Finite-system statistical mechanics for nucleons, hadrons and partons

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RECURSIVE TECHNIQUES

THE BACK AND FORTHS OF STATISTICAL NUCLEAR PHYSICS

- **Multi-Fragmentation**
- **Level Densities**
- **Hadron Gas**
- **QGP**

Collaborators:

S. Das Gupta, McGill
W. Bauer, MSU
S. Cheng, UCSF
S. Petriconi, MSU
J. Ruppert, Frankfurt
M. Skoby, U.Minn.Morris

The "OTHER" method for canonical ensembles

$$Z_{\mathrm{GC}}(\beta,\alpha) = \mathrm{Tr}\exp(-\beta H + \alpha Q)$$

$$Z_{\mathrm{C}}(\beta,Q) = \frac{1}{2\pi}\int d\alpha\, e^{-i\alpha Q} Z_{\mathrm{GC}}(\beta,i\alpha)$$

- Can even be applied to non-additive charges:
  H.-T. Elze and W. Greiner, PRA(86), PLB(86)
- Has been applied to QGP

Transforming Z_{GC} vs. Recursive Techinques

Old methods:

- Can be performed analytically for simple systems

Recursive techniques:

- Bose and Fermi statistics
- Arbitrary level densities
- Multiplicity distributions
- Exact (discrete, sums not integrals)

Fundamental Relation

K. Chase & A. Mekjian, PRC 1995, S.P. & S.Das Gupta, PRC 2000

- Consider Species *k* with mass a_k

$$Z_A = \sum_{\langle n_k a_k = A \rangle} \prod_k \frac{\omega_k^{n_k}}{n_k!}$$

$$= \sum_{\langle n_k a_k = A \rangle} \prod_k \frac{\omega_k^{n_k}}{n_k!} \frac{n_{k'} a_{k'}}{A}$$

$$= \frac{1}{A} \sum_k \omega_k a_k Z_{A-a_k}$$

- One can add other charges

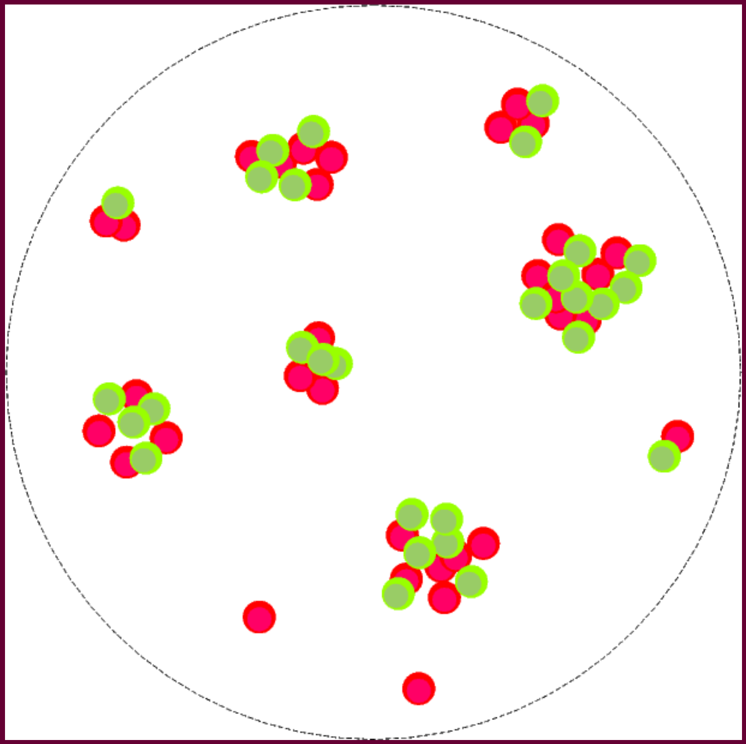
$$Z_{A,\vec{Q}} = \frac{1}{A} \sum_k a_k \omega_k Z_{A-a_k,\vec{Q}-\vec{q}_k}$$

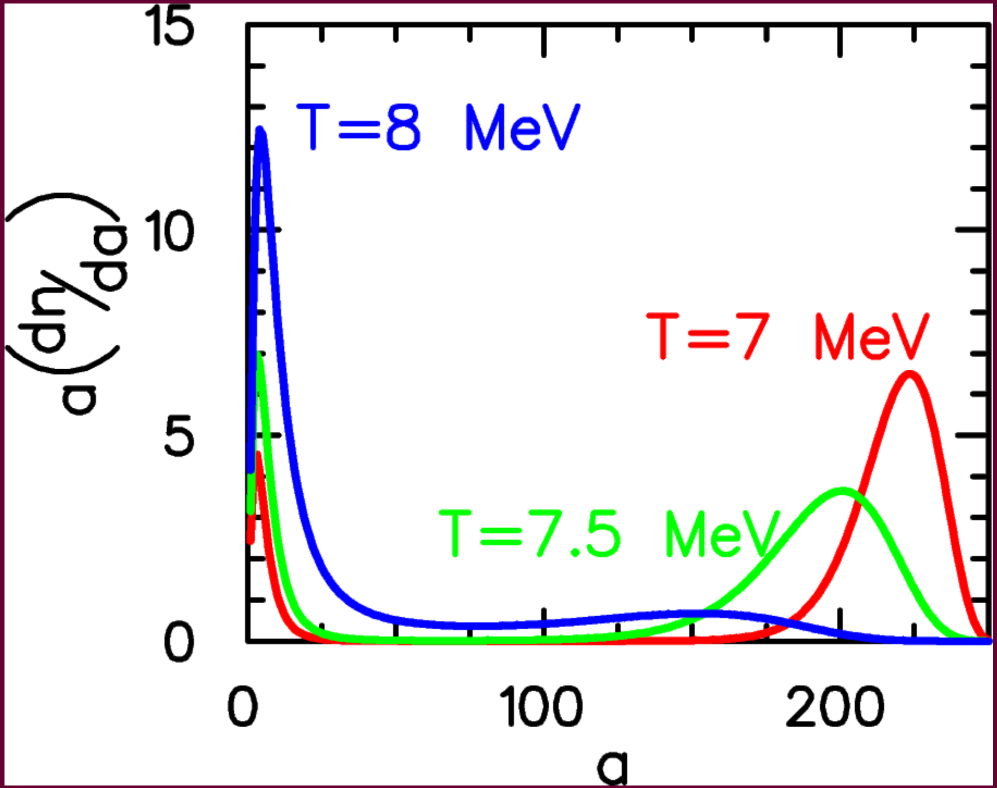
Multifragmentation

Population of species *k*

$$\langle n_k \rangle = \frac{\omega_k Z_{A-a_k}}{Z_A}$$

Mass Distribution, A=250

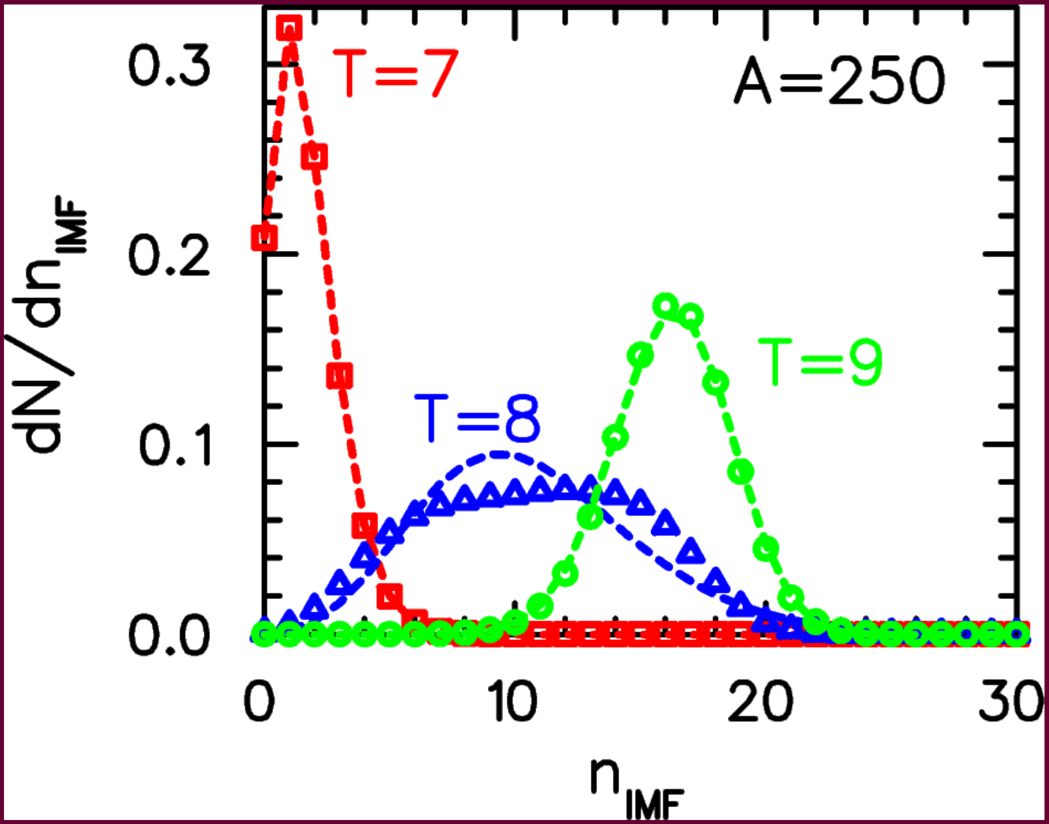




Multiplicity distributions

Treat the number *N* as a "charge"

$$P_A(N)=\frac{Z_{A,N}}{Z_A}=\frac{1}{AZ_A}\sum_k \omega_k a_k Z_{A-a_k,N-n_k}$$



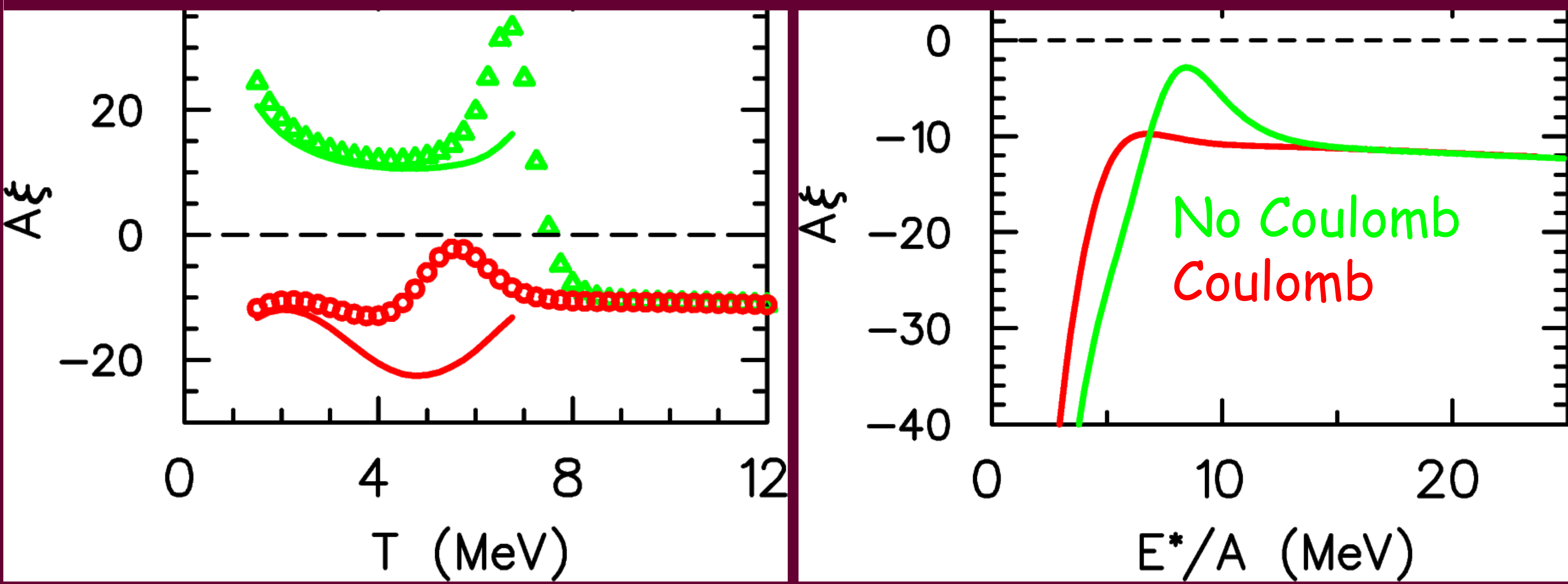
Microcanonical ensembles

Discretize *E* and treat it as a "charge"

$$N_A(E) = Z_{A,E} = \frac{1}{A}\sum_k \omega_k a_k Z_{A-a_k, E-\varepsilon_k}$$

FLUCTUATIONS in IMF PRODUCTION

Canonical Microcanonical



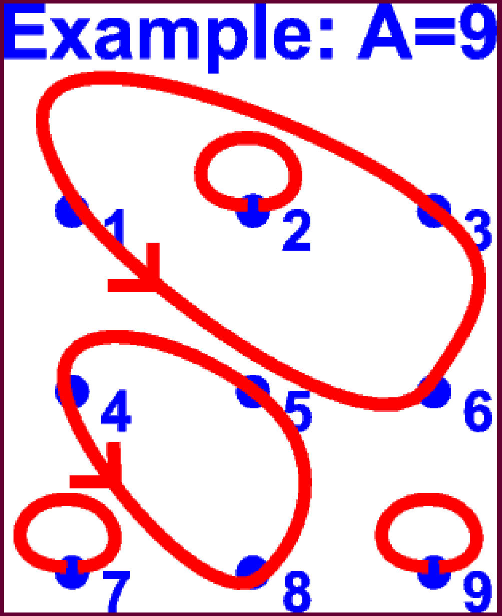
Fermi systems

S.P., PLB 93, PRL 2001

- Previous form, $\omega^n/n!$, neglects degenerate states.

  - Account for symmetrization with permutations

$$Z_A = \frac{1}{A!} \sum_{i_1 - i_A, P(i)} \left\langle i_1, i_2, \cdots i_A \,|\, e^{-\beta H} \,|\, P(i) \right\rangle (-1)^{N_P}$$



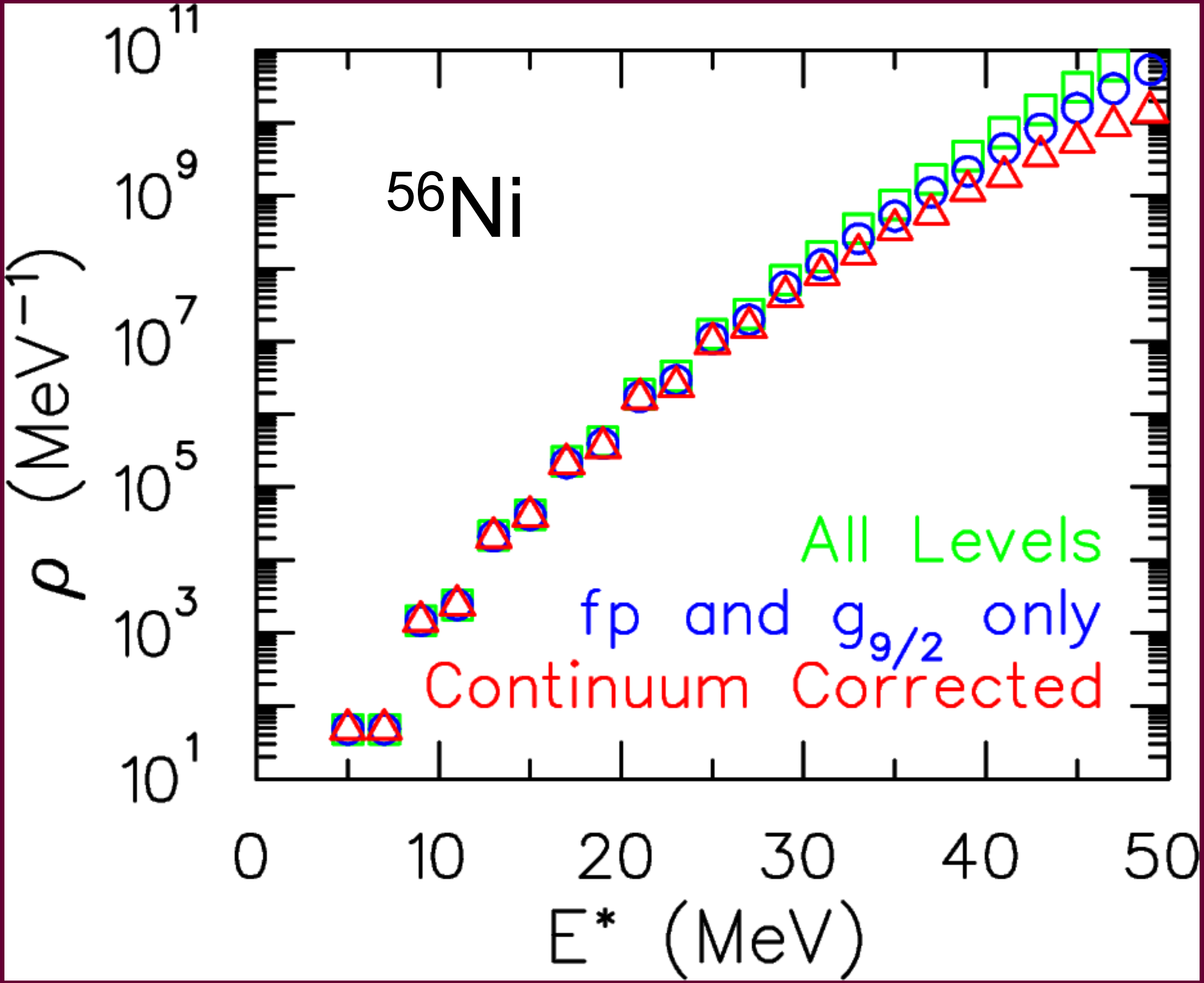
  - Arrange permutations into cycles,

$$Z_A = \frac{1}{A} \sum_{\ell} (-1)^{\ell+1} C_\ell Z_{A-\ell}$$

$$C_\ell = \sum_{i_1, i_2 \cdots} \left\langle i_1, i_2 \cdots i_\ell \,|\, e^{-\beta H} \,|\, i_\ell, i_1 \cdots i_{\ell-1} \right\rangle = \sum_i e^{-\ell \beta E_i}$$

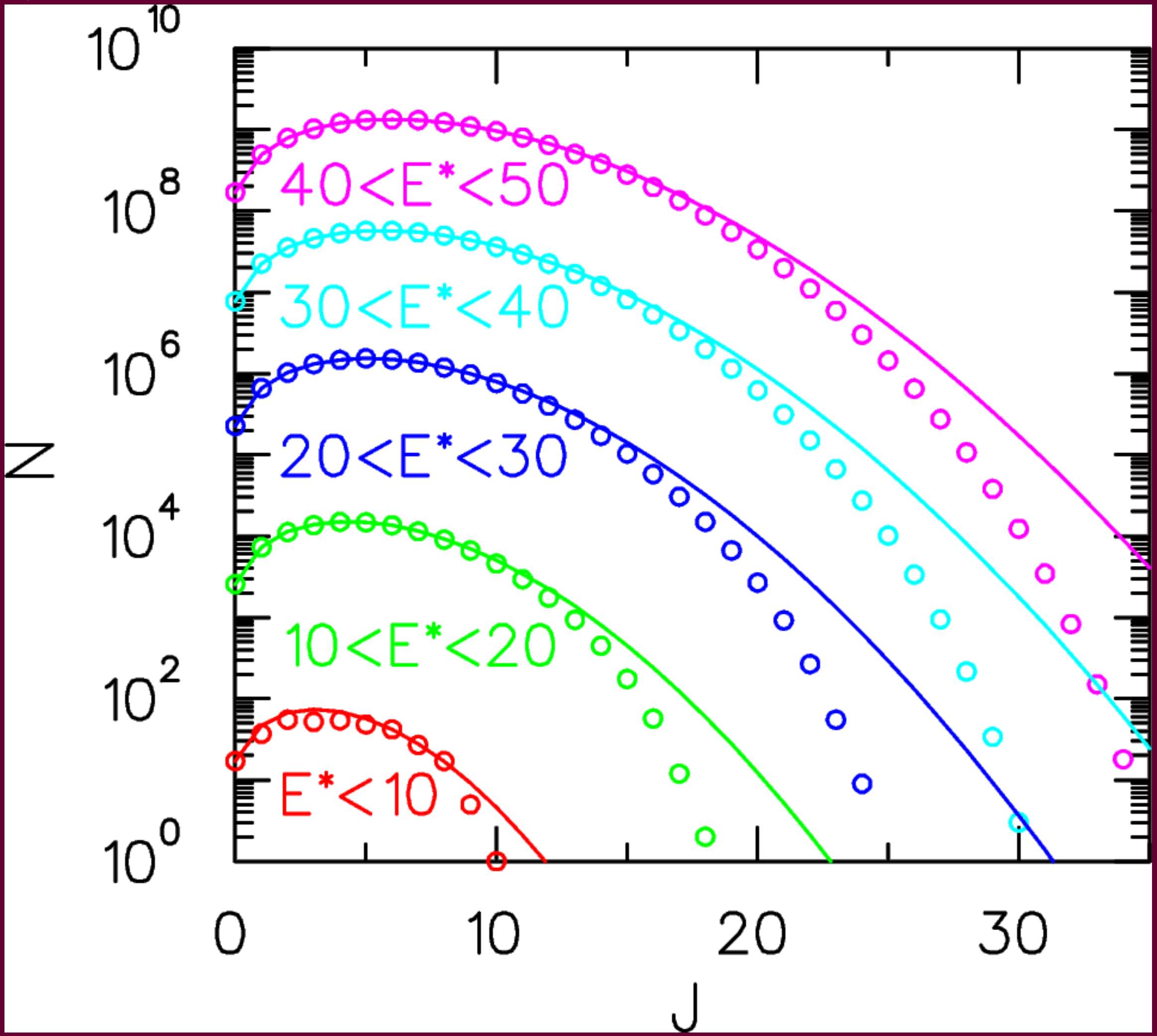
Level densities

Using microcanonical ensemble, calculate $N_A(E)$



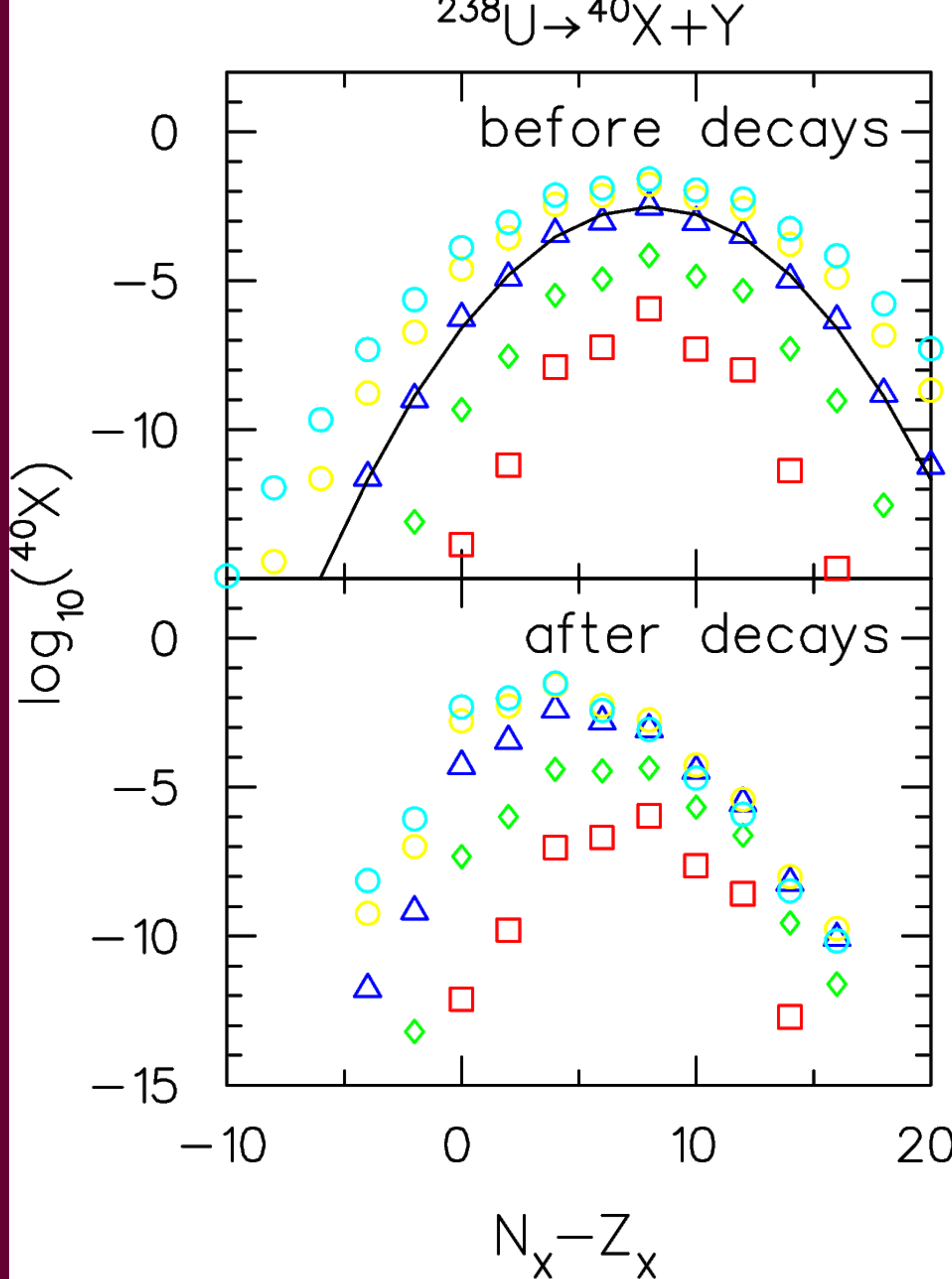
Angular momentum

1. Calculate $Z_{A,M}$
2. $Z_{A,J} = Z_{A,M=J} - Z_{A,M=J+1}$



Application: Rare isotope production

Incorporate:
- FRLDM ground states
- Excited states
- Fragment into all possible partitions
- Sequential decay



S.P., P.Underhill, W.Bauer
PRC 2001

Hadron gas

- Same formalism
- Conserve B, S, I, I_3, Q
- Symmetrization for pions
- Isospin treated like angular momentum
- Monte Carlo particles

Hadron gas, Monte Carlo

MONTE CARLO PROCEDURE:

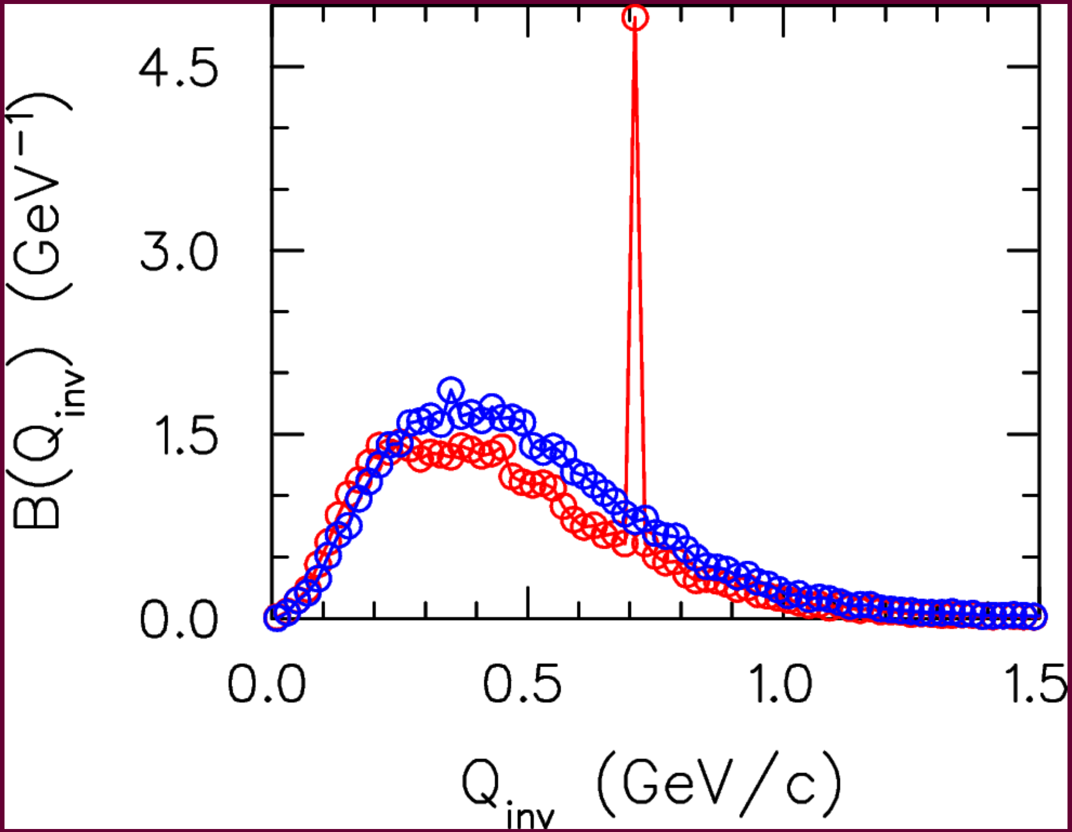
1. Pick Q=0 and A according to weight ~ Z_A
2. For A and Q:
   - Calculate weight(k)=$(a_k w_k/A)Z(A-a_k,Q-q_k)/Z(A,Q)$
   - Choose species according to weight
   - Reduce size: $A\to A-a_k$, $Q\to Q-q_k$
   - Repeat
3. Can not include Fermi/Bose statistics

Balance functions

S.P., S. Petriconi and M. Skoby, PRC 2003.

- Charge conservation is local
- Width determined largely by *T*
- Accounts for "lost" charge

$\pi^+\pi^-$ balance function



Isospin distributions: background

Random state:

$$P(N_0) = \frac{A!}{(A-N_0)!N_0!}\left(\frac{1}{3}\right)^{N_0}\left(\frac{2}{3}\right)^{A-N_0}$$

DCC state: (isosinglet in one quantum level)

$$|\eta\rangle = \frac{1}{\sqrt{N}}\left(2a_+^+a_-^+ - a_0^+a_0^+\right)^{A/2}|0\rangle$$

$$P(N_0) = \frac{1}{2\sqrt{AN_0}}$$

Bose-Gas with no isospin constraints:

- Broad distribution at high phase space density
- Pairwise isospin conservation further broadens distribution

S.P. and V.Zelevinsky, PRL 94

Isospin distributions: challenges

- Include all arrangements
- Conserving I
- Bose Einstein statistics
- Resonances
- Number of neutral pions, N_0,
  -- does not commute with I

Isospin distributions: solution

S.Cheng and S.P. PRC 2003

One can show (non-trivial)

$$P_{A,I,M}(N_0) = \frac{W_{A,I,M}(N_0)}{Z_{A,I}}$$

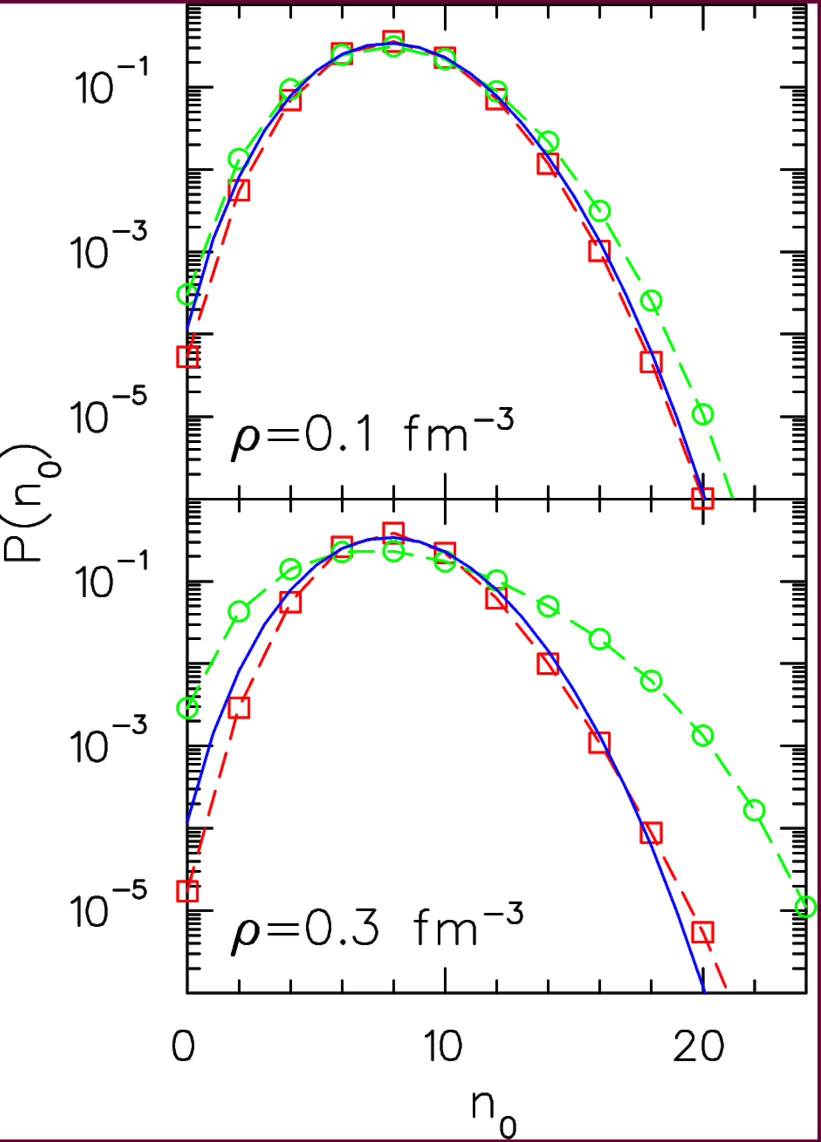
$$W_{A,I,M}(N_0) = \frac{1}{A}\sum_{\ell}\sum_{i,I',M',m} C_\ell(i,m;n_0) W_{A-\ell,I',M'}(N_0 - n_0)\langle I,M \mid i,m,I',M'\rangle^2$$

$$C_\ell(i,m;n_0) = \left(\sum_i e^{-\beta E_i}\right)\sum_\alpha \langle \alpha(n_0) \mid \mathrm{P}^i_m \mid \tilde{\alpha}(n_0)\rangle$$

- Isospin decomposition of cycle diagram for pions in one quantum state
- Calculated *brute force* with raising/lowering operators

Isospin distributions: results

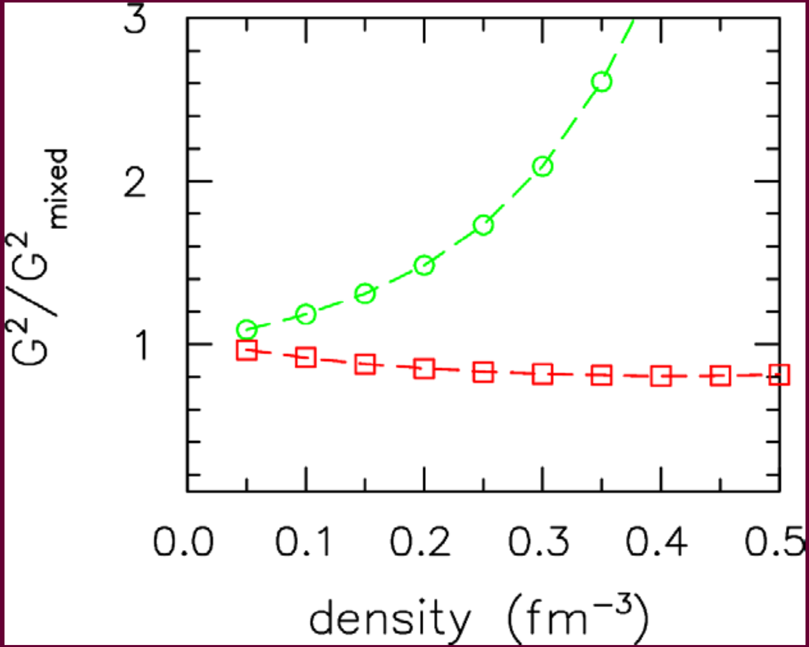
π, ρ, ω gas at T=150 MeV



Random

Isosinglet + symmetry

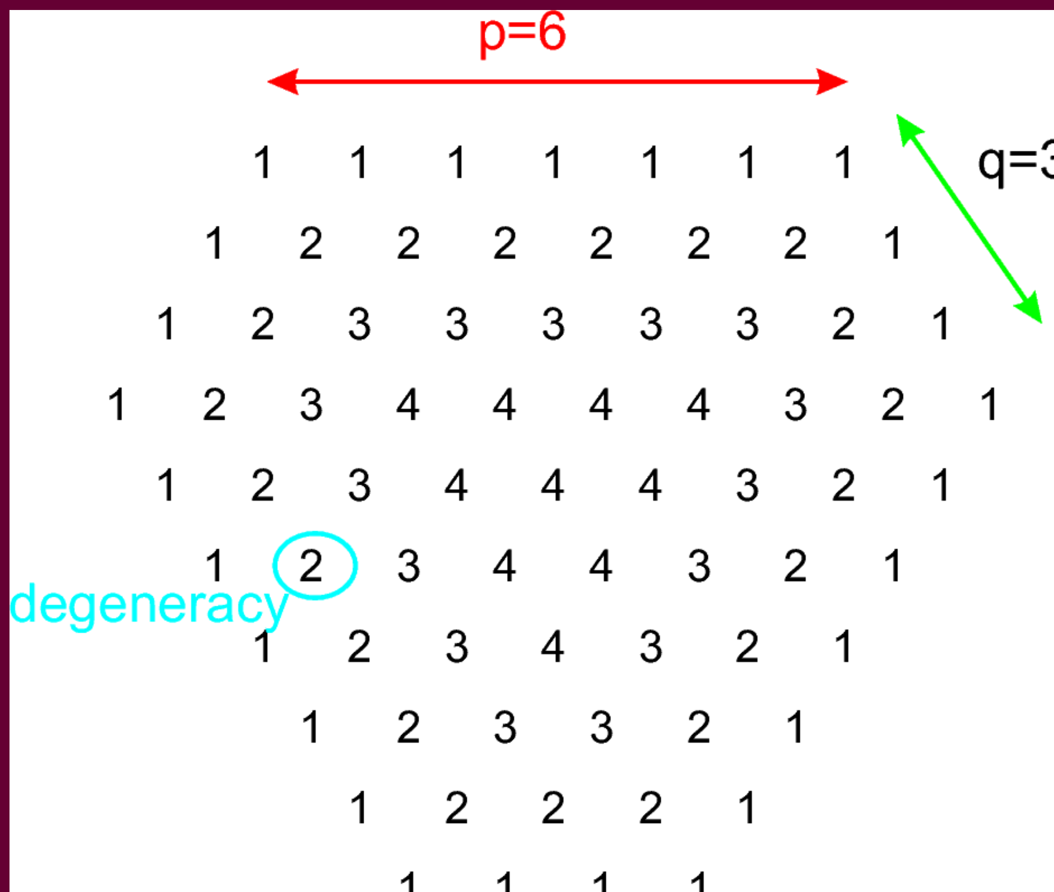
+ resonances



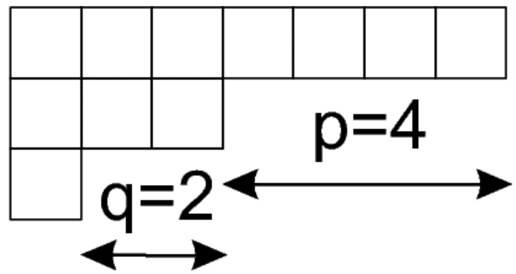
- Bose effects broaden distribution
- Resonances narrow distribution
- Resonances win

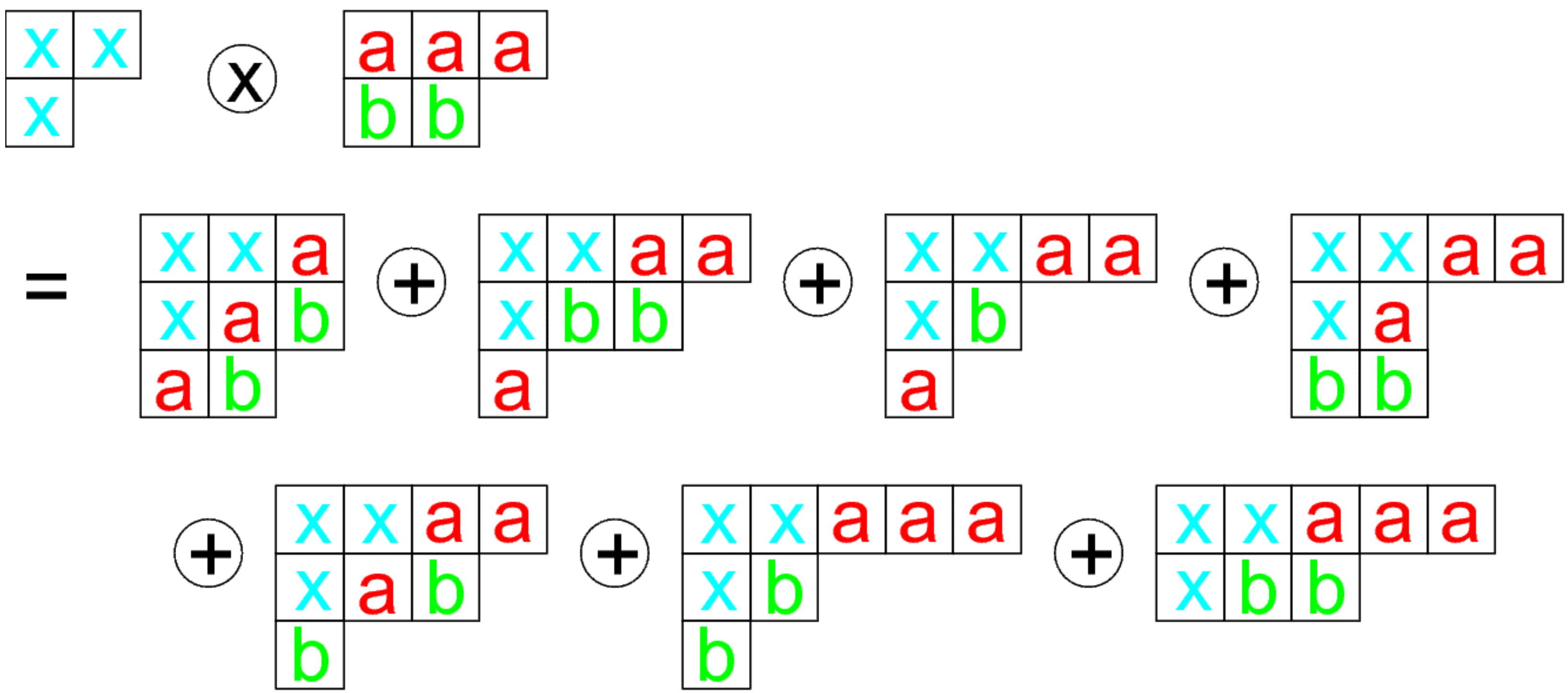
Conserving SU(3) color

- All systems are confined to color singlets
- Constraint should lower entropy
- Color multiplets are labeled (*p*,*q*)
- Singlet is (0,0), *q* is (1,0) anti-*q* is (0,1) gluon is (1,1)



Adding color multiplets





$$(1,1)\otimes(1,2)=(0,1)\oplus 2\bullet(1,2)\oplus(2,1)\oplus(2,0)\oplus(3,1)\oplus(2,3)$$

Recursion relations for gluons

$$Z(p,q)=\sum_A Z_A(p,q)$$

$$Z_A(p,q)=\frac{1}{A}\sum_{\ell,p_\ell,q_\ell,p',q'} C_\ell(p_\ell,q_\ell) Z_{A-\ell}(p',q')\beta(p_\ell,q_\ell,p',q';p,q)$$

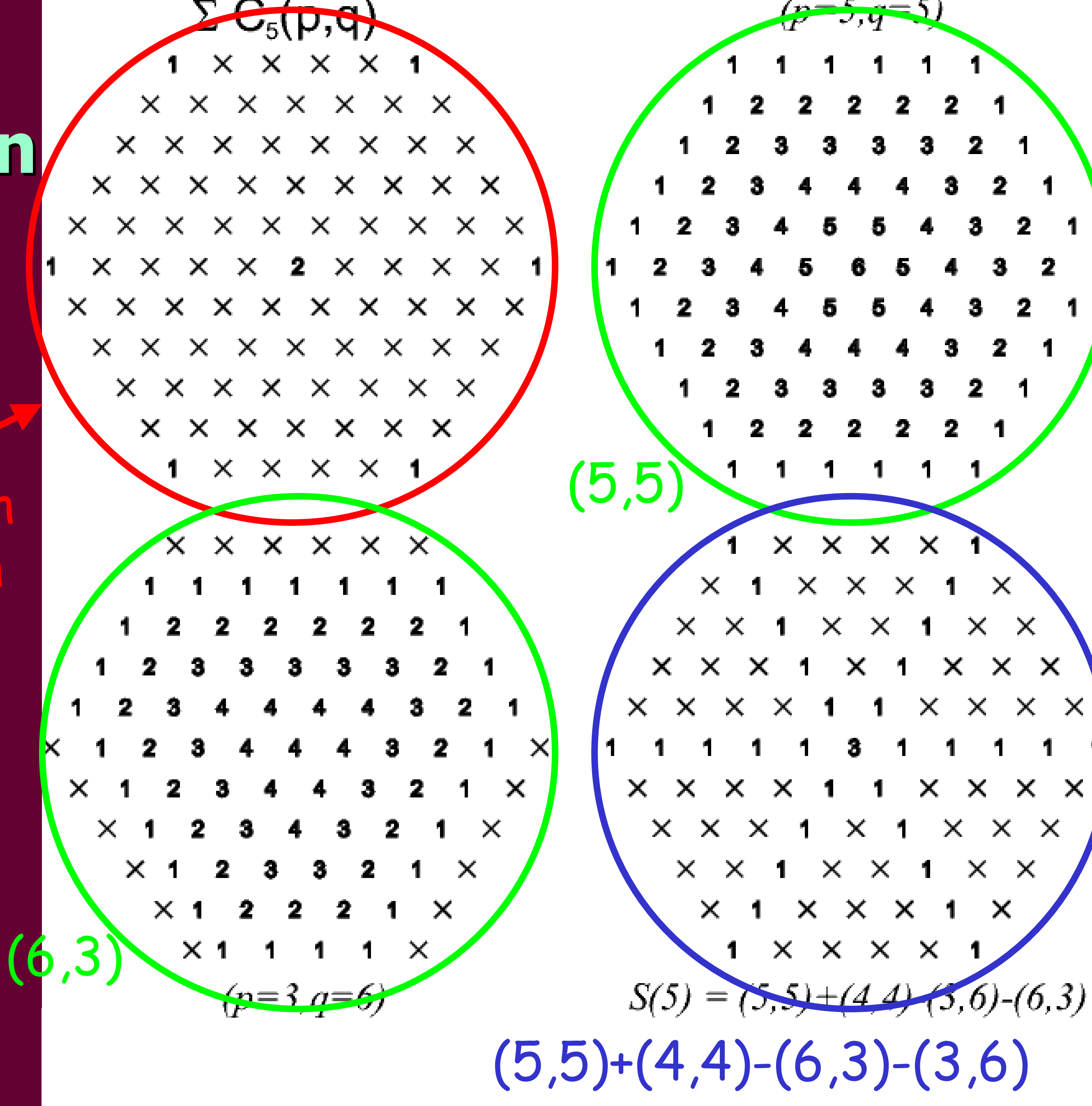
Must know Color decomposition of:

$$\sum_{a_1-a_\ell}\langle a_1a_2\cdots a_\ell\,|\,\mathrm{P}(p,q)\,|\,a_2a_3\cdots a_1\rangle$$

From Young-tableaux Addition rules

Color decomposition of cycle diagram

Cycle diagram with no (p,q) projection



Color decomposition of cycle diagram

Gluons:

$$C_\ell = (\ell,\ell) + (\ell-3,\ell) + (\ell,\ell-3) - (\ell-2,\ell+1) - (\ell+1,\ell-2) - (\ell-2,\ell-2)$$

Quarks:

$$C_\ell = (\ell,0) - (\ell-1,2) + (\ell-3,\ell-3)$$

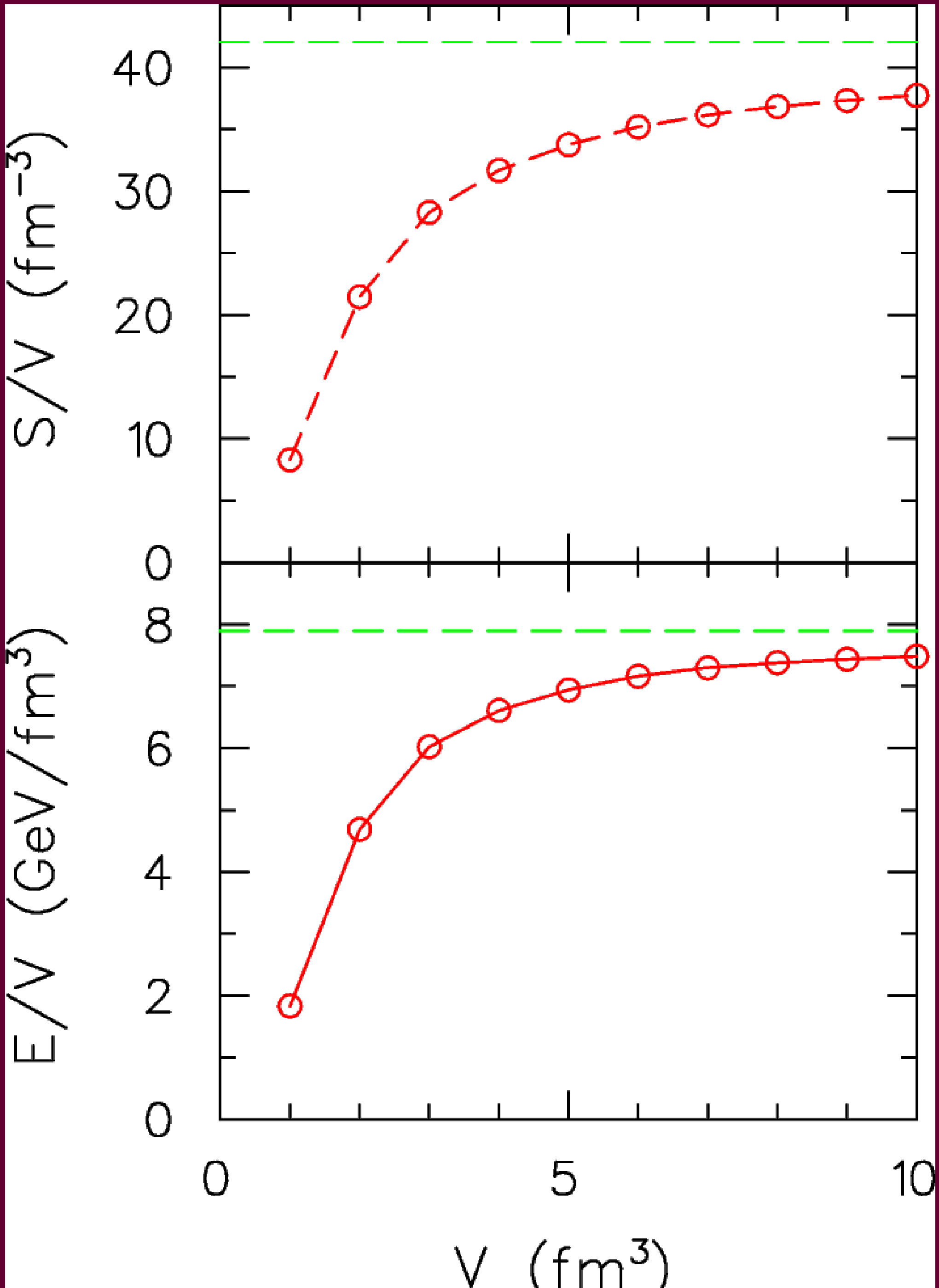
Calculating *Z* for parton gas

J.Ruppert and S.P., PRC 2003

- Calculate *Z(p,q)* for gluons
- Calculate $Z_A(p,q)$ for strange quarks
- Convolute strange/antistrange $Z_A(p,q)$s

$$Z(P,Q) = \sum_{A,p,q,\bar{p},\bar{q}} Z_A(p,q) Z_A(\bar{p},\bar{q}) \beta(p,q,\bar{p},\bar{q};P,Q)$$

- Calculate $Z_{A,I}(p,q)$ for up/down quarks
- Convolute up/down $Z_A(p,q)$ with antiup/antidown $Z_A(p,q)$
- Keep only *I=0* piece of up/down/antiup/antidown *Z*
- Convolute both quark segments
- Convolute quark sector with gluon sector
- Keep only *(p=0,q=0)* piece.



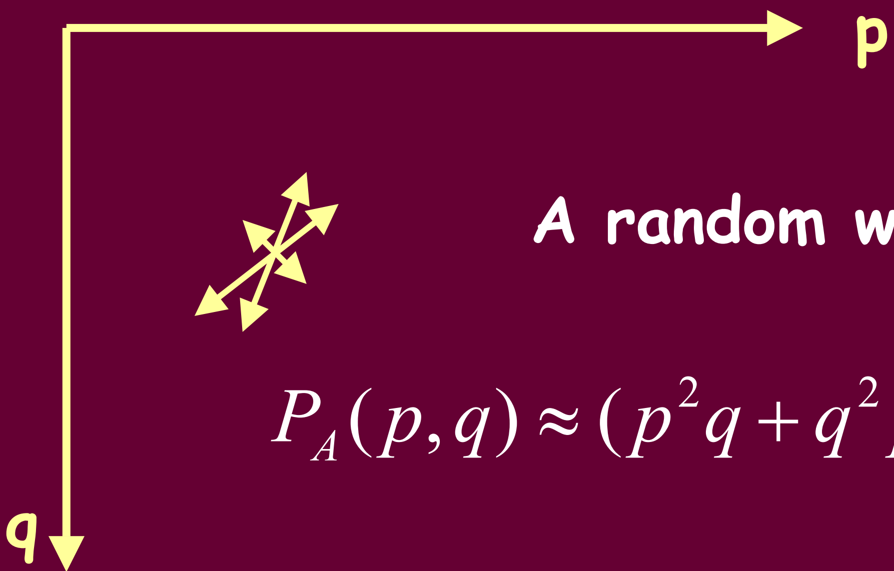
Parton gas: results

- Effects are important for V < 10 fm^3
- What are effective volumes at RHIC?

(20 fm^3 ?)

Aside: Why are the constraints so large for systems with 50 partons?

Now, consider the SU(3) case
Add *A* gluons (1,1)



A random walk in 2-d

$$P_A(p,q) \approx (p^2 q + q^2 p)\exp\left(-\frac{3(p+q)^2 + (p-q)^2}{2A\Delta^2}\right)$$

- P(p=0,q=0) ~ A^{-4}
- Entropy penalty ~ -4 log(A)
- For 20 gluons:
  1.153×10^{18} multiplets, chance of singlet = 1/122,558

Summary

- We can calculate anything
  "When you have a hammer,
  every problem looks like a nail." -K.H.

- Interactions are ignored
  Mean Field or 1st-order perturbation theory is easy
  Iterative perturbation theory? or BBGKY hierarchy?