

- Calculate matter correlation functions in the Hagedorn phase (neglecting the metric fluctuations)
- For fixed k, convert the matter fluctuations to metric fluctuations at Hubble radius crossing $t = t_i(k)$
- Evolve the metric fluctuations for $t > t_i(k)$ using the usual theory of cosmological perturbations

Extracting the Metric Fluctuations

Ansatz for the metric including cosmological perturbations and gravitational waves:

$$ds^2 = a^2(\eta)\big((1+2\Phi)d\eta^2 - [(1-2\Phi)\delta_{ij} + h_{ij}]dx^i dx^j\big)\,. \quad (9)$$

Inserting into the perturbed Einstein equations yields

$$\langle|\Phi(k)|^2\rangle = 16\pi^2 G^2 k^{-4} \langle\delta T^0{}_0(k)\delta T^0{}_0(k)\rangle\,, \quad (10)$$

$$\langle|h(k)|^2\rangle = 16\pi^2 G^2 k^{-4} \langle\delta T^i{}_j(k)\delta T^i{}_j(k)\rangle\,. \quad (11)$$

Power Spectrum of Cosmological Perturbations

Key ingredient: For thermal fluctuations:

$$\langle \delta\rho^2 \rangle = \frac{T^2}{R^6} C_V \, . \tag{12}$$

Key ingredient: For string thermodynamics in a compact space

$$C_V \approx 2 \frac{R^2/\ell_s^3}{T(1 - T/T_H)} \, . \tag{13}$$

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Power spectrum of cosmological fluctuations

$$P_{\Phi}(k) = 8G^2k^{-1} < |\delta\rho(k)|^2 > \quad (14)$$
$$= 8G^2k^2 < (\delta M)^2 >_R \quad (15)$$
$$= 8G^2k^{-4} < (\delta\rho)^2 >_R \quad (16)$$
$$= 8G^2 \frac{T}{\ell_s^3} \frac{1}{1 - T/T_H} \quad (17)$$

Key features:

- scale-invariant like for inflation
- slight red tilt like for inflation

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Spectrum of Gravitational Waves

R.B., A. Nayeri, S. Patil and C. Vafa, *Phys. Rev. Lett. (2007)*

$$P_h(k) = 16\pi^2 G^2 k^{-1} < |T_{ij}(k)|^2 > \quad (18)$$

$$= 16\pi^2 G^2 k^{-4} < |T_{ij}(R)|^2 > \quad (19)$$

$$\sim 16\pi^2 G^2 \frac{T}{\ell_s^3}(1 - T/T_H) \quad (20)$$

Key ingredient for string thermodynamics

$$< |T_{ij}(R)|^2 > \sim \frac{T}{l_s^3 R^4}(1 - T/T_H) \quad (21)$$

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Plan

Conclusions

- String Gas Cosmology: Model of cosmology of the very early universe based on new degrees of freedom and new symmetries of superstring theory.
- SGC $\rightarrow$ nonsingular cosmology
- SGC $\rightarrow$ natural explanation of the number of large spatial dimensions.
- SGC $\rightarrow$ new scenario of structure formation
- Scale invariant spectrum of cosmological fluctuations (like in inflationary cosmology).
- Spectrum of gravitational waves has a small blue tilt (unlike in inflationary cosmology).