Exact Solution to the Fragment Multiplicity distributions

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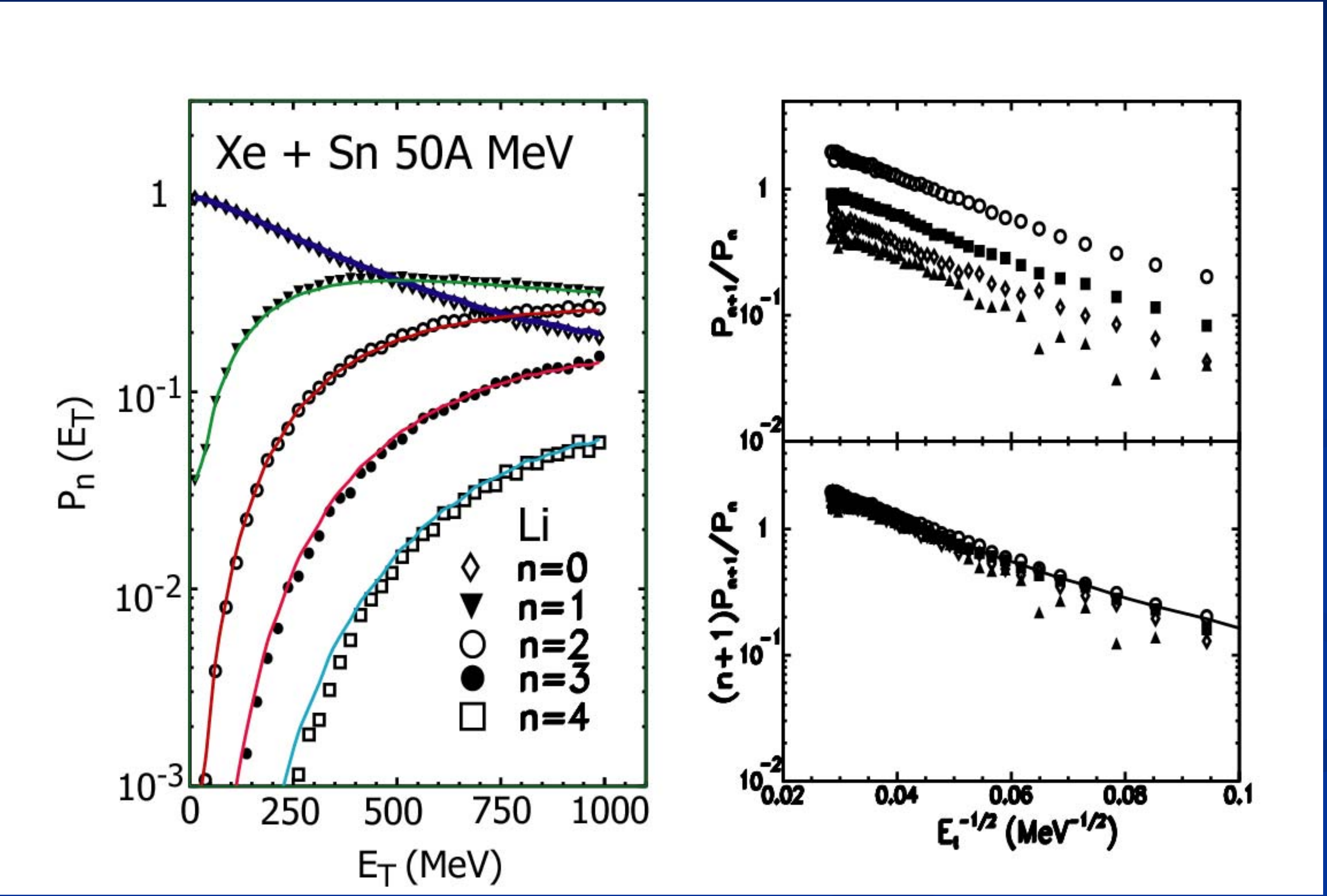
Outline

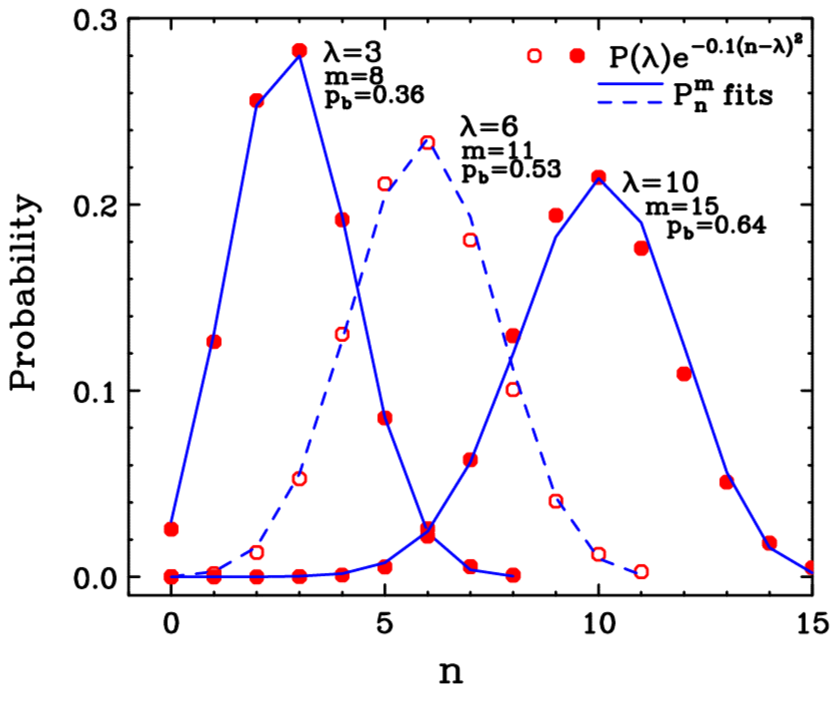
- Questions raised from experiment
  - Multiplicity distributions – poissionian or not
  - Interpretations (independent emission, system size)
- Iterative formula for multiplicity distributions
  - Iteration for partition function
  - Blocking method for complete distributions
- Systematics from multiplicity distribution
  - Single isotope and IMF
  - System size scaling
- Variance – phase transition – isospin asymmetry
- Summary

Questions from experiments: independent emission? Simple?



L. Beaulieu, et al., Nucl-ex/9805003

Different interpretations: part 1: Poisson or binomial



M. B. Tsang and P. Danielewicz, PRL 80, 1178 (1998)

1) L.G. Moretto, et al.,
=> Barrier emission
(W. Skulski, et al., multiplicity folding)

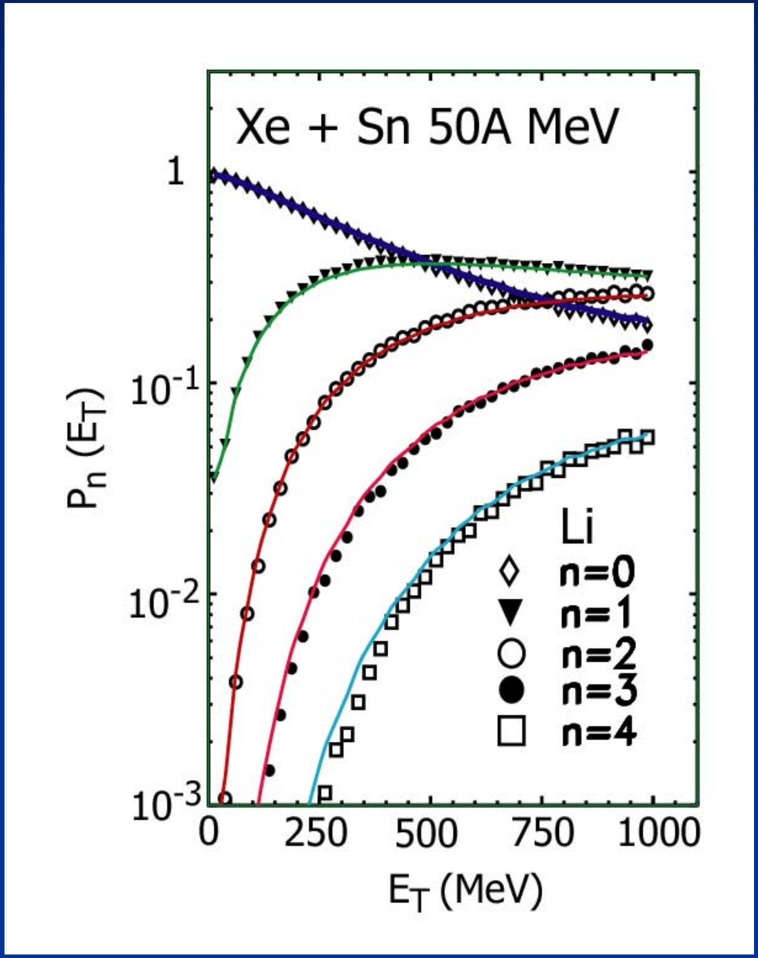
$$P_n = \frac{m!}{n!(m-n)!}p^n(1-p)^{m-n}.$$

2) B. Tsang, poisson to sub-poisson because of conservation law

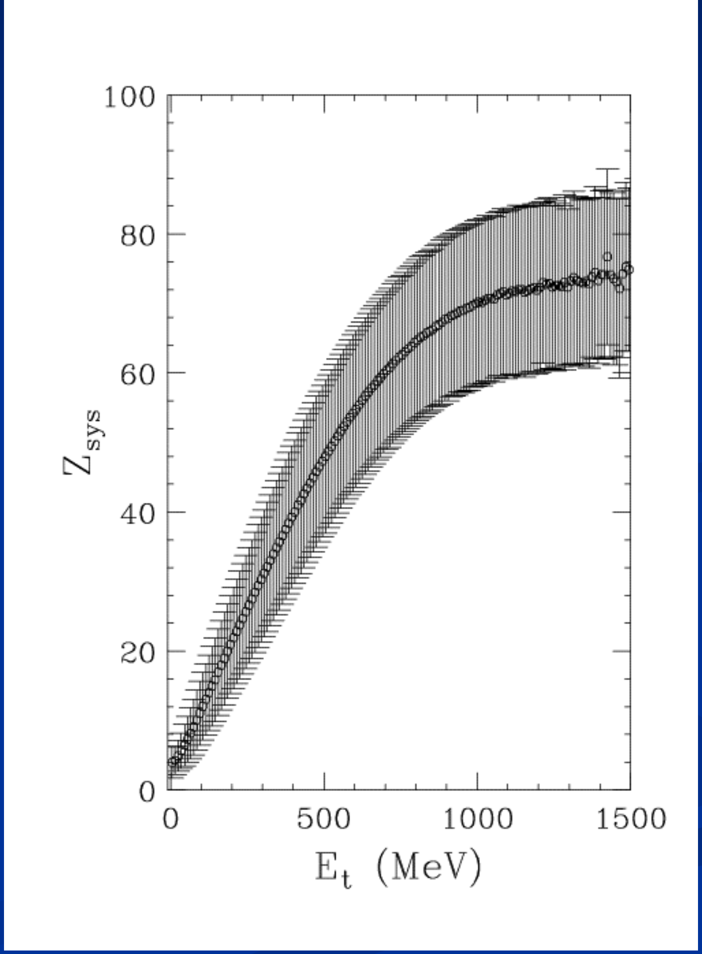
$$P_p(n) = \frac{\lambda^n}{n!}e^{-\lambda}$$

$$P_m(n,\alpha) = \frac{\lambda^n}{n!}e^{-\lambda}e^{-\alpha(n-\lambda)^2}$$

Different interpretations: part 2: size matters



L. Beaulieu, et al.,
Nucl-ex/9805003



Wolfgang Bauer, Scott Pratt,
Phys. Rev. C59 (1999) 2695-2698

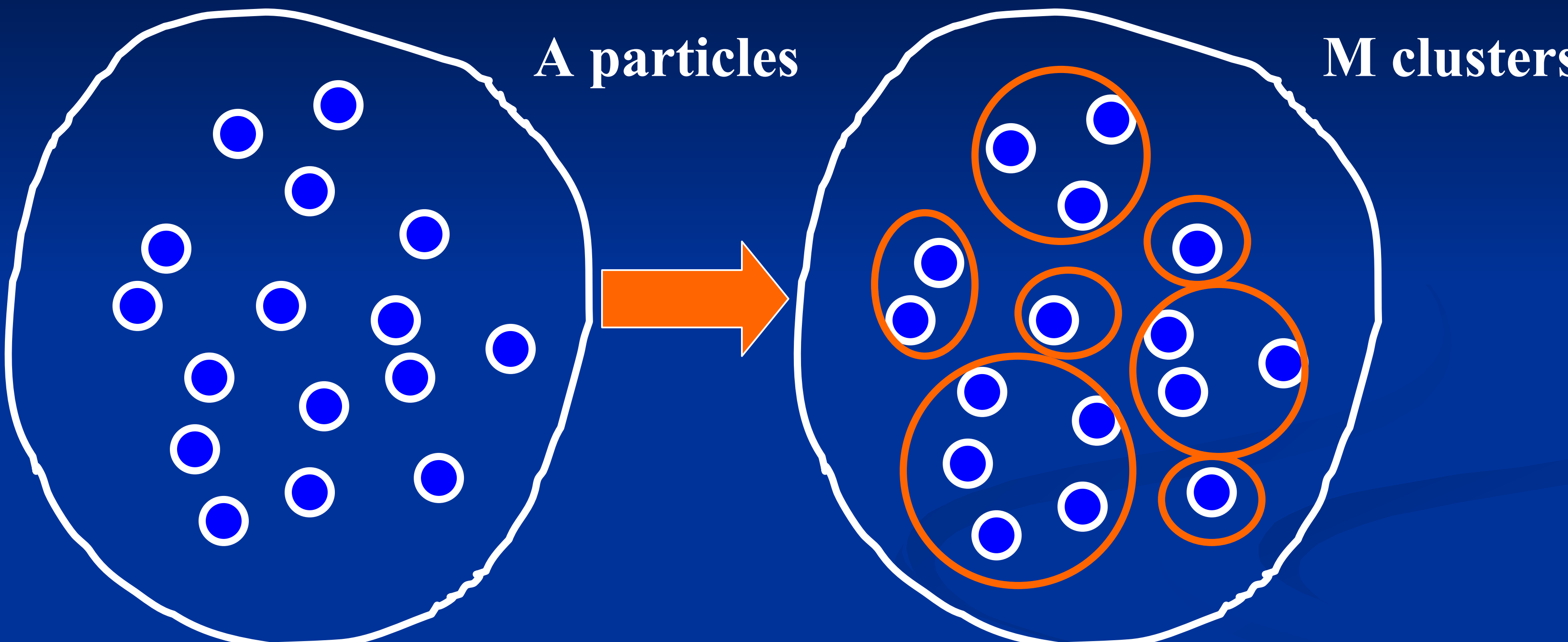
Exact Solvable Model: iterative formula

- Canonical partition problem
  - Particle number – conserved
  - Heat bath
- Iterative formula for partition function
- Blocking methods for multiplicity distributions

  – exact solution
- Incorporate decay and feeding effect

  – exact solution

Partition Problem

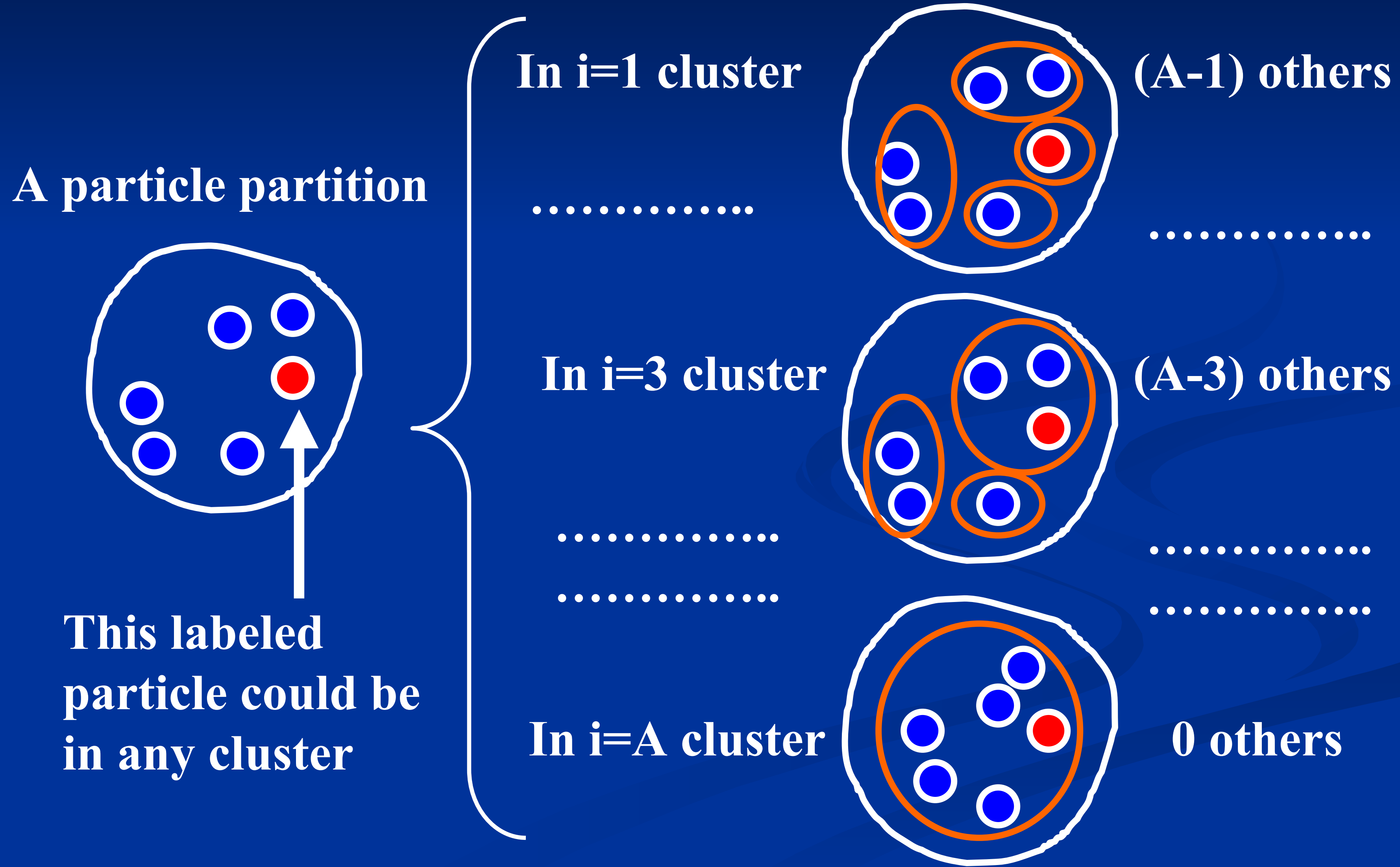


Find all the possible combinations

$$Q_A = \sum \prod_i \frac{(\omega_i)^{n_i}}{n_i!} \delta\left(\sum_i i \times n_i - A\right),$$

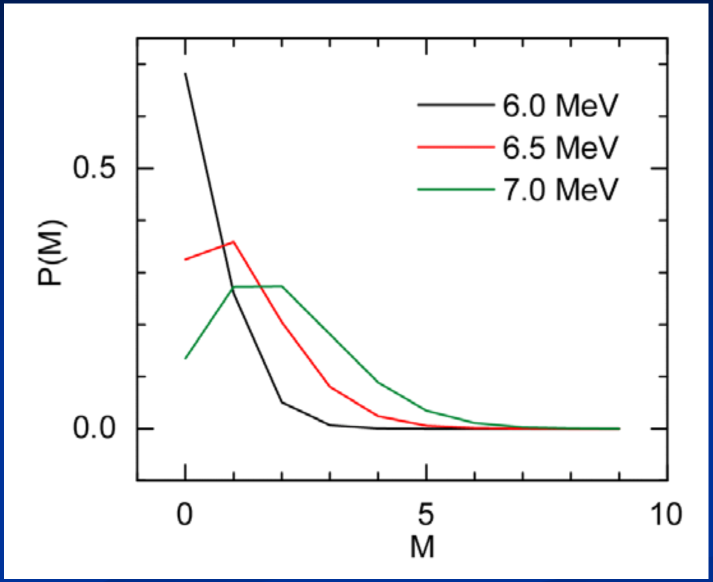
Iterative formula

$$Q_A = \frac{1}{A} \sum_{k=1}^{k=A} k \omega_k Q_{A-k} \,.$$



multiplicity distributions

Objective:
Find the multiplicity of a give cluster i, that is, the distribution of n_i .



For example, consider i=3:
$P(n_3=0)$, $P(n_3=1)$, $P(n_3=2)$, $P(n_3=3)$, $P(n_3=4)$, ….
Here the Total partition function does not help, we need to treat i=3 clusters separately, then the iterative formula does not work anymore!

$$Q_A = \sum \prod_i \frac{(\omega_i)^{n_i}}{n_i!} \delta(\sum_i i \times n_i - A) \, ,$$

Single Blocking method

Cluster size: (i=3 is special =>multiplicity distribution)

i = 1, 2, 3, 4, 5, 6, ……

Cluster formation probability:

ω1, ω2, ω3, ω4, ω5, ω6, …….

Now we have two separate partition problem

Partition: i=1,2,4,5,6,… ω1,ω2,ω3=0,ω4, … Use iterative formula	$\otimes$	Partition: i=3 ω3 Rebuild partition

$$P_n(k) = \frac{(\omega_k)^n}{n!}\frac{Q_{A-nk}(\omega_k = 0)}{Q_A},$$

Group Blocking method

Cluster size: (i=3, 4, 5=>multiplicity distribution)

i = 1, 2, 3, 4, 5, 6, ……

Cluster formation probability:

ω1, ω2, ω3, ω4, ω5, ω6, …….

Now we have two separate partition problem

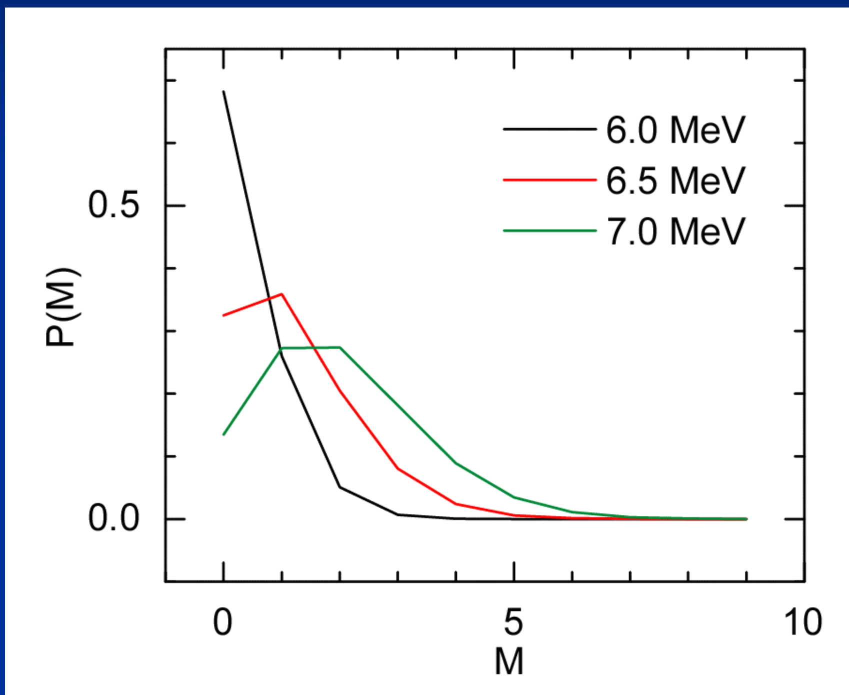
Partition: (group β)
i=1,2,6,…
ω1,ω2,ω3=0, ω4=0, ω5=0, ω6, …
Use iterative formula

$\otimes$

Partition: (group α)
i=3, 4, 5
ω3, ω4, ω5
Rebuild partition

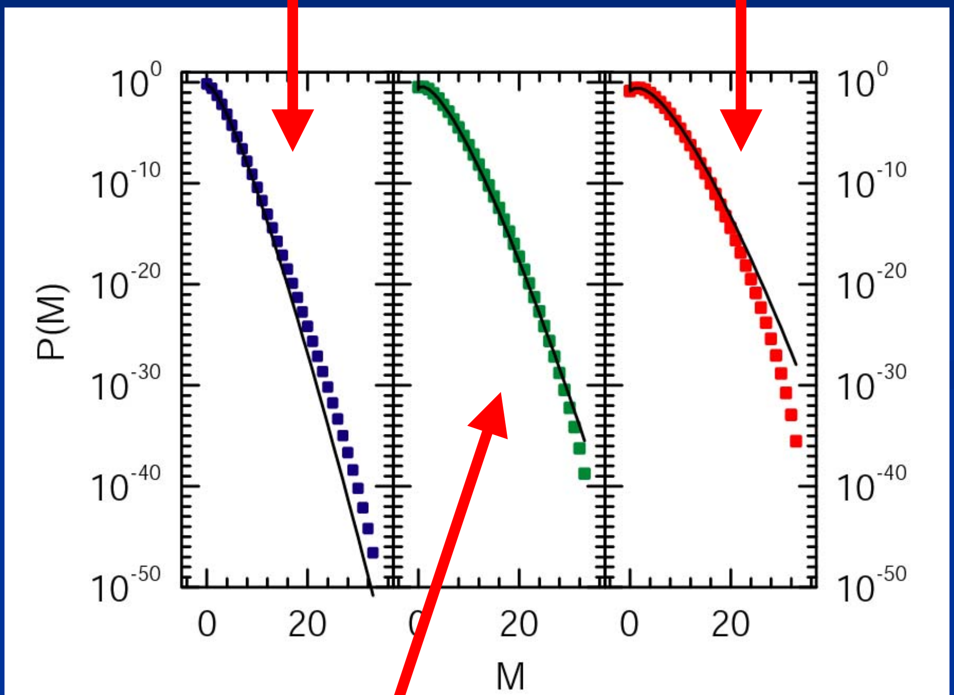
$$P(\alpha, M) = \frac{1}{Q_A} \sum_{A_1, A_2} Q_{A_1}(\alpha, M) Q_{A_2}(\beta) \delta(A_1 + A_2 - A) \, .$$

One component system



T=6 MeV,
Super-poissonian

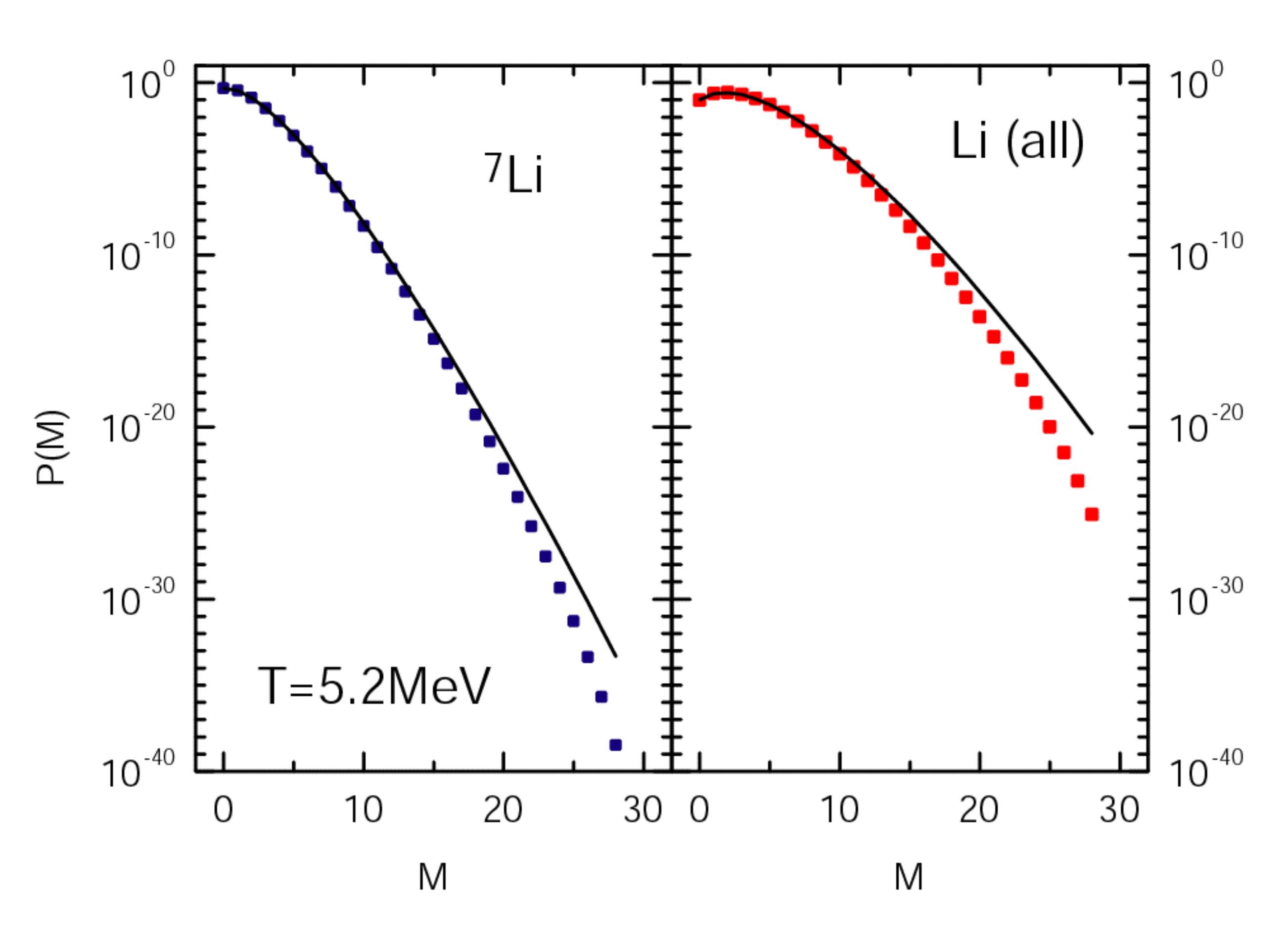
T=7 MeV
Sub-poissonian



Exact result:
Cluster we care: k=6,
Total system: A=200

T=6.5 MeV,
phase transition temperature
Poisson fit is very good

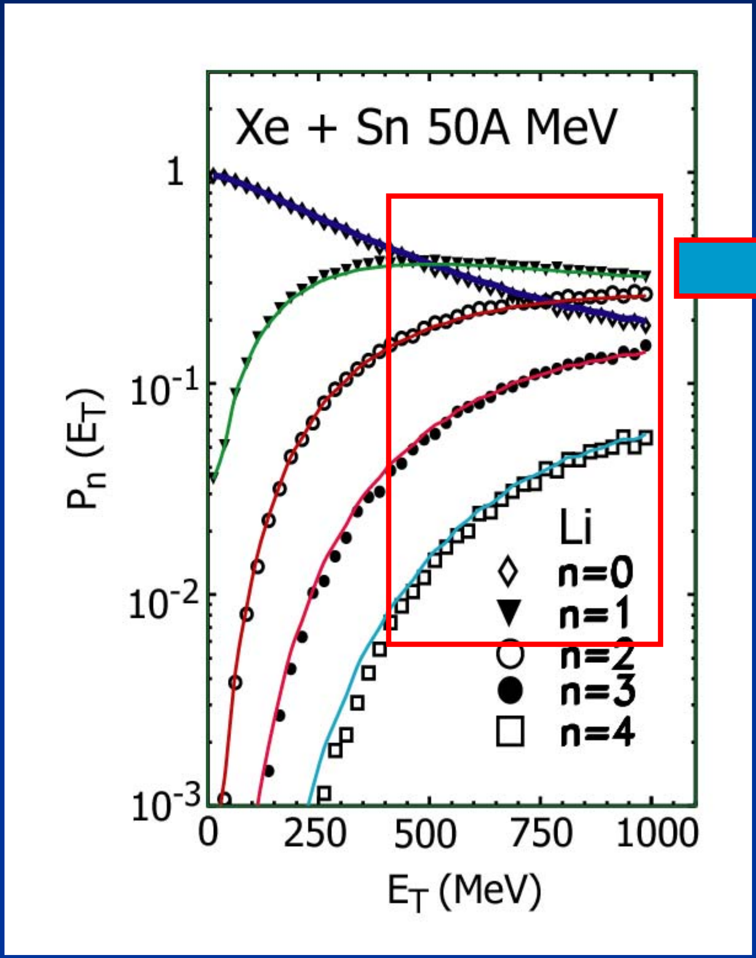
Two component system

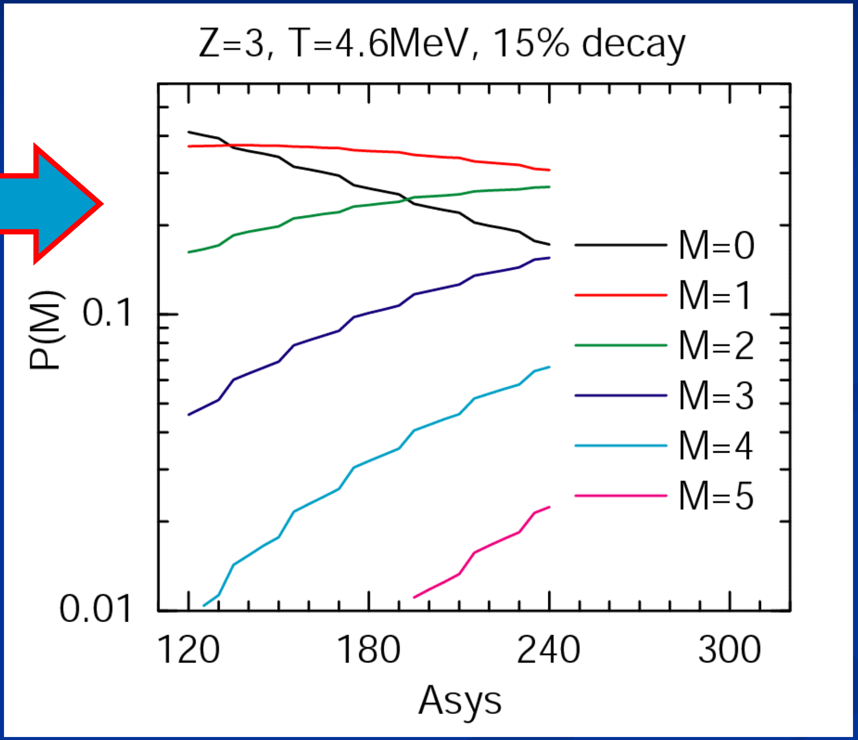


Multiplicity distribution is Sub-poissonian at all temperature in two comp system;

different than one comp system

Two component system: single element

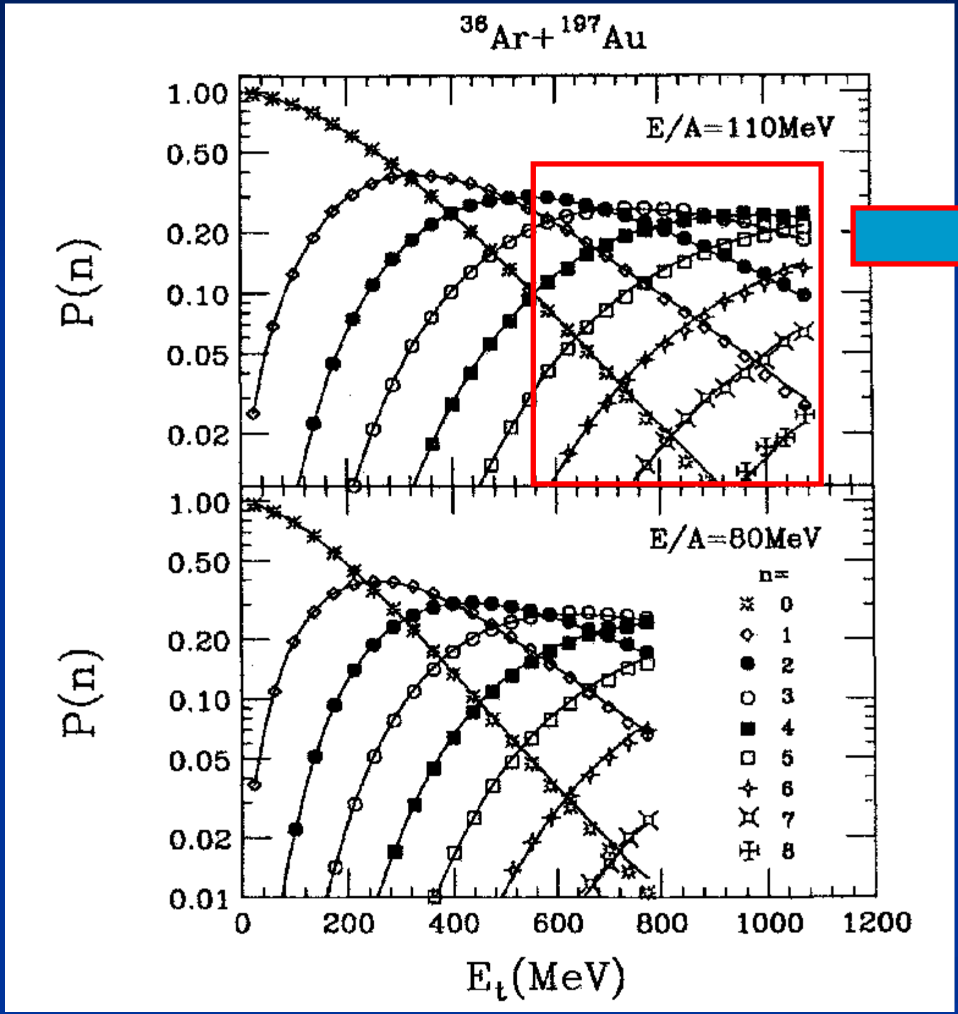


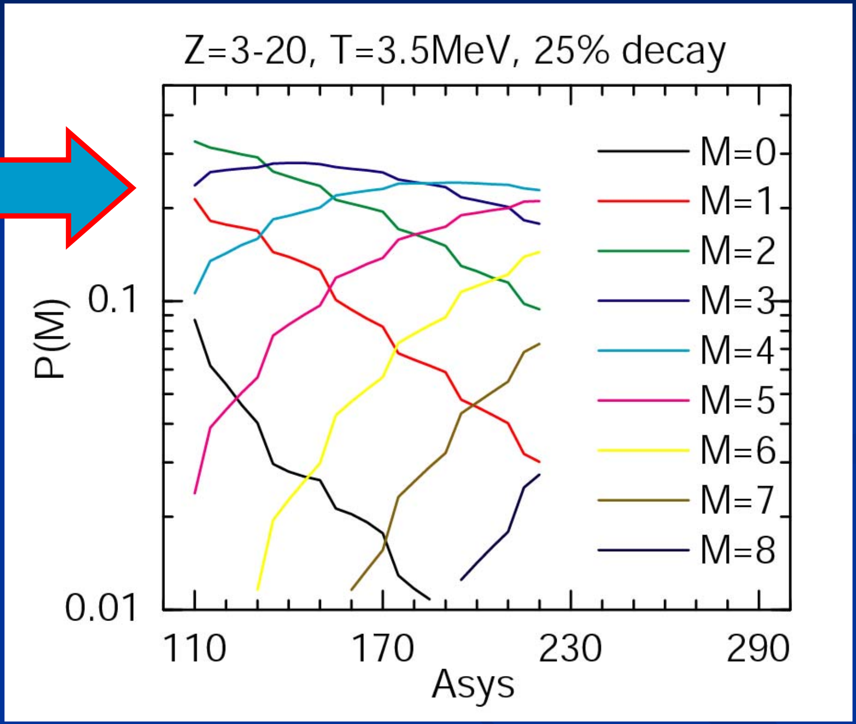


Exact solvable model calculation

Experimental data, L. Beaulieu, et al., Nucl-ex/9805003

Two component system:
All IMF (Z=3-20)





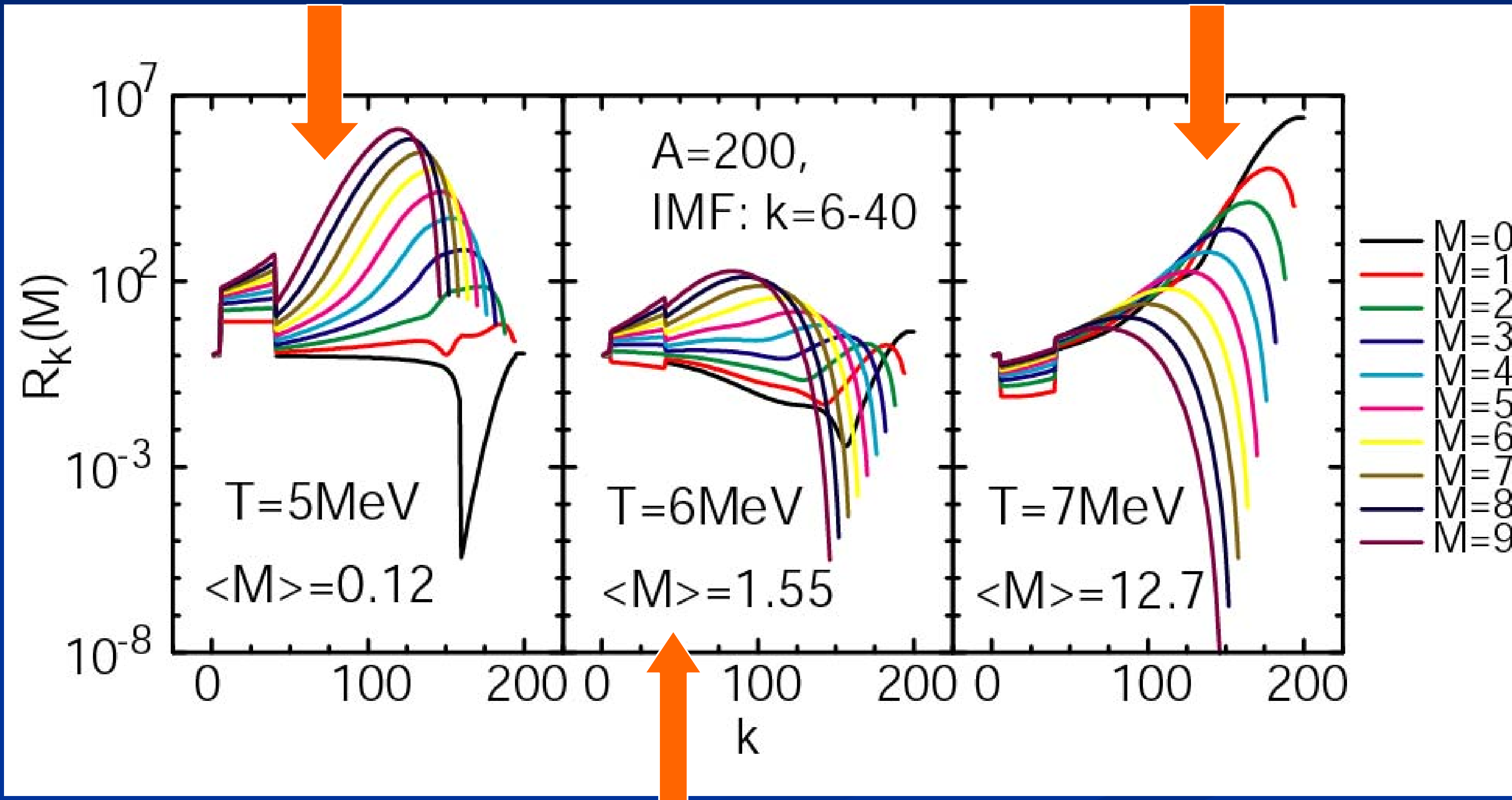
Exact solvable model calculation

Experimental data, L. G. Moretto, et al., Phys. Rev. Lett. 74, 1530–1533 (1995)

M-gated yield ratio

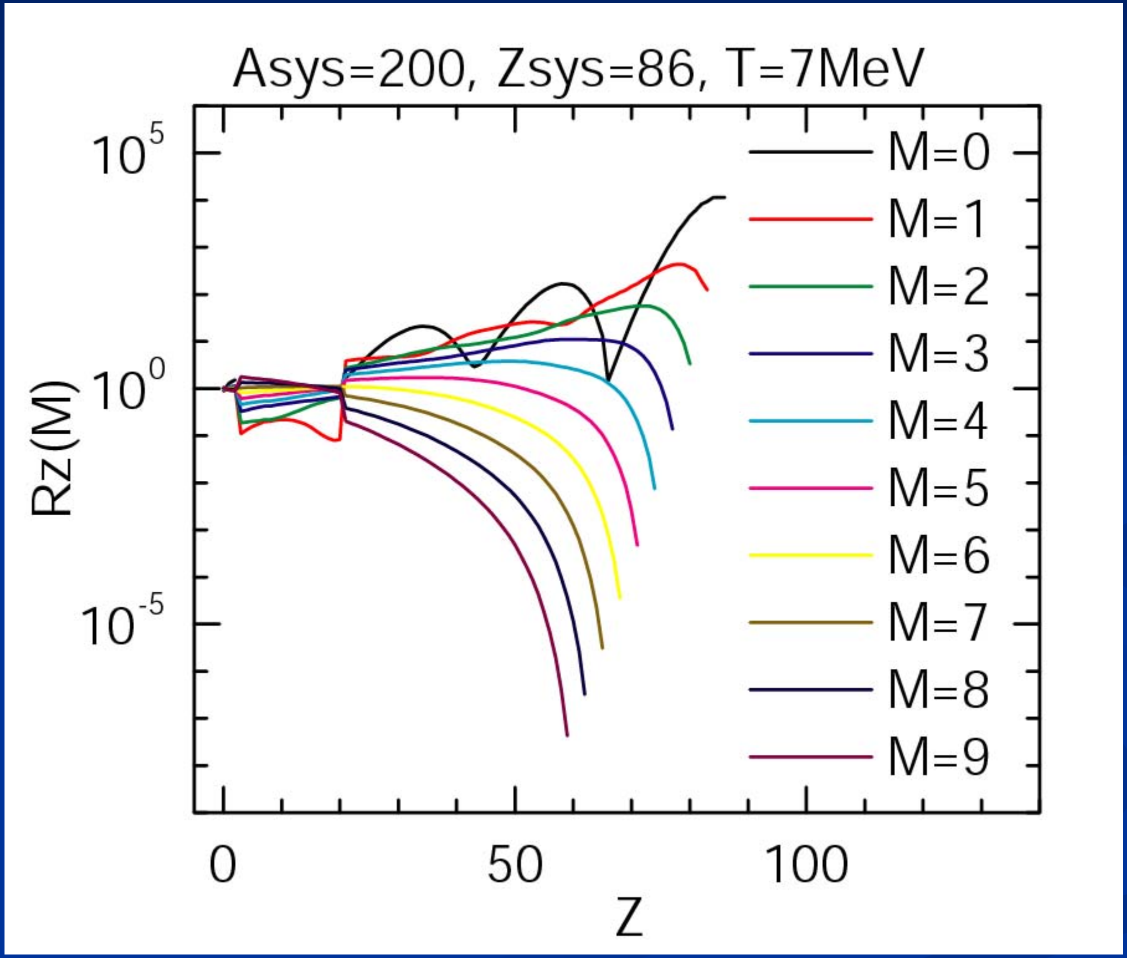
$$R_k(M) = \frac{\langle n_k(M) \rangle}{\langle n_k \rangle},$$

below transition temperature **above transition temperature**

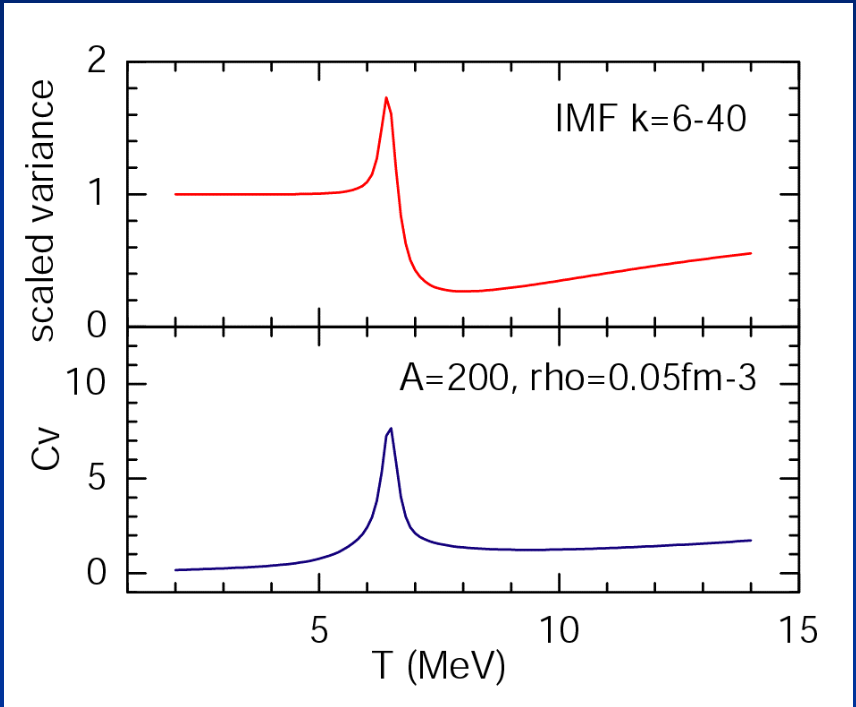


One-comp system: Close to phase transition temperature 6.5MeV

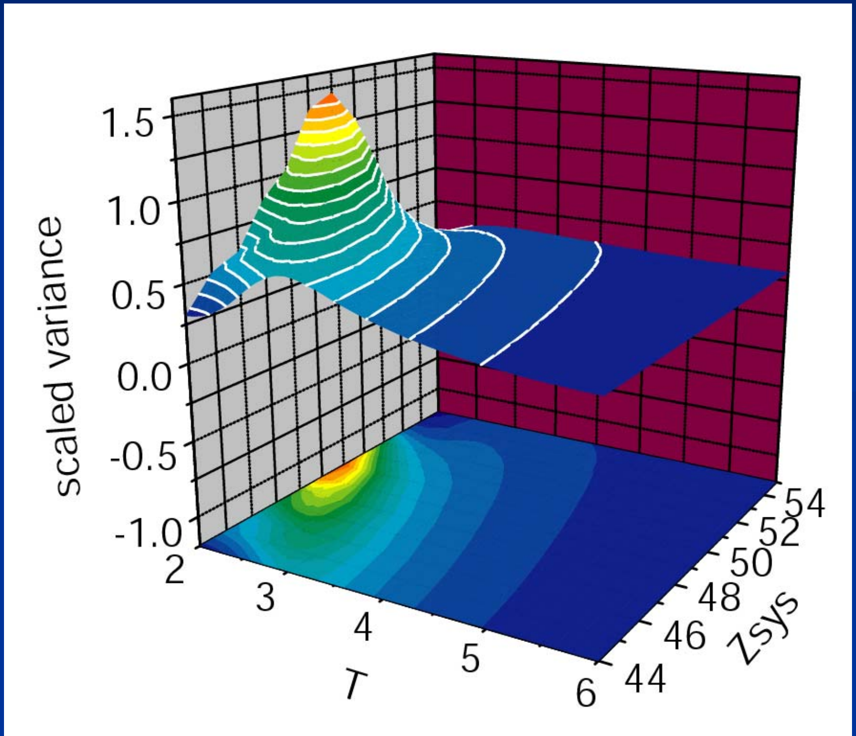
M-gated yield ratio: two-comp system



Variance: one and two component systems



One-comp system



Two-comp system
Asys=100

Summary

- Iterative formula for the canonical partition problem
  - extended for multiplicity distribution,
  - include decay and feeding
- Multiplicity distribution
  - Critical behavior in one-comp system, not in two comp system
  - System size scaling
- M-gated yield ratio – new signal for experiments
- Variance and critical point
  - Complications from isospin
- Future development
  - Complete investigation in the isospin and decay effects

Formula summary

1. Partition problem:

   exponential large number

2. Iterative formula:

   calculate partition function easily

3. Blocking methods:

   multiplicity distribution of single cluster and a group of clusters

4. Decay & feeding:

   complete solution to the multiplicity distribution

Formula summary

Partition problem

$$Q_A = \sum \prod_i \frac{(\omega_i)^{n_i}}{n_i!} \delta(\sum_i i \times n_i - A) \,,$$

Iterative formula

$$Q_A = \frac{1}{A} \sum_{k=1}^{k=A} k \omega_k Q_{A-k} \,.$$

Single blocking

$$P_n(k) = \frac{(\omega_k)^n}{n!} \frac{Q_{A-nk}(\omega_k = 0)}{Q_A} \,,$$

Group blocking

$$P(\alpha, M) = \frac{1}{Q_A} \sum_{A_1, A_2} Q_{A_1}(\alpha, M) Q_{A_2}(\beta) \delta(A_1 + A_2 - A) \,.$$

Decay & feeding

$$Q_A^{D,F}(\alpha, M) = \sum Q_{A_1}(\alpha', M) Q_{A_2}(\beta') \, \delta(A_1 + A_2 - A) \,.$$