SUPERALLOWED β-DECAY: THE DETERMINATION OF V_{ud}

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- Summary on V_{ud}
- Details on determination V_{ud} from nuclear decays
  - radiative corrections
  - isospin-symmetry breaking corrections

CKM Matrix

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ud} \\ V_{cd} & V_{cs} & V_{cd} \\ V_{td} & V_{ts} & V_{td} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Unitary:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

$$\sim 95\% \quad \sim 5\% \quad \sim 0\%$$

V_{ud} STATUS

1. $\underline{V_{ud} \text{ from nuclear } 0^+ \to 0^+ \text{ decays}}$

$$V_{ud} = 0.9738(4) \qquad \sum_i |V_{ui}|^2 = 0.9966(14)$$

- Fails to meet unitarity by 2.2σ
- Error <u>not</u> statistical, but theoretical
- Are nuclear-structure corrections under control?

2. $\underline{V_{ud} \text{ from neutron decay}}$

$$V_{ud} = 0.9745(16) \qquad \sum_i |V_{ui}|^2 = 0.9978(33)$$

- Consistent with nuclear decay and unitarity
- Error mainly due to accuracy of β-asymmetry expt.
- No nuclear-structure dependent correction

3. $\underline{V_{ud} \text{ from pion beta decay}}$

$$V_{ud} = 0.9670(160) \qquad \sum_i |V_{ui}|^2 = 0.9833(310)$$

- Large error; difficulty of measuring 10^{-8} BR
- No nuclear-structure dependent correction

V_{us} STATUS

PDG '04

$$V_{us} = 0.2200(26) \qquad \sum_i |V_{ui}|^2 = 0.9966(14)$$

E865: PR **91**, 261802 (2003)

$$V_{us} = 0.2272(30) \qquad \sum_i |V_{ui}|^2 = 0.9999(16)$$

$$K^+ \to \pi^0 e^+ \nu_e$$

Included in the PDG average value.
But is inconsistent with older K_{e3}^+ and K_{e3}^0 decay measurements.
Experiments in progress

Hyperon decay data not included; SU(3) symmetry-breaking corrections are significant and problematic.

FERMI $0^+ \rightarrow 0^+$ DECAYS

$$ft = \frac{K}{G_V^2 \langle M_F \rangle^2} \qquad G_V = G_F V_{ud}$$

$$\langle M_F \rangle = \langle f|\tau_+|i\rangle$$

$$= \sqrt{2} \quad \text{for } T = 1 \text{ states}$$

Thus if:

(a) G_V is a true constant

ie. NOT renormalised in nuclear medium (CVC)

(b) Isospin is an exact symmetry

Then:

$$ft = \text{constant} \qquad \text{for given isospin } T$$

However to reach this result, two theoretical electromagnetic corrections have to be applied:

(a) radiative corrections

$$t \rightarrow t(1+\underset{\substack{\text{nucleus}\\\text{dependent}}}{\delta_R})(1+\underset{\substack{\text{nucleus}\\\text{independent}}}{\Delta_R}) \qquad \begin{array}{l}\delta_R \sim 1.5\% \\ \Delta_R \sim 2.4\%\end{array}$$

(b) isospin-symmetry breaking correction

$$\langle M_F \rangle^2 \rightarrow \langle M_F \rangle^2 (1-\delta_C) \qquad \delta_C \sim 0.5\%$$

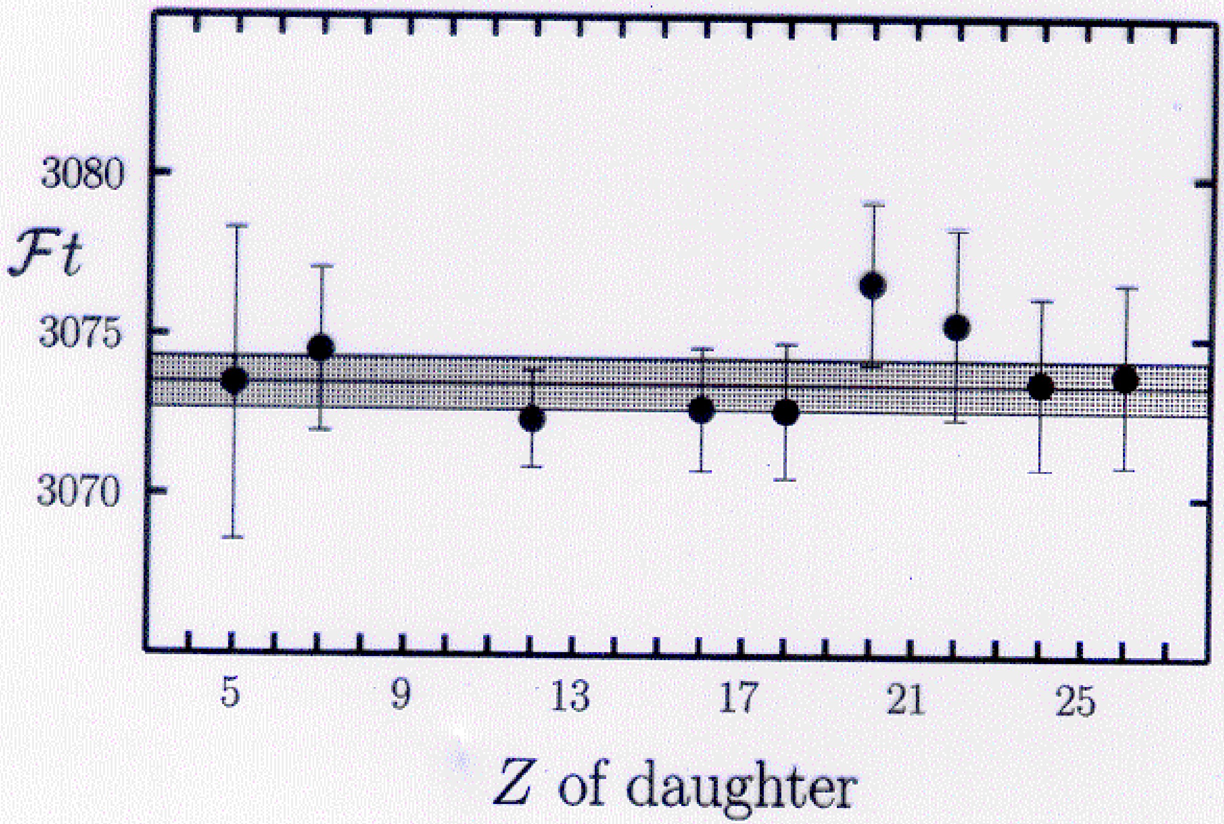
So the amended formula is

$$\begin{aligned}\mathcal{F}t \;&\equiv\; ft(1+\delta_R)(1-\delta_C) = \frac{K}{G_V^2(1+\Delta_R)\langle M_F\rangle^2} \\ &=\; \text{constant}\end{aligned}$$

The radiative correction can be further divided into terms that depend trivially on the nucleus (*eg.* Z, end-point energy), and terms that depend on the details of nuclear structure

$$\delta_R = \underset{\substack{\text{structure}\\\text{independent}}}{\delta_R'} \quad + \quad \underset{\substack{\text{structure}\\\text{dependent}}}{\delta_{NS}}$$

Then:



3080
$\mathcal{F}t$
3075
3070
5
9
13
17
21
25
Z of daughter

CKM matrix element

$$V_{ud}^2 = \frac{K}{2G_F^2(1+\Delta_R)\overline{\mathcal{F}t}}$$

where

K = known constant = $2\pi^3 \ln 2(\hbar c)^6/(m_e c^2)^5$
G_F = weak interaction coupling constant, from μ-decay
Δ_R = nucleus-independent radiative correction, $\Delta_R \sim 2.4\%$
$\overline{\mathcal{F}t}$ = best-fit value from $0^+ \to 0^+$ decays

Hence

$$V_{ud}^2 = 0.9482 \pm 0.0008$$

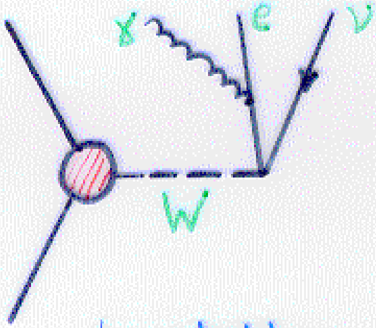
Alternatively, from unitarity of the CKM matrix

$$V_{ud}^2 = 1 - V_{us}^2 - V_{ub}^2 = 0.9516 \pm 0.0011$$

Discrepancy of about 2σ.

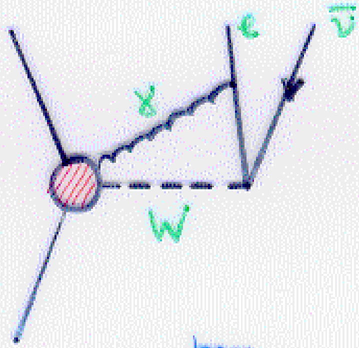
$$\frac{V_{ud}^2(unitarity)}{V_{ud}^2(0^+ \to 0^+)} = 1.0035 \pm 0.0015$$

radiative corrections need to be shifted <u>downwards</u>
Eg. $\delta_R \simeq 1.5\% \to 1.2\%$; $\Delta_R \simeq 2.4\% \to 2.1\%$
or
isospin-symmetry breaking correction shifted <u>upwards</u>
Eg. $\delta_C \simeq 0.5\% \to 0.8\%$



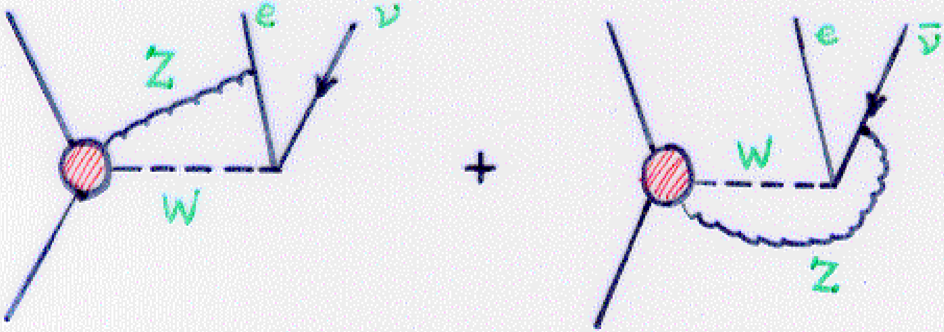
bremsstrahlung

$\mathcal{M}_B$



box

$\mathcal{M}_V^{box} + \mathcal{M}_A^{box}$



$\equiv \mathcal{M}$

Recall $\quad V_{ud} = \dfrac{\beta\text{-decay}}{\mu\text{-decay}}$

∴ Any radiative correction that is universal has no impact on

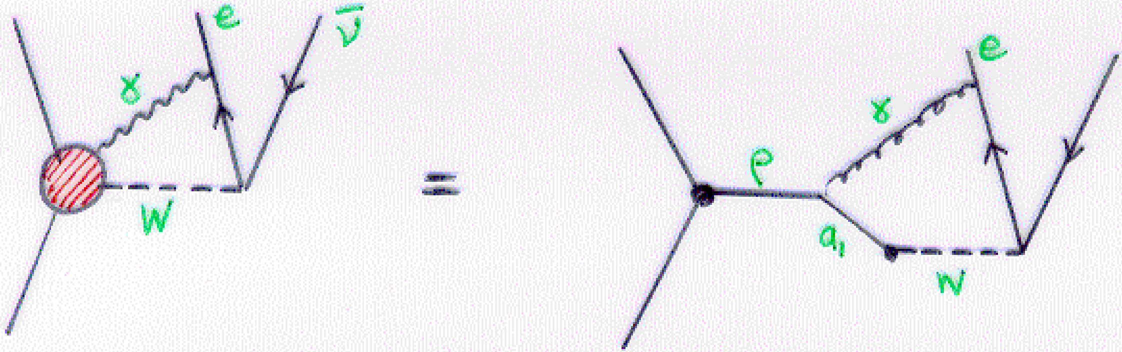
Consider only non-universal terms :

$\equiv \frac{\alpha}{2\pi}$

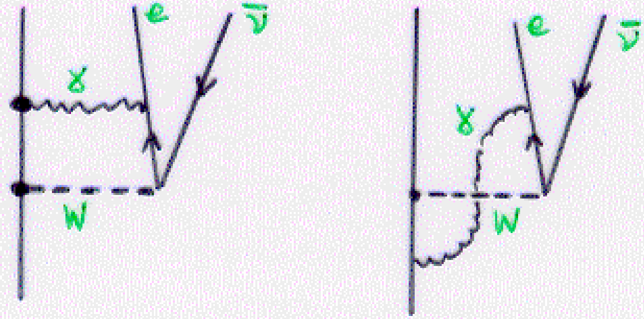
$$\mathcal{M}_B + \mathcal{M}_V^{box} = \frac{\alpha}{4\pi}\left[\, 3\ln\left(\frac{m_W}{m_p}\right) + \bar{g}(E_m) \,\right] \mathcal{M}_0$$

$$\mathcal{M}_A^{box} = \quad ?$$

The Gamow-Teller piece



a_1-
dom



Bo
gr

$$\mathcal{M}_A^{box} = \frac{\alpha}{4\pi}\left[\ln\frac{m_W}{m_A} + 2C\right]\mathcal{M}_0$$

Born graphs

Sirlin recommends:

$m_{a_1}/2 \leq m_A \leq 2m_{a_1}$ $\qquad$ m_{a_1} = mass of a_1 me

This range of values for m_A is largest contributor to er

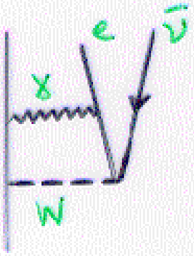
Born graph calculation

Use the standard nuclear physics technique of :

a) evaluate the Feynman graph using Dirac spinors for nucleon
   and non-interacting fermion propagators

b) make nonrelativistic reduction $\Rightarrow$ operator between Pauli spin

c) evaluate expectation value of operator with shell-model wfn

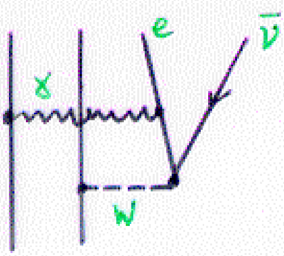


$\Rightarrow$ C_1 (one-body) $\Rightarrow$ $C_{Born} < SM | \tau_+$

Same for a
nuclei of

$$C_{Born} = 3 g_A (\mu_p + \mu_n) \times \text{Loop In}$$

$$= 0{\cdot}881 \pm 0{\cdot}030$$



$\Rightarrow$ C_2 (two-body) $\Rightarrow$ $\langle SM | C_2 | SM \rangle$

Requires shell-model calcul
Nuclear-structure depen

Putting all terms together:

$$|\mathcal{M}_0 + \mathcal{M}_B + \mathcal{M}_V^{box} + \mathcal{M}_A^{box} + \mathcal{M}_Z^{box}|^2 \equiv |\mathcal{M}_0|^2 \left(1 + \underset{\text{nucleus dependent}}{\delta_R} + \underset{\text{nuc... inde...}}{\Delta_R} \right)$$

$$\delta_R = \underset{\mathcal{M}_B + \mathcal{M}_V^{box}(k < M_p)}{\frac{\alpha}{2\pi}\, \overline{g(E_e, E_0)}} + \underset{\mathcal{M}_A^{box}}{\delta_{NS}}$$

$$\Delta_R = \frac{\alpha}{2\pi} \left[\underset{\mathcal{M}_V^{box}(k > M_p)}{3 \ln\left(\frac{m_W}{m_p}\right)} + \underbrace{\ln\left(\frac{m_W}{m_A}\right) + 2C_{Born}}_{\mathcal{M}_A^{box}} \underset{\mathcal{M}_Z^{box}}{- 4 \ln\left(\frac{m}{m}\right.} \ldots \right.$$

$$= \frac{\alpha}{2\pi} \left[4 \ln\left(\frac{m_Z}{m_p}\right) + \ln\left(\frac{m_p}{m_A}\right) + 2C_{Born} \right]$$

Nucleus-dependent terms

$$\delta_R = \frac{\alpha}{2\pi}\left[\overline{g(E_e, E_0)} + \delta_2(Z\alpha^2) + \delta_3(Z^2\alpha^3)\right] + \delta_{NS}$$

Sirlin's function averaged over electron spectrum

Nuclear-str... dependence of...

Typical values:

$$\delta_R(\%) = \underbrace{0.95}_{\text{Firm}} + \underbrace{0.43 + 0.05 - 0.03}_{\text{Less secure}}$$

$$= 1.4\%$$

To obtain unitarity of CKM with present experimental data requi...

$$\delta_R^{\text{unitarity}}(\%) \simeq 1.1\%$$

ie. less secure results to be reduced by a factor of 3.

Nucleus-independent terms

$$\Delta_R = \frac{\alpha}{2\pi}\left[\underbrace{4\ln\left(\frac{m_Z}{M_p}\right)}_{\text{Firm}} + \underbrace{\ln\left(\frac{M_p}{m_A}\right) + 2C_{\text{Born}} + \cdots}_{\text{Less secure}}\right]$$

$\Delta_R(\%)$ ... 2.1 ... + (0.30 ± 0.08) = 2...

COULOMB CORRECTION, δ_C

Beta decay in nuclei described by one-body operator

$$F = \sum_{\alpha\beta} \langle\alpha|\,\tau_+\,|\beta\rangle\; a_\alpha^+ a_\beta$$

Matrix element in many-body system

$$\langle M_F\rangle = \sum_{\alpha\beta} \langle f|\,a_\alpha^+ a_\beta\,|i\rangle\; \langle\alpha|\,\tau_+\,|\beta\rangle$$

shell-model one-body density matrix elements evaluated in many-body states.

|

Add one-body and two-body charge-dependent terms to shell-model Hamiltonian

Single-particle mat

|

$$\delta_{\alpha\beta} \int R^{\text{proton}}_{n_\alpha \ell_\alpha}\; R^{\text{neutron}}_{n_\alpha \ell_\alpha}$$

Define

$$\langle M_F\rangle^2 = 2\,(1 - \delta_C)$$

$$\delta_C = \delta_{IM} + \delta_{RO}$$

Isospin Mixing | Radial Overlap

$$\simeq 0.1\% + 0.4\%$$

typical

Damgaard model (1969)

Change in proton wavefunction caused by presence of C
potential, V_c. Described in 1st order perturbation
as an admixture with state with one more radial no

$$\psi_\ell = \psi_{n\ell} + x\, \psi_{n+1,\ell}$$

$$x = \langle \psi_{n+1,\ell} | V_c | \psi_{n\ell} \rangle / (-2\hbar\omega)$$

$$V_c(r) = \frac{Ze^2}{2R}\left(3 - \frac{r^2}{R^2}\right)$$

The constant term gives no contribution to x

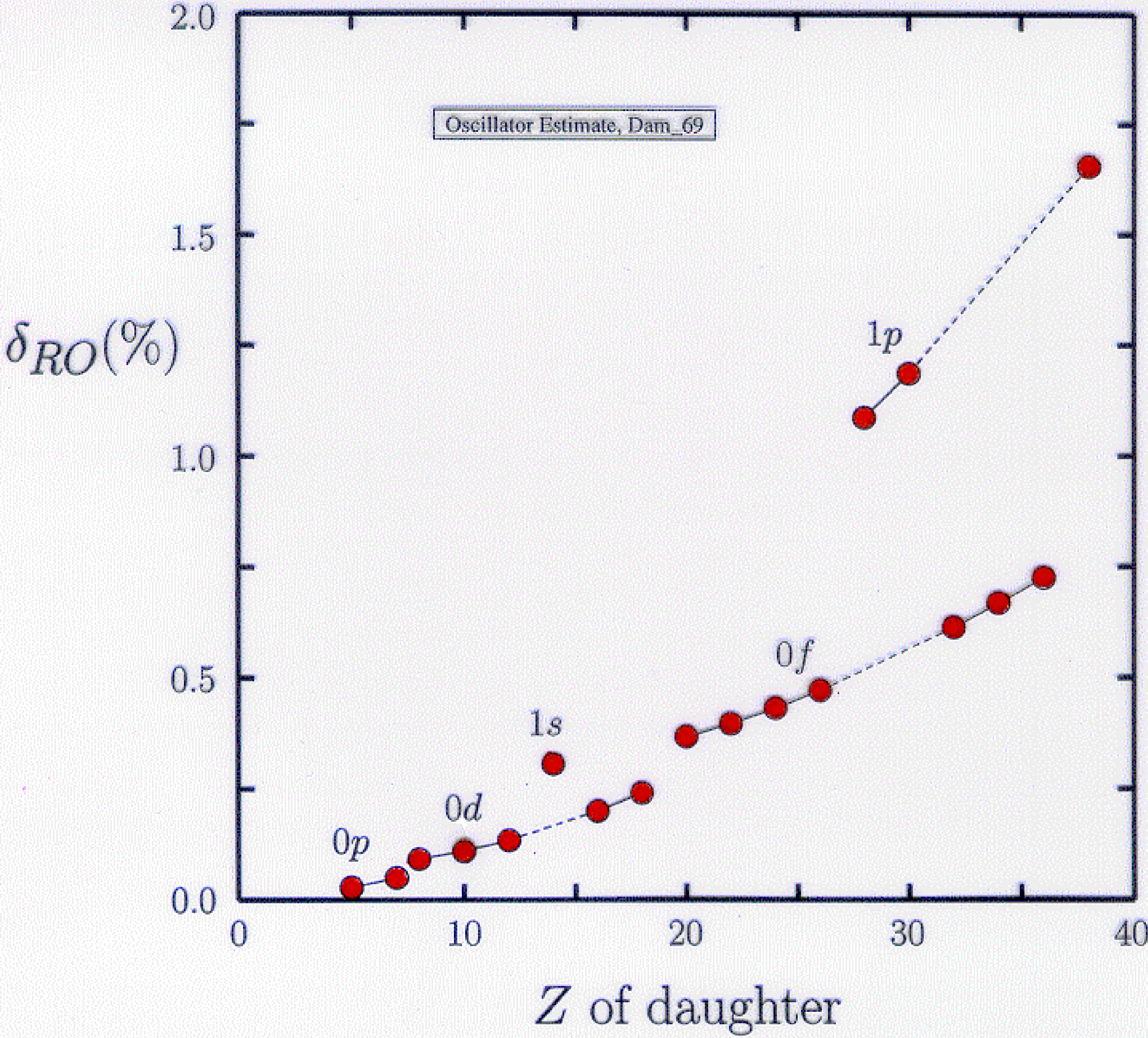
$$x = \frac{Ze^2}{2R^3} \frac{1}{2\hbar\omega} \langle \psi_{n+1,\ell} | r^2 | \psi_{n\ell} \rangle$$

$$= \frac{Z\alpha}{4R^3} \frac{(\hbar c)^3}{(\hbar\omega)^2 mc^2} \left[(n+1)(n+\ell+3/2) \right]^{1/2}$$

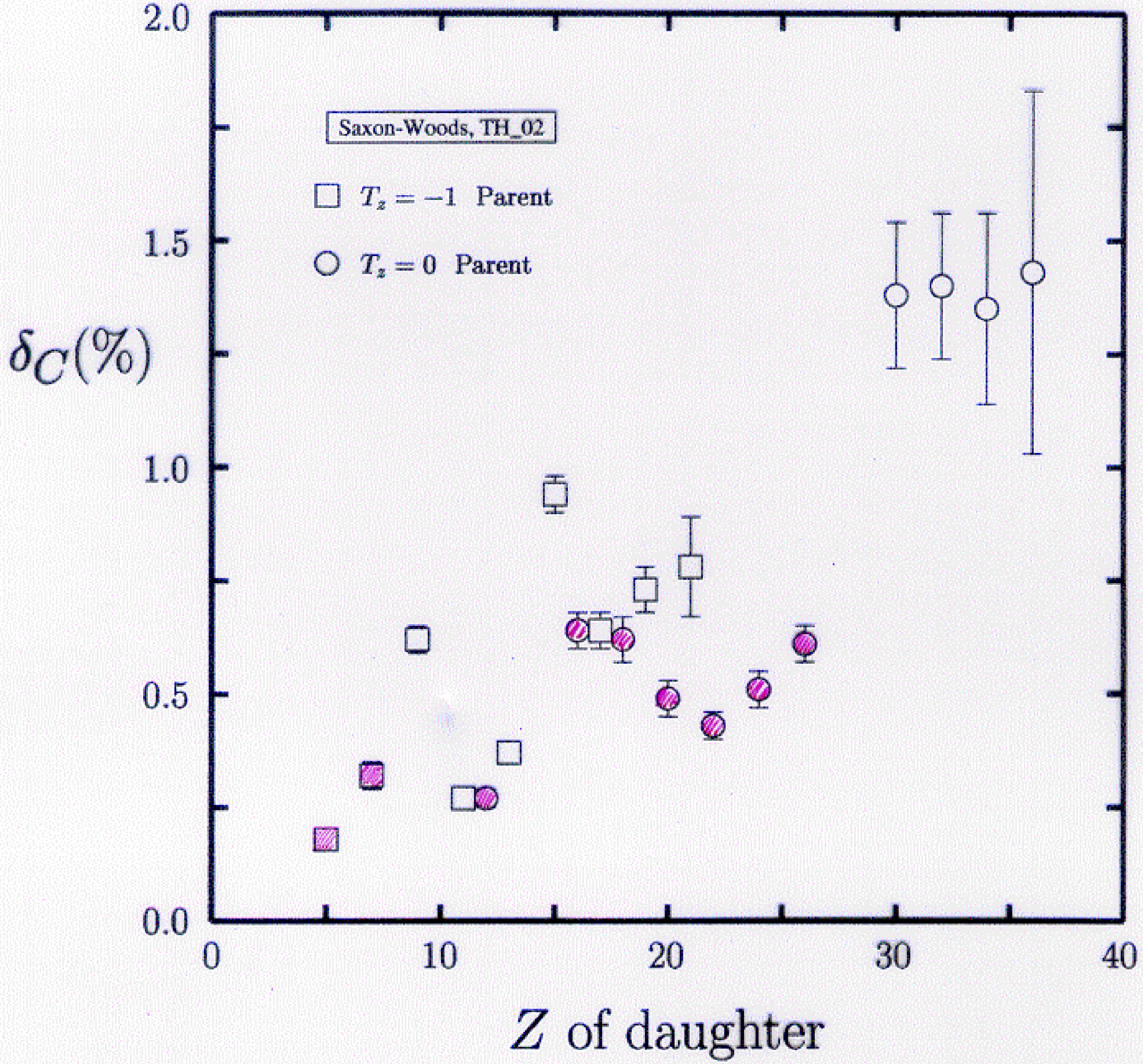
Fermi matrix element

$$|M_F|^2 = M_0^2 (1 - x^2) \equiv M_0^2 (1 - \delta_c)$$

$$\delta_c = \frac{(Z\alpha)^2}{16 R^6} \frac{(\hbar c)^6}{(\hbar\omega)^4 (mc^2)^2} (n+1)(n+\ell+3/2)$$



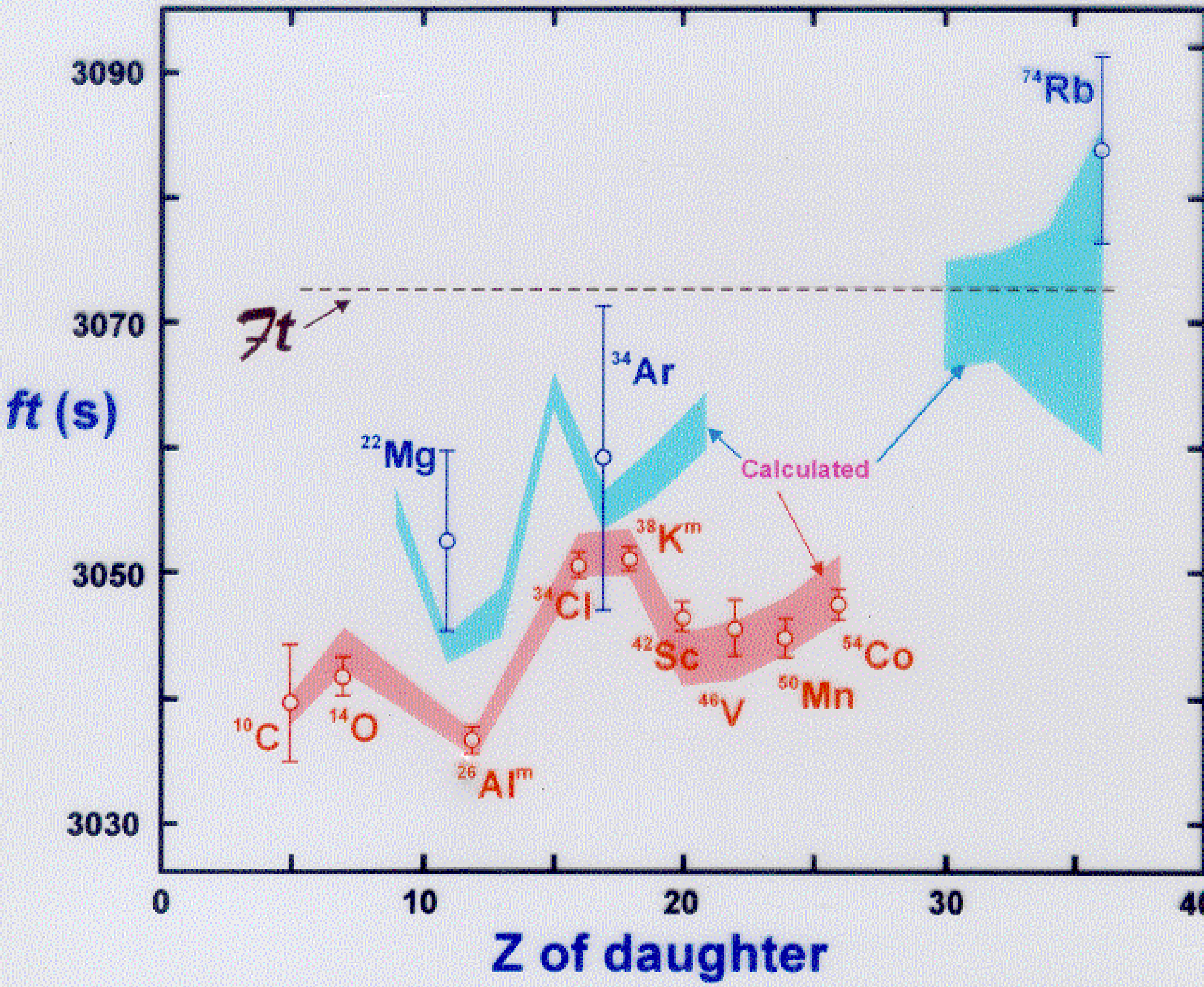
Oscillator Estimate, Dam_69
$\delta_{RO}(\%)$
2.0
1.5
1.0
0.5
0.0
0
10
20
30
40
Z of daughter
0p
0d
1s
0f
1p



Saxon-Woods, TH_02
$T_z = -1$ Parent
$T_z = 0$ Parent
$\delta_C(\%)$
2.0
1.5
1.0
0.5
0.0
0
10
20
30
40
Z of daughter

EXPERIMENTAL TEST OF CORRECTIONS

$$\text{Calculated } ft\text{-value} = \frac{\overline{\mathcal{F}t}}{(1 + \delta'_R + \delta_{NS})(1 - \delta_C)}$$



Improvements anticipated soon in:

δ_C - corrections : Summary

1. For $A \leq 54$, <u>all</u> calculations give $\delta_C \lesssim 0.5\%$ on

2. The more recent δ_C calculations have tended to p
smaller δ_C values.

3. To obtain CKM unitarity, require larger values of δ
typically $\delta_C \sim 0.8\%$

No calculational evidence for such a value.

Achilles' Heel.

CONCLUSIONS

1. The failure of $0^+ \rightarrow 0^+$ data to give V_{ud} consistent with unitarity could be overcome if:

   (i) the radiative correction is decreased
   (ii) the isospin-symmetry breaking correction is increased

However:

$$\text{rad. corr.} = \text{'Firm'} + \text{'Less Secure'} \quad \text{terms}$$

'Less Secure' terms need to be wrong by a factor of three.

Further:

Coulomb corrections for $A \leq 54$ calculated in many models.
All find $\delta_C \simeq 0.5\%$.
Require $\delta_C \sim 0.8\%$.

2. The V_{ud} from nuclear decays

$$V_{ud} = 0.9738(4) \qquad \sum_i |V_{ui}|^2 = 0.9966(14)$$

Discrepancy with unitarity of 2σ

Note: Error here is theoretical, not statistical

Question: Is the value of V_{us} secure?